

## Exclusive Decays of Beauty Hadrons

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### Abstract

The principal difficulty in deducing weak interaction properties from experimental measurements of  $B$ -decays lies in controlling the strong interaction effects. In this talk I review the status of theoretical calculations of the amplitudes for exclusive leptonic and semileptonic decays, in the latter case with special emphasis on the extraction of the  $V_{cb}$  and  $V_{ub}$  matrix elements.

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# 1 Introduction

In this lecture I will review the status of theoretical calculations of exclusive  $B$ -decays. It is intended that this talk should complement those presented at this conference by N. Uraltsev [1] (theory of heavy quark physics), A. Ali [2] (rare  $B$ -decays) and M. Gronau [3] ( $CP$ -violation). The two main topics which will be discussed here are:

- i) *Leptonic Decays* in which the  $B$ -meson decays into leptons, e.g.  $B \rightarrow \tau\nu_\tau$ . These are the simplest to consider theoretically (see sec. 2). Their observation at future  $b$ -factories would have a significant impact on the phenomenology of beauty decays.
- ii) *Semileptonic Decays* in which the  $b$ -quark decays into a lighter quark + leptons. Examples of such decays include  $B \rightarrow (D \text{ or } D^*) + l\nu_l$  and  $B \rightarrow (\pi \text{ or } \rho) + l\nu_l$ , which are being used to determine the  $V_{cb}$  and  $V_{ub}$  matrix elements of the CKM-matrix (see sec. 3). Many of the theoretical issues concerning these decays are relevant also for rare decays, such as  $B \rightarrow K^*\gamma$ .

*Non-Leptonic Decays* in which the  $B$ -meson decays into two or more hadrons, such as  $\bar{B}^0 \rightarrow \pi^- D^+$ , are considerably more complicated to treat theoretically, and with our current level of understanding require model assumptions. I will not discuss them further in this talk (see however the talk by Gronau [3]).

In studying the decays of  $B$ -mesons, we are largely interested in extracting information about the properties and parameters of the weak interactions, and in looking for possible signatures of physics beyond the standard model. The most important theoretical problem in interpreting the experimental results, is to control the strong interaction effects which are present in these decays. This is a non-perturbative (and hence very difficult) problem, and is the main subject of this talk. The main theoretical tools that are used to quantify the effects are lattice simulations and QCD sum rules, combined with the formalism of the heavy quark effective theory (HQET) where appropriate.

As with any problem in non-perturbative quantum field theory, the exploitation of all available symmetries is very important. For the case of heavy quark physics, the use of the spin-flavour symmetries that are present when the masses of the heavy quarks are  $\gg \Lambda_{QCD}$ , leads to considerable simplifications (see refs. [1] and [4, 5] for recent reviews and references to the original literature). In particular, as will be seen in the following sections, the use of heavy quark symmetries and the HQET is particularly helpful for  $B$ -decays.

It is not appropriate in this lecture to present a detailed critical review of the systematic errors present in lattice simulations (see ref. [6] for a recent review). Since many of the results below are based on lattice simulations, it is, however, necessary to mention at least the main source of uncertainty present in the calculations of quantities in  $B$ -physics. The number of space time points on a lattice is

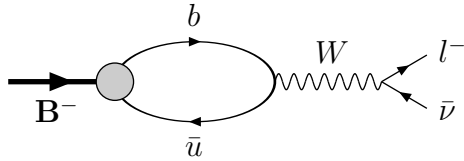


Figure 1: Diagram representing the leptonic decay of the  $B$ -meson.

limited by the available computing resources. One therefore has to compromise between two competing requirements: (i) that the lattice be sufficiently large in physical units to contain the particle(s) whose properties are being studied, i.e. the length of the lattice in each direction should be  $\gg 1$  fm, and (ii) that the spacing between neighbouring lattice points,  $a$ , be sufficiently small to avoid errors due to the granularity of the lattice (called “lattice artefacts” or “discretization errors” in the literature), i.e.  $a^{-1} \gg \Lambda_{QCD}$ . Much effort is currently being devoted to reducing the discretization effects by constructing “improved” (or even “perfect” [7]) lattice actions and operators following the approach of Symanzik [8]. Typical values of  $a^{-1}$  in current simulations are about 2–3 GeV, i.e. the lattice spacings are larger than the Compton wavelength of the  $b$ -quark, and the propagation of a  $b$ -quark on such lattices cannot be studied directly. The results presented below are obtained by extrapolating those computed directly for lighter quarks (with masses typically around that of the charm quark). In addition, calculations can be performed in the HQET and the results obtained in the infinite mass limit can then be used to guide this extrapolation. I should also add that, except where explicitly stated to the contrary, the results below have been obtained in the quenched approximation, in which sea-quark loops are neglected. This approximation is very gradually being relaxed, as computing resources and techniques are improved.

The second non-perturbative method which is used extensively to compute amplitudes for  $B$ -decays is QCD sum rules [9]. In this approach, correlation functions are calculated at intermediate distances, keeping a few terms in the Operator Product Expansion (OPE), and by using dispersion relations are related to spectral densities. The evaluation of the systematic uncertainties, such as those due to the truncation of the perturbation series and OPE or to the specific models that are used for the continuum contribution to the spectral densities, is a very complicated issue; see refs. [4, 5] and the papers cited below for any discussion of this important question.

I now review the status of leptonic and semileptonic decays of  $B$ -mesons in turn.

## 2 Leptonic Decays

Leptonic decays of  $B$ -mesons, see fig. 1, are particularly simple to treat theoretically<sup>2</sup>. The strong interaction effects are contained in a single unknown number, called the decay constant  $f_B$ . Parity symmetry implies that only the axial component of the  $V-A$  weak current contributes to the decay, and Lorentz invariance that the matrix element of the axial current is proportional to the momentum of the  $B$ -meson (with the constant of proportionality defined to be  $f_B$ ):

$$\langle 0 | A_\mu(0) | B(p) \rangle = i f_B p_\mu . \quad (1)$$

Knowledge of  $f_B$  would allow us to predict the rates for the corresponding decays:

$$\Gamma(B \rightarrow l\nu_l + l\nu_l \gamma) = \frac{G_F^2 V_{ub}^2}{8\pi} f_B^2 m_l^2 m_B \left(1 - \frac{m_l^2}{m_B^2}\right)^2 (1 + O(\alpha)) , \quad (2)$$

where the  $O(\alpha)$  corrections are also known.

In addition to leptonic decays, it is expected that the knowledge of  $f_B$  would also be useful for our understanding of other processes in  $B$ -physics, particularly for those for which “factorization” might be thought to be a useful approximation. For example, in  $B-\bar{B}$  mixing, the strong interaction effects are contained in the matrix element of the  $\Delta B = 2$  operator:

$$M = \langle \bar{B}^0 | \bar{b}\gamma_\mu(1 - \gamma_5)q \bar{b}\gamma_\mu(1 - \gamma_5)q | B^0 \rangle . \quad (3)$$

It is conventional to introduce the  $B_B$ -parameter through the definition

$$M = \frac{8}{3} f_B^2 M_B^2 B_B . \quad (4)$$

In the vacuum saturation approximation (whose precision is difficult to assess a priori)  $B_B = 1$ . It appears that  $B_B$  is considerably easier to evaluate than  $f_B$ , e.g. recent lattice results (for the matrix element  $M$  of the operator renormalized at the scale  $m_B$  in the  $\overline{MS}$  scheme) include  $B(m_b) = 0.90(5)$  and  $0.84(6)$  [10] and  $0.90(3)$  [11]. Thus it is likely that the uncertainty in the value of the matrix element  $M$  in eq. (3) is dominated by our ignorance of  $f_B$ .

**$f_{D_s}$ :** Since experimental results are beginning to become available for  $f_{D_s}$ , I will start with a brief review of the decay constants of charmed mesons. Many lattice computations of  $f_D$  have been performed during the last ten years, and my summary of the results is [12]<sup>3</sup>

$$f_D = 200 \pm 30 \text{ MeV} , \quad (5)$$

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<sup>2</sup>For simplicity the presentation here is for the pseudoscalar  $B$ -meson. A parallel discussion holds also for the vector meson  $B^*$ .

<sup>3</sup>The rapporteur at the 1995 Lattice conference summarized the results for the decay constants as  $f_D \simeq f_B \simeq 200 \text{ GeV} \pm 20\%$  [13].

using a normalization in which  $f_{\pi^+} \simeq 131$  MeV. The value of the decay constant is found to decrease as the mass of the light valence quark is decreased (as expected), so that  $f_{D_s}$  is 7–15% larger than  $f_D$ ,  $f_{D_s} = 220 \pm 35$  MeV. As an example of the many lattice results which have been published for  $f_{D_s}$ , I give here the two new ones presented at this year’s international symposium on lattice field theory. The MILC collaboration found  $f_{D_s} = 211 \pm 7 \pm 25 \pm 11$  MeV, where the first error is statistical, the second an estimate of the systematic uncertainties within the quenched approximation, and the third an estimate of the quenching errors [14]. The JLQCD collaboration found  $f_{D_s} = 216 \pm 6^{+22}_{-15}$  MeV, where the second error is systematic (within the quenched approximation) [15]. These results illustrate the fact that the errors are dominated by systematic uncertainties, and the main efforts of the lattice community are being devoted to controlling these uncertainties.

It is very interesting to compare the lattice prediction of  $220 \pm 35$  MeV with experimental measurements for  $f_{D_s}$ . The 1996 Particle Data book [16] quotes the results  $f_{D^+} < 310$  MeV and

$$f_{D_s^+} = 232 \pm 45 \pm 20 \pm 48 \text{ MeV} \quad \text{WA75} \quad (6)$$

$$f_{D_s^+} = 344 \pm 37 \pm 52 \pm 42 \text{ MeV} \quad \text{CLEO} \quad (7)$$

$$f_{D_s^+} = 430^{+150}_{-130} \pm 40 \text{ MeV} \quad \text{BES} . \quad (8)$$

More recently the CLEO result has been updated [17] ( $f_{D_s^+} = 284 \pm 30 \pm 30 \pm 16$  MeV) and the E653 collaboration has found [18]  $f_{D_s^+} = 194 \pm 35 \pm 20 \pm 14$  MeV. Combining the four measurements of  $f_{D_s}$  from  $D_s \rightarrow \mu\nu$  decays, the rapporteur at this year’s ICHEP conference found [19]

$$f_{D_s} = 241 \pm 21 \pm 30 \text{ MeV} . \quad (9)$$

In spite of the sizeable errors, the agreement with the lattice prediction is very pleasing and gives us further confidence in the predictions for  $f_B$  and related quantities.

**f<sub>B</sub>:** For sufficiently large masses of the heavy quark, the decay constant of a heavy–light pseudoscalar meson ( $P$ ) scales with its mass ( $M_P$ ) as follows:

$$f_P = \frac{A}{\sqrt{M_P}} \left[ \alpha_s(M_P)^{-2/\beta_0} \{1 + O(\alpha_s(M_P))\} + O\left(\frac{1}{M_P}\right) \right] , \quad (10)$$

where  $A$  is independent of  $M_P$ . Using the scaling law (10), a value of about 200 MeV for  $f_D$  would correspond to  $f_B \simeq 120$  MeV. Results from lattice computations, however, indicate that  $f_B$  is significantly larger than this and that the  $O(1/M_P)$  corrections on the right-hand side of eq. (10) are considerable. My summary of the lattice results is [12] (see also footnote 3):

$$f_B = 180 \pm 40 \text{ MeV} . \quad (11)$$

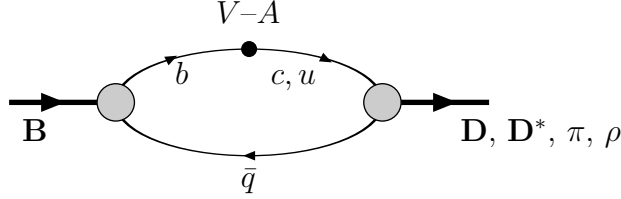


Figure 2: Diagram representing the semileptonic decay of the  $B$ -meson.  $\bar{q}$  represents the light valence antiquark, and the black circle represents the insertion of the  $V-A$  current with the appropriate flavour quantum numbers.

The coefficient of the  $O(1/M_P)$  corrections is found to be typically between 0.5 and 1 GeV.

Present lattice studies of heavy–light decay constants are concentrating on relaxing the quenched approximation, on calculating the  $O(1/M_P)$  corrections in eq. (10) explicitly, and on reducing the discretization errors through the use of improved actions and operators. The results obtained using QCD sum rules are in very good agreement with those from lattice simulations (see, for instance, ref. [4] and references therein, and ref. [20]).

### 3 Semileptonic Decays

For the remainder of this talk I will discuss semileptonic decays of  $B$ -mesons, considering in turn the two cases in which the  $b$ -quark decays semileptonically into a  $c$ -quark or a  $u$ -quark, see fig. 2. In both cases it is convenient to use space-time symmetries to express the matrix elements in terms of invariant form factors (I use the helicity basis for these as defined below). When the final state is a pseudoscalar meson  $P = D$  or  $\pi$ , parity implies that only the vector component of the  $V-A$  weak current contributes to the decay, and there are two independent form factors,  $f^+$  and  $f^0$ , defined by

$$\begin{aligned} \langle P(p_P) | V^\mu | B(p_B) \rangle &= f^+(q^2) \left[ (p_B + p_P)^\mu - \frac{M_B^2 - M_P^2}{q^2} q^\mu \right] \\ &+ f^0(q^2) \frac{M_B^2 - M_P^2}{q^2} q^\mu, \end{aligned} \quad (12)$$

where  $q$  is the momentum transfer,  $q = p_B - p_P$ . When the final-state hadron is a vector meson  $V = D^*$  or  $\rho$ , there are four independent form factors:

$$\begin{aligned} \langle V(p_V) | V^\mu | B(p_B) \rangle &= \frac{2V(q^2)}{M_B + M_V} \epsilon^{\mu\gamma\delta\beta} \epsilon_{\beta P B \gamma P V \delta}^* \\ \langle V(p_V) | A^\mu | B(p_B) \rangle &= i(M_B + M_V) A_1(q^2) \epsilon^{*\mu} - \end{aligned} \quad (13)$$

$$i \frac{A_2(q^2)}{M_B + M_V} \varepsilon^* \cdot p_B (p_B + p_V)^\mu + i \frac{A(q^2)}{q^2} 2M_V \varepsilon^* \cdot p_B q^\mu , \quad (14)$$

where  $\varepsilon$  is the polarization vector of the final-state meson, and  $q = p_B - p_V$ . Below we shall also discuss the form factor  $A_0$ , which is given in terms of those defined above by  $A_0 = A + A_3$ , with

$$A_3 = \frac{M_B + M_{D^*}}{2M_{D^*}} A_1 - \frac{M_B - M_{D^*}}{2M_{D^*}} A_2 . \quad (15)$$

### 3.1 Semileptonic $B \rightarrow D$ and $B \rightarrow D^*$ Decays

$B \rightarrow D^*$  and, more recently,  $B \rightarrow D$  decays are used to determine the  $V_{cb}$  element of the CKM matrix. Theoretically they are relatively simple to consider, since the heavy quark symmetry implies that the six form factors are related, and that there is only one independent form factor  $\xi(\omega)$ , specifically:

$$\begin{aligned} f^+(q^2) &= V(q^2) = A_0(q^2) = A_2(q^2) \\ &= \left[ 1 - \frac{q^2}{(M_B + M_D)^2} \right]^{-1} A_1(q^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \xi(\omega) , \end{aligned} \quad (16)$$

where  $\omega = v_B \cdot v_D$ . Here the label  $D$  represents the  $D$ - or  $D^*$ -meson as appropriate. In this leading approximation the pseudoscalar and vector mesons are degenerate. The unique form factor  $\xi(\omega)$ , which contains all the non-perturbative QCD effects, is called the Isgur–Wise (IW) function. Vector current conservation implies that the IW-function is normalized at zero recoil, i.e. that  $\xi(1) = 1$ . This property is particularly important in the extraction of the  $V_{cb}$  matrix element.

The relations in eq. (16) are valid up to perturbative and power corrections. The theoretical difficulty in making predictions for the form factors lies in calculating these corrections with sufficient precision.

The decay distribution for  $B \rightarrow D^*$  decays can be written as:

$$\begin{aligned} \frac{d\Gamma}{d\omega} &= \frac{G_F^2}{48\pi^3} (M_B - M_{D^*})^2 M_{D^*}^3 \sqrt{\omega^2 - 1} (\omega + 1)^2 \cdot \\ &\left[ 1 + \frac{4\omega}{\omega + 1} \frac{M_B^2 - 2\omega M_B M_{D^*} + M_{D^*}^2}{(M_B - M_{D^*})^2} \right] |V_{cb}|^2 \mathcal{F}^2(\omega) , \end{aligned} \quad (17)$$

where  $\mathcal{F}(\omega)$  is the IW-function combined with perturbative and power corrections. It is convenient theoretically to consider this distribution near  $\omega = 1$ . In this case  $\xi(1) = 1$ , and there are no  $O(1/m_Q)$  corrections (where  $Q = b$  or  $c$ ) by virtue of Luke’s theorem [21], so that the expansion of  $\mathcal{F}(1)$  begins like:

$$\mathcal{F}(1) = \eta_A \left( 1 + 0 \frac{\Lambda_{QCD}}{m_Q} + c_2 \frac{\Lambda_{QCD}^2}{m_Q^2} + \dots \right) , \quad (18)$$

where  $\eta_A$  represents the perturbative corrections. The one-loop contribution to  $\eta_A$  has been known for some time now, whereas the two-loop contribution was evaluated this year, with the result [22]:

$$\eta_A = 0.960 \pm 0.007 , \quad (19)$$

where we have taken the value of the two loop contribution as an estimate of the error.

The power corrections are much more difficult to estimate reliably. Neubert has recently combined the results of refs. [23]–[25] to estimate that the  $O(1/m_Q^2)$  terms in the parentheses in eq. (18) are about  $-0.055 \pm 0.025$  and that

$$\mathcal{F}(1) = 0.91(3) . \quad (20)$$

In considering eq. (20), the fundamental question that has to be asked is whether the power corrections are sufficiently under control. There are differing, passionately held views on this subject. The opinion of G. Martinelli and myself is that the uncertainty in eq. (20) is underestimated [26]. The power corrections are proportional to matrix elements of higher-dimensional operators. These have either to be evaluated non-perturbatively or to be determined from some other physical process. In either case, before the matrix element can be determined a subtraction of large terms is required (since higher-dimensional operators in general contribute to non-leading terms). The “large” terms are usually only known in perturbation theory at tree level, one-loop level or exceptionally at two-loop level. Therefore the precision of such a subtraction is limited. Moreover the definition of the higher-dimensional operators, and hence the value of their matrix elements, depend significantly on the treatment of the higher-order terms of the perturbation series for the coefficient function of the leading twist operator (this series not only diverges, but is not summable by any standard technique). These arguments are expanded, with simple examples and references to the original literature, in ref. [26]. Considerable effort is being devoted at present to improving the theoretical control over power corrections.

Bearing in mind the caveat of the previous paragraph, the procedure for extracting the  $V_{cb}$  matrix element is to extrapolate the experimental results for  $d\Gamma/d\omega$  to  $\omega = 1$  and to use eq. (17) with the theoretical value of  $\mathcal{F}(1)$ . See for example the results presented by Artuso at this conference [27].

Having discussed the theoretical status of the normalization  $\mathcal{F}(1)$ , let us now consider the shape of the function  $\mathcal{F}(\omega)$ , near  $\omega = 1$ . A theoretical understanding of the shape would be useful to guide the extrapolation of the experimental data, and also as a test of our understanding of the QCD effects. We expand  $\mathcal{F}$  as a power series in  $\omega - 1$ :

$$\mathcal{F}(\omega) = \mathcal{F}(1) \left[ 1 - \hat{\rho}^2 (\omega - 1) + \hat{c} (\omega - 1)^2 + \dots \right] , \quad (21)$$



where [28]

$$\hat{\rho}^2 = \rho^2 + (0.16 \pm 0.02) + \text{power corrections} , \quad (22)$$

and  $\rho^2$  is the slope of the IW-function. What is known theoretically about the parameters in eqs. (21) and (22)? Bjorken [29] and Voloshin [30] have derived lower and upper bounds, respectively, for the  $\rho^2$ :

$$\frac{1}{4} < \rho^2 < \frac{1}{4} + \frac{\bar{\Lambda}}{2E_{min}} , \quad (23)$$

where  $\bar{\Lambda}$  is the binding energy of the  $b$ -quark in the  $B$ -meson, and  $E_{min}$  is the difference in masses between the ground state and the first excited state. There are perturbative corrections to the bounds in eq. (23) [31], on the basis of which Korchemsky and Neubert [32] conclude that

$$0.5 < \rho^2 < 0.8 . \quad (24)$$

Values of  $\rho^2$  obtained using QCD sum rules and lattice simulations are presented in table 1. The theoretical results are broadly in agreement with the experimental measurements, e.g. in fig. 3 we show the comparison of the lattice results from ref. [37] with the data from the CLEO collaboration [38].

$\rho^2$	Method
$0.84 \pm 0.02$	QCD sum rules [33]
$0.7 \pm 0.1$	QCD sum rules [34]
$0.70 \pm 0.25$	QCD sum rules [35]
$1.00 \pm 0.02$	QCD sum rules [36]
$0.9^{+0.2+0.4}_{-0.3-0.2}$	Lattice QCD [37]

Table 1: Values of the Slope of the IW-function of a heavy meson, obtained using QCD sum rules or Lattice QCD.

Recently, using analyticity and unitarity properties of the amplitudes, as well as the heavy quark symmetry, Caprini and Neubert have derived an intriguing result for the curvature parameter  $\hat{c}$  [39]:

$$\hat{c} \simeq 0.66 \hat{\rho}^2 - 0.11 . \quad (25)$$

This result implies that one of the two parameters in (21) can essentially be eliminated, simplifying considerably the extrapolation to  $\omega = 1$ . Earlier attempts to exploit similar methods gave weaker bounds on the parameters.

Finally in this section I consider  $B \rightarrow D$  semileptonic decays, which are beginning to be measured experimentally [27] with good precision. Theoretically the first complication is that the  $1/m_Q$  corrections do not vanish at  $\omega = 1$ . However, as pointed out by Shifman and Voloshin [40], these corrections would

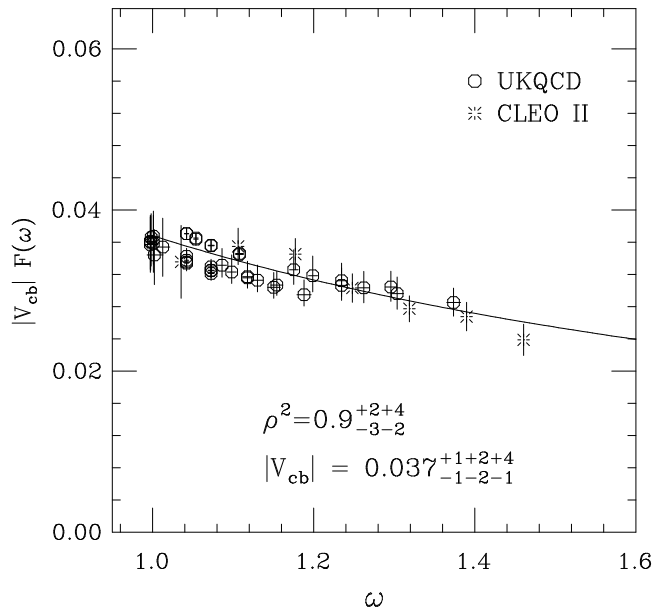


Figure 3: Fit of the UKQCD lattice results for  $|V_{cb}|\mathcal{F}(\omega)$  [37] to the experimental data from the CLEO collaboration [38].

vanish in the limit in which the  $b$ - and  $c$ -quarks are degenerate. This leads to a suppression factor

$$S = \left( \frac{M_B - M_D}{M_B + M_D} \right)^2 \simeq 0.23 \quad (26)$$

in the  $1/m_Q$  corrections, which reduces their size considerably. Ligeti, Nir, and Neubert estimate the  $1/m_Q$  corrections to be between approximately  $-1.5\%$  to  $+7.5\%$  [41]. The  $1/m_Q^2$  corrections for this decay have not yet been studied systematically.

### 3.2 Semileptonic $B \rightarrow \rho$ and $B \rightarrow \pi$ Decays

In this subsection I consider the semileptonic decays  $B \rightarrow \rho$  and  $B \rightarrow \pi$ . They decays are currently being studied experimentally, with the goal of extracting the  $V_{ub}$  matrix element.

Heavy quark symmetry is less predictive for heavy $\rightarrow$ light decays than it is for heavy $\rightarrow$ heavy ones. In particular, as we have seen in the preceding subsection, the normalization condition  $\xi(1) = 1$  was particularly useful in the extraction of  $V_{cb}$ . There is no corresponding normalization condition for heavy $\rightarrow$ light decays. Heavy quark symmetry does, however, give useful scaling laws for the behaviour of the form factors with the mass of the heavy quark at fixed  $\omega$ :

$$V, A_2, A_0, f^+ \sim M^{\frac{1}{2}}; \quad A_1, f^0 \sim M^{-\frac{1}{2}}; \quad A_3 \sim M^{\frac{3}{2}}. \quad (27)$$

Each of the scaling laws in eq. (27) is valid up to calculable logarithmic corrections.

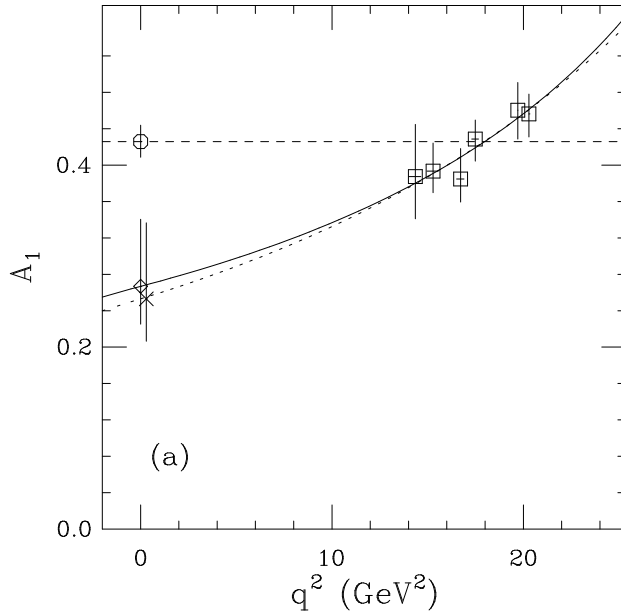


Figure 4: The form factor  $A_1(q^2)$  for the decay  $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ . Squares are measured lattice data, extrapolated to the  $B$  scale at fixed  $\omega$ . The three curves and points at  $q^2 = 0$  have been obtained by fitting the squares using the three procedures described in the text: constant (dashed line and octagon), pole (solid line and diamond) and dipole (dotted line and cross).

Several groups have tried to evaluate the form factors using lattice simulations [42]–[44] (for a review see ref. [45]). The results that I will use for illustration are taken from the UKQCD collaboration, who have attempted to study the  $q^2$  dependence of the form factors.

From lattice simulations we can only obtain the form factors for part of the physical phase space. In order to keep the discretization errors small, we require that the three-momenta of the  $B$ ,  $\pi$  and  $\rho$  mesons be small in lattice units. This implies that we can only determine the form factors at large values of momentum transfer  $q^2 = (p_B - p_{\pi,\rho})^2$ . Fortunately, as we will see below, for  $B \rightarrow \rho$  decays, this region of momentum space is appropriate for the extraction of  $V_{ub}$ .

As an example, I show in fig. 4 the values of the  $A_1$  form factor from ref. [44]. These authors evaluate the form factors for four different values of the mass of the heavy quark (in the region of that of the charm quark), and then extrapolate them, using the scaling laws in eq. (27), to the  $b$ -quark. The squares in fig. 4 represent the extrapolated values, and as expected they are clustered at large values of  $q^2$ . In order to estimate them over the full kinematical range some assumption about the  $q^2$  behaviour is required. Fig. 4 also contains three such extrapolations in  $q^2$ , performed assuming that:

- i)  $A_1$  is independent of  $q^2$  (dashed line). The extrapolated value of  $A_1(0)$  is denoted by an octagon, and the  $\chi^2/\text{dof}$  is poor for this fit.

ii) The behaviour of  $A_1(q^2)$  satisfies pole dominance, i.e. that  $A_1$  is given by

$$A_1(q^2) = \frac{A_1(0)}{(1 - q^2/M_n)^n} , \quad (28)$$

with  $n = 1$  (solid line).  $A_1(0)$  and  $M_1$  are parameters of the fit, but the value of  $M_1$  is in the range expected for the  $1^+ b\bar{u}$  resonance. The extrapolated value of  $A_1(0)$  is denoted by the diamond.

iii) The behaviour of  $A_1(q^2)$  takes the dipole form (28) with  $n = 2$  (dotted line). This is almost indistinguishable from the pole fit. The extrapolated value of  $A_1(0)$  is now denoted by a cross.

The  $\chi^2/\text{dof}$  for the pole and dipole fits are both very good.

The UKQCD collaboration [44] comment that for  $b \rightarrow \rho$  decays in particular, the fact that the lattice results are obtained at large values of  $q^2$  is not a serious handicap to the extraction of the  $V_{ub}$  matrix element. Indeed they advocate using the experimental data at large values of  $q^2$  (as this becomes available during the next few years) to extract  $V_{ub}$ . To get some idea of the precision that might be reached they parametrize the distribution by:

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu})}{dq^2} = 10^{-12} \frac{G_F^2 |V_{ub}|^2}{192\pi^3 M_B^3} q^2 \lambda^{\frac{1}{2}}(q^2) a^2 (1 + b(q^2 - q_{max}^2)) , \quad (29)$$

where  $a$  and  $b$  are parameters to be determined from lattice computations, and the phase-space factor  $\lambda$  is given by  $\lambda(q^2) = (M_B^2 + M_\rho^2 - q^2)^2 - 4m_B^2 M_\rho^2$ . Already from their current simulation the UKQCD collaboration are able to obtain  $a^2$  with good precision [44]

$$a^2 = 21 \pm 3 \text{ GeV}^2 . \quad (30)$$

Although  $b$  is obtained with less precision,

$$b = (-8_{-6}^{+4}) 10^{-2} \text{ GeV}^{-2} , \quad (31)$$

the fits are less sensitive to this parameter at large  $q^2$ . The prediction for the distribution based on these numbers is presented in fig. 5, and the UKQCD collaboration estimate that they will be able to determine  $V_{ub}$  with a precision of about 10% or better.

Although, in this case, the difficulty of extrapolating lattice results from large values of  $q^2$  to smaller ones may not have significant implications for extracting physical information, this is not always the case. Already for  $B \rightarrow \pi$  decays, using results at large values of  $q^2$  restricts the precision with which  $V_{ub}$  can be extracted. This problem is even more severe for the penguin-mediated rare decay  $B \rightarrow K^* \gamma$ , where the physical process occurs at  $q^2 = 0$ . Much effort is being devoted to this extrapolation, trying to include the maximum number of constraints from heavy

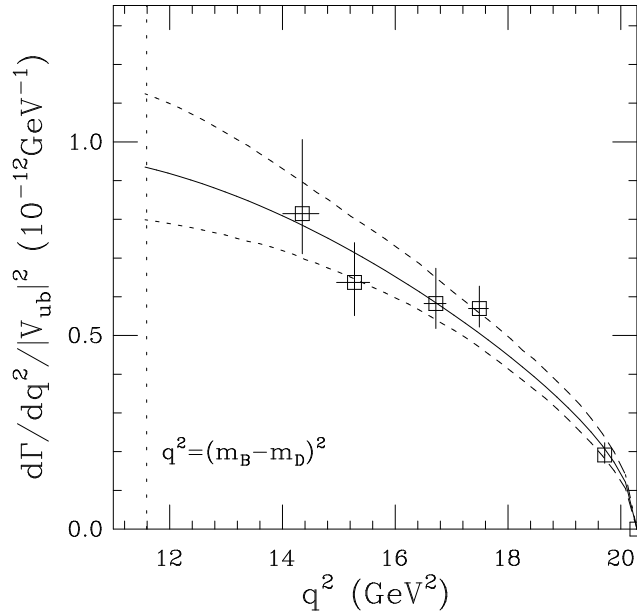


Figure 5: Differential decay rate as a function of  $q^2$  for the semileptonic decay  $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ . Squares are measured lattice data, solid curve is fit from eq. (29) with parameters given in eqs. (30) and (31). The vertical dotted line marks the charm threshold.

quark symmetry (as discussed above) and elsewhere [46]. A simple example of such a constraint for  $B \rightarrow \pi$  semileptonic decays is that at  $q^2 = 0$ , the two form factors  $f^+$  and  $f^0$  must be equal. Similar constraints exist for other processes.

An interesting approach to the problem of the extrapolation to low values of  $q^2$  has been suggested by Lellouch [47]. By combining lattice results at large values of  $q^2$  with kinematical constraints and general properties of field theory, such as unitarity, analyticity and crossing, he is able to tighten the bounds on form factors at all values of  $q^2$ . This technique can, in principle, be used with other approaches, such as sum rules, quark models, or even in direct comparisons with experimental data, to check for compatibility with QCD and to extend the range of results.

Ball and Braun have recently re-examined  $B \rightarrow \rho$  decays using light-cone sum rules [48], extending the earlier analysis of ref. [49]. Consider, for example, the graph of fig. 6, which represents a contribution to the decay amplitude. For large heavy-quark masses and small  $q^2$  there are two competing contributions of the same order (e.g.  $O(m_Q^{-3/2})$  for the form factor  $A_1$ ). The first one comes from the region of phase space in which the momentum of the gluon ( $g$ ) is of the order of  $\sqrt{m_b \Lambda_{QCD}}$ , so that this contribution corresponds to small transverse separations and can be treated in perturbation theory (the non-perturbative effects are contained in the wave functions at the origin, i.e. in the decay constants). This is similar to the treatment of hard exclusive processes, such as the form factors of the pion and the proton at large momentum transfers. However, there is a second

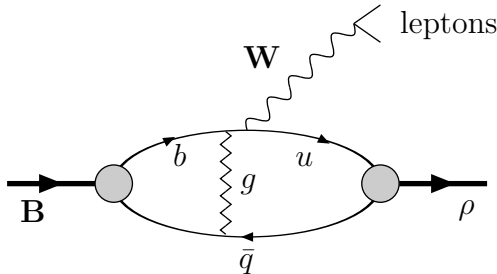


Figure 6: Representation of a contribution to the semileptonic  $B \rightarrow \rho$  decay.

contribution in which the  $\rho$ -meson is produced in a very asymmetric configuration with most of the momentum carried by one of the quarks. In this case there are no hard propagators. For most other hard exclusive processes the “end-point” contribution is suppressed by a power of the large momentum transfer. Although, in principle, for  $m_Q$  very large, the end-point is suppressed by Sudakov factors [50], this suppression is not significant for the  $b$ -quark. The end-point contribution has to be included and treated non-perturbatively, since it comes from the region of large transverse separations. This is the motivation for introducing light-cone sum rules [49], based on an expansion of operators of increasing twist (rather than dimension). The non-perturbative effects are contained in the light-cone wave function of the  $\rho$ -meson, and the leading twist contribution to this wave function was recently re-examined in ref. [51].

An interesting consequence of the analysis of the previous paragraph is a set of scaling laws for the behaviour of the form factors with the mass of the heavy quark at fixed (low)  $q^2$ , rather than at fixed  $\omega$  as in eq. (27). An example of fixed  $q^2$  scaling laws is:

$$A_1(0) \Theta M_P^{3/2} = \text{const}(1 + \gamma/M_P + \delta/M_P^2 + \dots) , \quad (32)$$

where  $M_P$  is the mass of the heavy pseudoscalar meson. The factor  $\Theta$  contains the perturbative logarithmic corrections.

Some of the results of Ball and Braun are presented in fig. 7, where the form factors  $A_1, A_2$  and  $V$  are plotted as functions of  $q^2$ . The results are in remarkable agreement with those from the UKQCD collaboration, in the large  $q^2$  region where they can be compared.

## 4 Conclusions

The principal difficulty in deducing weak interaction properties from experimental measurements of  $B$ -decays lies in controlling the strong interaction effects. These are being studied using non-perturbative methods such as lattice simulations or

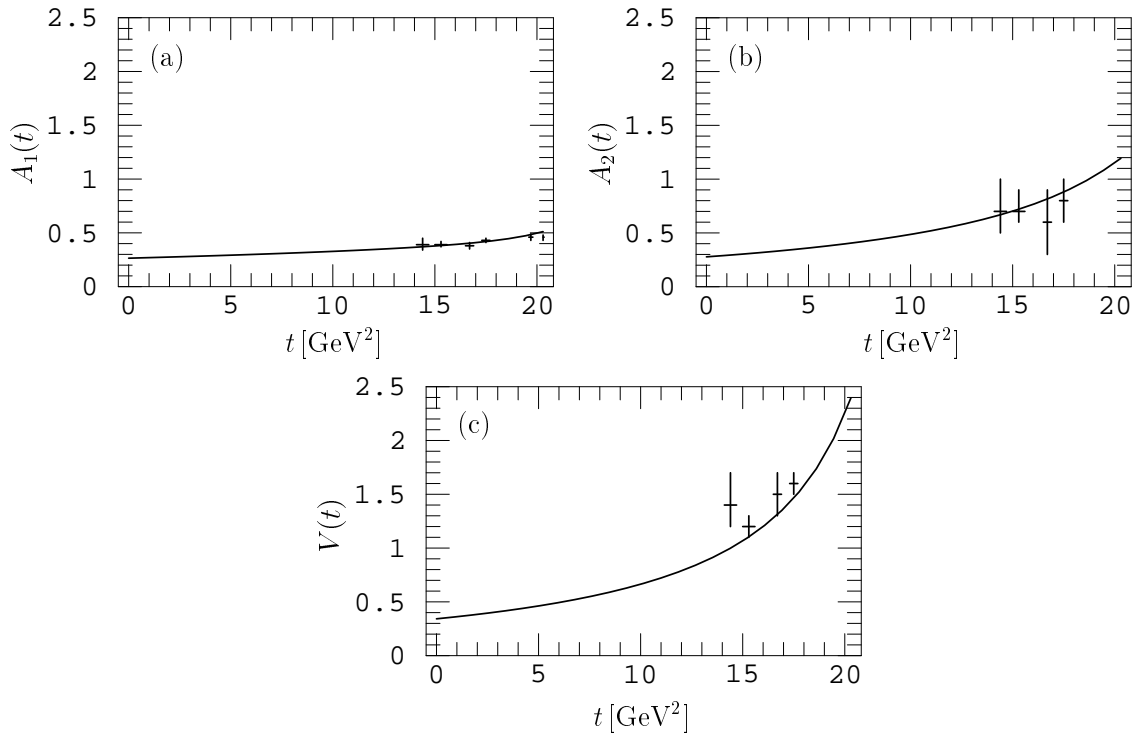


Figure 7: Results for the form factors  $A_1(q^2)$ ,  $A_2(q^2)$  and  $V(q^2)$  for  $B \rightarrow \rho$  semileptonic decays as a function of  $t = q^2$  [48]. The curves correspond to the results obtained with light-cone sum rules by Ball and Braun [48], and the points to the results from the UKQCD collaboration [44].

QCD sum rules. Considerable effort and progress is being made in reducing the systematic uncertainties present in lattice computations.

Although both the theoretical and experimental errors on the value of  $f_{D_s}$  are still sizeable, it is nevertheless very pleasing that they are in agreement. It is also satisfying that the values of  $V_{cb}$  extracted from exclusive and inclusive measurements are in good agreement. The theoretical uncertainties for the two processes are different, and the agreement is evidence that they are not significantly underestimated.

It has been argued that  $B \rightarrow \rho$  decays at large  $q^2$ , where the evaluation of the relevant form factors using lattice simulations is reliable, will soon provide a determination of  $V_{ub}$  at the 10% level or better [44]. It will also be interesting to observe developments of the light-cone approach to these decays.

Many lattice computations of  $f_B$  have been performed using static heavy quarks ( $m_Q = \infty$ ), and serve as a very valuable check of the consistency of the extrapolation of the results obtained with finite heavy-quark masses. Such checks have not been performed yet for many other quantities in  $B$ -physics; this is an important omission, which should be put right.

This talk has been about the decays of  $B$ -mesons. Detailed experimental and theoretical studies are also beginning for the  $\Lambda_b$ -baryon. For example, the first lattice results for the Isgur–Wise function of the  $\Lambda_b$  has been presented in ref. [52].

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