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Centre d'Economie de la Sorbonne

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CES Working Papers

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2017.03

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://centredeconomiesorbonne.univ-paris1.fr/
ISSN : 1955-611X

Sophisticated Bidders in Beauty-Contest Auctions*

Stefano Galavotti† Luigi Moretti‡ Paola Valbonesi§

Abstract

We study bidding behavior by firms in beauty-contest auctions, i.e. auctions in which the winning bid is the one which gets closest to some function (average) of all submitted bids. Using a dataset on public procurement beauty-contest auctions, we show that firms' observed bidding behavior departs from equilibrium and can be predicted by a *sophistication* index, which captures the firms' accumulated capacity of bidding close to optimality in the past. We show that our empirical evidence is consistent with a Cognitive Hierarchy model of bidders' behavior. We also investigate whether and how firms learn to bid strategically through experience.

JEL classification: C70; D01; D44; D83; H57.
Keywords: cognitive hierarchy; auctions; beauty-contest; public procurement.

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*We are indebted to Francesco Decarolis for providing us with his codes. We would like to thank for their valuable comments: Malin Arve, Riccardo Camboni, Ottorino Chillemi, Decio Coviello, Klenio Barbosa, Gordon Klein, Luciano Greco, Marco Pagnozzi, Tim Salmon, Giancarlo Spagnolo, Alessandra Bianchi and participants at the "Workshop on Economics of Public Procurement" (Stockholm, June 2013), International Conference on "Contracts, Procurement, and Public-Private Arrangements" (Chaire EPPP IAE Panthéon-Sorbonne, Florence, June 2013), 54th Annual Conference of the Italian Economic Association (Bologna, October 2013), "Workshop on How do Governance Complexity and Financial Constraints affect Public-Private Contracts? Theory and Empirical Evidence" (Padova, April 2014), EARIE Conference (Milan, August 2014), seminar at PSE & U. Paris I Panthéon-Sorbonne (February 2015), 1st Berkeley-Paris Organizational Economics Workshop (April 2015), seminar at the Department of Economics, Leicester University (October 2015).

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1 Introduction

The competition among firms in market economies generates winners and losers: some firms survive, grow up and pay dividends to their shareholders, others have poor performances or go bankrupt and exit the market. Why does this happen? Is it because, though all firms are making their optimal decisions, the winners have some structural or informational advantage over the losers? Or is it simply because the losers are making the wrong decisions, or, in game-theoretic language, they are not playing their equilibrium strategies?

Answering to this question is typically difficult, as we rarely observe all the fine details of the game that firms are actually playing. Moreover, even though we can replicate market games in controlled lab-experiment, it is questionable whether and to what extent these insights can be generalized to real-world situations where stakes are large.

In this paper to address the above question using field data from a peculiar procurement auction market: average bid auctions. These auctions resemble beauty-contest games in that the winning bid is the one which gets closest to some function (average) of all submitted bids. Average bid auctions have very precise Nash equilibrium predictions which are essentially unaffected by variables that are often unobservable: in equilibrium, either all or – possibly – most bids should be equal. This makes it an ideal setting to investigate possible deviations from equilibrium.

Using an original dataset of procurement average bid auctions in the Italian region of Valle d'Aosta, we observe that actual bids significantly depart from equilibrium, being characterized by a systematic heterogeneity. Starting from the consideration that the peculiar rules of these auctions call for refined strategic thinking by bidders, as a bidder has to anticipate the behavior of all other bidders (whereas in a first-price auction, it is sufficient to guess the distribution of the highest competing bid), we hypothesize that the observed heterogeneity could be the result of the interaction of firms with different abilities in performing an iterated process of strategic reasoning, in the spirit of the Cognitive Hierarchy (CH, henceforth) model by Camerer et al. (2004). Applied to our context, this model predicts that more sophisticated firms, being able to formulate more accurate beliefs about how others are going to bid, make "better" bids, i.e. closer to the truly optimal one. We estimate an empirical reduced form model which shows that, in accordance with the main prediction of the CH model, the firm's sophistication index, measured by the accumulated capacity of bidding well in the past, is strongly and positively correlated to the goodness of that firm's bid, measured by the (negative of the) distance from the truly optimal bid. This result is robust to several specifications of the empirical model; most importantly, it is also confirmed when we focus our analysis on a sample of auctions awarded with a new average bid format, which includes a stochastic component. Interestingly, our evidence shows that a significant learning process is at work: firms become better strategic bidders as they participate in more and more auctions of the same format; instead, sophistication acquired in one format does not significantly affects performance in the other.

This paper mainly contributes to two strands of literature. First, we relate to two recent papers which fit structural econometric CH model on real data. In particular, Goldfarb and Yang (2009) study the decision by Internet Service Providers whether or not to adopt the then new 56K modem technology in 1997. Goldfarb and Xiao (2011) investigate the choice by U.S. managers of competitive local exchange carriers (CLECs) to enter local telephone markets after the Telecommunication Act in 1996. Both papers uncover significant heterogeneity of sophistication among managers, with more sophisticated managers less likely to adopt the new technology or to enter markets with more competitors. They also show that the level of sophistication is higher for firms operating in larger cities, with more competitors or in markets with more educated populations (Goldfarb and Yang) and for more experienced, better educated managers (Goldfarb and Xiao). Both these papers assume a CH model, but do not address whether their model fits better than an equilibrium model.[1] In our paper, instead, we do not assume any structural model but show that the capacity of firms of making better decisions has a systematic component which goes in a direction coherent with a CH model.

Second, our paper contributes to a recent empirical and experimental literature on average bid auctions. Decarolis (2014) and Bucciol et al. (2013) empirically compare the performances of average bid and first-price auctions for the procurement of public works in Italy. These papers show that the first-price is in general associated with lower awarding prices but worse performances in terms of cost and time overruns in the completion time. Conley and Decarolis (2016) argue that the average bid auction is weak to collusion as the members of a cartel, by placing coordinated bids, may pilot the average, thus increasing the probability that one of them wins. Using a dataset (different from ours) of Italian average bid procurement auctions, they find that a large fraction of auctions (no less than 30%) is likely to be affected by the presence of cartels; thus, they conclude that the observed deviations from Nash equilibrium are mostly due to a cooperative behavior by bidders. Our paper suggests a complementary explanation to the observed bidding behavior in this type of auctions, but based on a non-cooperative argument. Nevertheless, we provide and discuss some arguments supporting the robustness of our findings to the possible presence of collusion. Chang et al. (2015) experimentally investigate whether a simple average bid auction can be an effective alternative to first-price auctions for an auctioneer concerned with reducing winner's curse phenomena in common value settings. Their results suggest a positive answer: prices are higher in the average bid than in the first-price auction, thus reducing losses and virtually eliminating default problems. Interestingly, in the average bid auction, subjects do not coordinate on high prices as the Nash equilibrium would predict; rather, they follow a bidding strategy which is strictly increasing in their cost signal. The authors argue that a level-k model would qualitatively generate bidding functions with this shape, but would predict larger bids than observed. They propose an almost-equilibrium explanation to their evidence: while subjects with intermediate signals do best-respond to the behavior of the others, subjects with extreme signals misinterpret the informative content of their signal and bid suboptimally.

The rest of the paper is organized as follows: in Section 2, we illustrate the auction formats considered, describe our dataset and present some preliminary descriptive evidence; in Section 3, we show that our evidence is clearly inconsistent with Nash equilibrium and obtain a testable prediction from a CH model; this prediction leads to the empirical analysis, provided in Section 4; Section 5 offers a discussion of our results, with further supporting evidence and robustness checks; Section 6 briefly concludes.

[1] Brown et al. (2012) use a CH model to explain empirical evidence on box-office premiums associated to cold-opened movies, i.e. movies that are not shown to critics prior to their release. In their paper, consumers, not firms, have limited capacity of strategic thinking and firms exploit the consumers' naïveté to extract more surplus by not disclosing information on the low quality movies.

2 Auction formats and descriptive evidence

Since 1998, the large majority of public works in Italy are procured by means of average bid auctions: these are auctions in which the winner is not the firm that offers the best (i.e. lowest) price, but the one whose offer is closest to some endogenous function (average) of all submitted offers. Participating firms submit a (sealed) price consisting of a percentage discount on the reserve price set by the Contracting Authority (CA).[2] Once the CA has verified the firms' legal, fiscal, economic, financial and technical requirements, the winning firm is determined according to the following mechanism (see Figure 1, top panel): discounts are ordered from the lowest to the highest and a first average ($A1$) is computed by averaging all bids except the 10% highest and lowest bids.[3] Then, a second average ($A2$) is computed by averaging all bids strictly above $A1$ (again disregarding the 10% highest bids). The winning bid is the one immediately *below* $A2$. In the event that all bids are equal, the winner is chosen randomly. We call this auction format "Average Bid", or simply AB.[4]

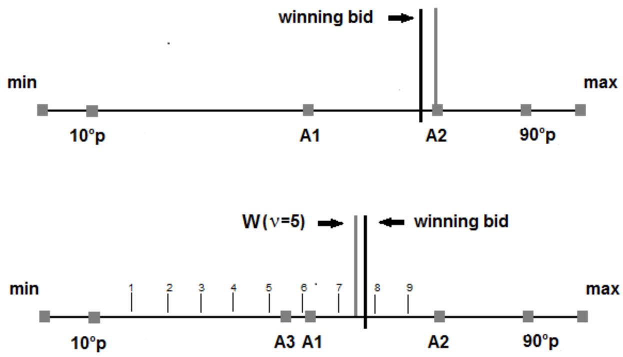

Figure 1 – AB (top panel) and ABL (bottom panel) auction.

Our dataset collects auctions for public works issued by the Regional Government of Valle d'Aosta in the period 2000-2009 (data are from Moretti and Valbonesi, 2015). It contains all bids submitted in each auction, together with several detailed information at the firm- and auction-level: for each participating firm, we know the identity (i.e. company name) and some characteristics such as size, location, number of pending public procurement projects, and subcontracting position (mandatory or optional, see Moretti and Valbonesi, 2015, for a discussion on this); for each auction, we have information on the reserve price, the task of the

[2]Hence, a higher discount means a lower price paid by the CA. In the rest of the paper, we will use the terms bids and discounts interchangeably.

[3]For example, if there are 20 bids, the 2 lowest and the 2 highest bids are not considered in the computation of $A1$. When this 10% is not an integer, the number of neglected bids is obtained by rounding up: for example, if there are 25 bids, the 3 lowest and the 3 highest bids are not considered.

[4]The AB format has been compulsory in Italy until June 2006 for all contracts with a reserve price below 5 mln euro. The ratio behind the choice of the AB format instead of the first-price was the consideration that the former, by softening price competition, would have generated higher awarding prices, thereby reducing the likelihood that, when the ex-post cost of realizing the project turns out to be larger than expected, the winning firm declares bankruptcy or asks for a renegotiation of the contract (for more on the trade-off between price and performance in first-price and average bid auctions, see, among others, Cameron, 2000, Albano et al., 2006, Bucciol et al., 2013, Decarolis, 2014). After 2006, every CA has been allowed to choose between AB and first-price auction (but since October 2008, first-price auctions are compulsory for contracts above 1 mln euro).

tendered project and the estimated duration of the work.

An interesting feature of our dataset is that it covers a change in the auction format. In fact, while public works before 2006 were awarded through the AB format described before, since 2006, and *only* in Valle d'Aosta, a new average bid awarding mechanism has been introduced. The new format differs from the previous one as it includes a stochastic component; for this reason, we call it "Average Bid with Lottery" auction, or simply ABL. The ABL auction works as follows (see Figure 1, bottom panel): given the average $A2$ computed as in AB, a random number (ω) is extracted from the set of nine equidistant numbers between the lowest bid above the first decile of bids and the bid immediately below $A2$. Averaging ω with $A2$, the winning threshold W is obtained and the winning bid is the one closest from *above* to W, provided this bid does not exceed $A2$. Otherwise, the winner will be the bid equal or closest from below to W. Again, if all bids are identical, the winner is chosen randomly. To be precise, if we denote by $d_{10\%}$ the discount immediately above the first decile of the bid distribution and by d_{A2} the discount immediately below $A2$, then the winning threshold is $W = [A2 + \omega]/2$ where $\omega = d_{10\%} + (d_{A2} - d_{10\%})\nu/10$ and ν can be any integer between 1 and 9. Hence, the winning threshold will necessarily fall within an interval whose lower and upper bounds are $[A2 + d_{10\%} + (d_{A2} - d_{10\%})/10]/2$ and $[A2 + d_{10\%} + (d_{A2} - d_{10\%})9/10]/2$, respectively. For reasons that will be clear later on in the paper, we denote the lower bound of this interval by $A3$.

Figure 2 shows non-parametric kernel density estimation of the bid distributions in the AB and ABL formats (dashed line for AB and straight line for ABL). For each auction, discounts have been re-scaled using a min-max normalization (the lowest discount in an auction takes value 0, while the highest takes value 1).

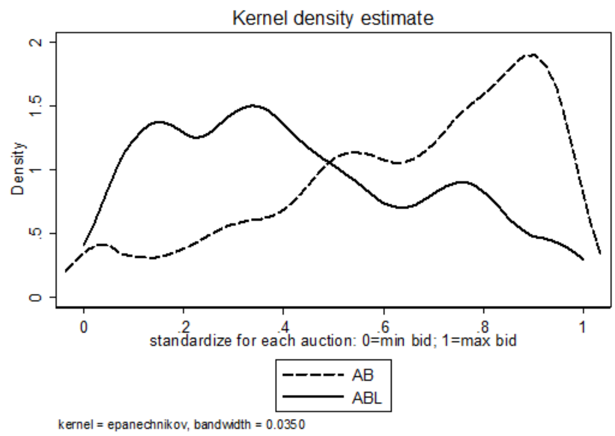

Figure 2 – Discounts in AB and ABL: Kernel density estimation.

Figure 2 highlights two relevant features. First, in either formats, bids are clearly neither uniformly, nor normally distributed. Second, the distributions are clearly asymmetric and different across the two formats: in AB, most bids are concentrated in the right end of the support of the distribution of bids; in ABL, most bids are concentrated below the midpoint of the support.

3 Theory: Equilibrium vs. Cognitive Hierarchy

The descriptive evidence presented in Figure 2 suggests that the bidding behavior by firms in our dataset is characterized by some regularities. In this section, we first investigate whether this evidence can be consistent with the standard notion adopted to model bidders' behavior in auctions, i.e. Nash equilibrium. To this end, we consider the following (static) model: a single contract is auctioned off through an AB or ABL auction. There are n risk neutral firms that participate in the auction. Firm i's cost of completing the job is given by $c_i(x_1, x_2, \ldots, x_n)$, where x_i is a cost signal privately observed by firm i (x_i is the *type* of firm i). We assume that firm i's cost is separable in her own and other firms' signals and linear in x_i, i.e. that $c_i(x_i, x_{-i}) = a_i x_i + \Gamma_i(x_{-i})$, with $a_i > 0$, $\partial\Gamma_i/\partial x_j \geq 0$, for all $i, j \neq i$. Firm i's signal is distributed according to a cumulative distribution function $F_i(x_i)$, with full support $[\underline{x}_i, \overline{x}_i]$ and density $f_i(x_i)$. Signals are independent. The cost functions as well as the signals' distributions are common knowledge.[5] Firms submit sealed bids formulated in terms of percentage discounts over the reserve price R. The winning firm in AB and ABL is determined according to the rules described in the previous section. For convenience, the theoretical analysis presented here is carried on under the restriction that all firms always participate in the auction, because they always find it worthwhile to do so. Relaxing it would not alter the results, at least qualitatively.

Under the above assumptions, we obtain rather sharp predictions on the (Bayes-) Nash equilibria in the two formats, that we summarize in the following proposition:[6]

PROPOSITION 1 [EQUILIBRIUM].

(i) *In the AB auction, there is a unique equilibrium in which all firms submit a* 0*-discount (irrespective of their signals).*

(ii) In the ABL auction, there exists a continuum of equilibria in which all firms make the same discount d (irrespective of their signals), where d guarantees a positive expected profit even to the firm with the highest expected cost.

(iii) In any equilibrium of the ABL auction, there is a set K of firms of cardinality $k \geq n - \tilde{n}$ that bid $\bar{d}$ with strictly positive probability, where $\bar{d}$ is the largest conceivable bid in the equilibrium and $\tilde{n}$ denotes the smallest integer greater than or equal to $n/10$. Moreover, if there is (at least) one firm $i \in K$ such that $\mathrm{PW}_i(\bar{d}, \delta_{-i}) \geq \mathrm{PW}_i(\bar{d} - \varepsilon, \delta_{-i})$ for $\varepsilon \to 0^+$, the probability that at least $n - \tilde{n} - 1$ firms do bid $\bar{d}$ must be larger than $\sum_{j=n-\tilde{n}-1}^{k-1} r^j / \sum_{j=0}^{k-1} r^j$, where r solves $\sum_{j=1}^{k-(n-\tilde{n}-1)} r^j = T$, where $T = (n-\tilde{n})(n-\tilde{n}-2)/(n-\tilde{n}-1)$.

The common feature of the equilibria of both types of auctions is that they all display a very large degree of pooling, both across firms and within firms. The intuition of this result is straightforward: consider a configuration where bids are largely differentiated across and/or within firms (e.g., all bidding functions are strictly decreasing). In this case, the firm/type

[5] Notice that the model allows for (ex-ante) heterogeneity across firms. Notice also that this model encompasses the pure private model ($\Gamma_i = 0$, for all i) and the pure common value model ($c_i = c_j$, for all i, j) as special cases.

[6] For the proofs, see Appendix A. Point *(i)* was already proved in Decarolis (2009) for the symmetric private value case.

that makes the highest bid will have a very low (if not zero) probability of winning and would rather reduce her bid to increase it. But if she does so unilaterally, another firm/type will become the highest bidder and would rather reduce her bid as well. This process of "escaping from being the highest bidder" comes to an end only when even the highest bidder itself has a sufficiently large probability of winning.

In the AB case, this occurs only when all participating firms make a 0-discount (Proposition 1-*(i)*). In fact, the firm/type that makes the highest bid $\bar{d}$ can win the AB auction only if the winning threshold W coincides with $\bar{d}$ itself and nobody else makes a lower bid (the winning bid is the one closest from below to W), i.e. only when all firms bid $\bar{d}$. However, if everybody bids $\bar{d} > 0$, any firm still has an incentive to deviate downward: doing so, her probability of winning would jump from $1/n$ to 1. Only when all firms make a 0-discount, such a downward deviation is not admissible and we have reached an equilibrium.

In the ABL case, instead, the incentive for the highest bidder to deviate downward stops when there is a sufficiently large probability that a large fraction of the other firms makes the same highest bid as well. This discrepancy with respect to the AB format is not due to the different way in which W is computed, but rather to the fact that in ABL the winning bid is the one closest from above (rather than below) to W. To see this, consider a firm/type that makes the highest equilibrium bid $\bar{d}$. Now, if less than $n - \tilde{n} - 1$ of the other firms bid $\bar{d}$, the winning threshold will certainly be below $\bar{d}$: the highest bid $\bar{d}$ can be a winning bid (the winning bid is the one closest from above to W, if there is one), but a slight downward deviation $\bar{d} - \varepsilon$ would certainly be profitable: the deviating firm would win in the same circumstances as when she bids $\bar{d}$, but now she would be the sole winner in case of winning. If, instead, at least $n - \tilde{n} - 1$ of the other firms bid $\bar{d}$, the winning threshold will certainly coincide with $\bar{d}$, and a lower bid will give a 0-probability of winning the auction. Hence, bidding $\bar{d}$ can be an equilibrium bid only if the probability that at least $n - \tilde{n} - 1$ bid $\bar{d}$ is sufficiently large. Proposition 1-*(iii)* states this intuition and provides an explicit lower bound to this probability valid when equilibrium satisfies a (mild) restriction: for at least one firm/type that bids $\bar{d}$, the probability of winning should not increase if this firm slightly deviated downward (notice that this restriction is necessarily satisfied when at least one firm has private cost or when at least one firm's bidding function is continuous at $\bar{d}$). To get an idea of the size of this lower bound, notice that, if $n = 20$, the probability of observing at least 17 equal bids (at $\bar{d}$) must be grater than 90.1%.[7]

The previous argument should also make it clear why, in these auctions, cost signals do not matter much. The point is that, unlike a standard auction where a higher bid always increases the probability of winning and thus stronger bidders – those with better signals – will bid higher, here to increase the probability of winning a bidder has to make a bid which is neither too high, nor too low; hence, having a better signal is much less of an advantage than in a standard auction. As a consequence, the cost structure plays a less important role in shaping the equilibrium.[8]

[7]With interdependent costs, the restriction in Proposition 1-*(iii)* is not necessarily satisfied, as one firm may still find it unprofitable to slightly deviate downward even if doing so her probability of winning increases: this might be the case if the downward deviation increases the probability of winning when the other firms has worse signals and thus increases the expected cost upon winning. However, if the cost functions are not dramatically affected by a marginal change in the other firms' signals, then the lower bound should be of the same order as the one stated in Proposition 1-*(iii)*.

[8]This line of reasoning applies not only to production costs but, more generally, to any other element that can affect the competitiveness of a firm in the auction. For example, Decarolis (2014) and Zheng (2001) argue that, in procurement first-price auctions where the true production costs are known only ex-post, riskier firms

Now, looking at our data, we can rather safely claim that the theoretical predictions described above are inconsistent with the empirical evidence. In the AB auction, bids are far from being equal (the standard deviation of the distribution of bids is, on average, 4.6%) and are significantly greater than zero (the average discount is 18.0%), while equilibrium predicts all bids equal to zero (Proposition 1-*(i)*).[9] In the ABL auction, we can safely reject the all-equal-bids equilibrium of Proposition 1-*(ii)* (in ABL, the average standard deviation of the distribution of bids is 3.6%).[10] Moreover, according to Proposition 1-*(iii)*, in ABL we should expect to observe a concentration of bids in the right tail of the distribution. This is clearly at odds with our descriptive evidence, according to which the typical bid distribution in an ABL auction has its mode below the midpoint of the range of bids.

Thus, we conclude that Nash equilibrium does not seem to be a correct modeling hypothesis for the bidding behavior of firms in our dataset. Although we do reject the equilibrium hypothesis that *all* firms are bidding optimally, our intuition is that *some* of them are doing so, while others are not. One model that supports this intuition is the CH model. This model has been introduced by Stahl and Wilson (1994, 1995) and further developed and applied by, among others, Camerer et al. (2004). Strictly related to the CH model is the level-k model introduced by Nagel (1995) and applied to first- and second-price auctions by Crawford and Iriberri (2007) and Gillen (2009). The CH model has proved to be particularly fruitful in explaining experimental evidence in beauty-contest games (see the thorough survey by Crawford et al., 2013). Since average bid auctions are nothing but incomplete information versions of beauty-contest games, this model is a natural candidate to explain our evidence.

The CH model holds that individuals (players) involved in strategic situations differ by their level of *sophistication*, i.e. their ability of performing an iterated process of strategic thinking. The proportion of each level in the population is given by a frequency distribution $P(k)$, where $k = 0, 1, 2, \ldots$ is the level of sophistication. Level-0 players are completely unsophisticated and simply play randomly (according to some probability distribution, in general uniform); a level-k player, with $k \geq 1$, believes that her opponents are distributed, according to a normalized version of $P(k)$, from level-0 to level-$(k-1)$ and chooses her optimal strategy given these beliefs. For example, a level-1 player believes that all her opponents are of level-0; a level-2 player believes that her opponents are a mixture of level-0 and level-1 players, where the proportion of level-0 players is $P(0)/(P(0) + P(1))$; and so on. In other words, a level-k player's strategy is optimal *conditional on her beliefs*, but since her beliefs do not contemplate the presence of players of the same or higher level, the resulting strategy will in general be suboptimal. Clearly, a player with a higher level of sophistication has in mind a more comprehensive picture of how other players think and play; hence, we expect

– those with lower default costs – make higher discounts, thus generating an adverse selection effect. Our model immediately adapts to an environment where firms differ in their default costs, thus predicting that this adverse selection effect almost disappears in average bid auctions.

[9] Hence, if firms had always played the Nash equilibrium, the CA would have paid much more for the works. In particular, given that, in our sample, the average reserve price is about 1 million euro and the average winning is 18%, the average additional payment by the CA for each project would have been about 180,000 euro. Moreover, given that in the Nash equilibrium, the winner is chosen randomly, the expected market share and market power of each firm would have been the same.

[10] In the ABL auction, given the multiplicity of equilibria, there is a potential problem of coordination, and one could object that our evidence is just the result of a coordination failure. However, this explanation does not seem fully convincing: first, it would apply to the ABL format only, leaving the observed behavior in AB unexplained; second, even restricting this explanation to the ABL case, the observed regular asymmetry in the distribution of bids would raise the following question: why do many firms reach a good coordination on relatively low discounts, whereas other firms seems totally unable to coordinate?

her strategy to be closer to the optimal one.

The logic behind the CH model seems particularly appropriate in our context. In an average bid auction, all bids affect the position of the winning threshold. Therefore, it is crucial to have correct guesses on how all other firms are going to bid. But predicting the behavior of all other firms involves answering a complicated chain of questions of the kind: what bid b will a firm make if she thinks others are going to bid a? And what bid c will a firm make if she anticipates that others are going to bid b because they think others are going to bid a? And so on. Firms who are able to push this chain of reasoning further will have an advantage over those who perform less steps of such reasoning, in the sense that they will end up with more precise predictions on the actual behavior of others. As a consequence, they are expected to make better (i.e. closer to optimality) bids.

Solving the CH model in our context for a generic number of firms is problematic.[11] Therefore we rely on an asymptotic analysis: this seems sufficiently appropriate in our case, given that the number of participating firms in our dataset is, on average, relatively large (about 53 in AB, about 83 in ABL); moreover, we believe that the intuition provided by the asymptotic analysis applies in general, at least under standard hypothesis about the behavior of level-0 firms.

Now, let $P(k) = p_k$, $k \geq 0$, be the proportion of level-k firms and suppose that level-0 firms (independently) choose their bids from the bid distribution $G_0(d)$ with density $g_0(d)$ and full support $[\underline{d}, \overline{d}]$. Solving the CH-model for $n \to \infty$, we obtain the following main prediction:[12]

Proposition 2 [Cognitive Hierarchy]. *In the AB auction, in the limit, the (expected) distance of a firm's bid from* $A2$ *is strictly decreasing in her level of sophistication. In the ABL auction, in the limit, the (expected) distance of a firm's bid from* $A3$ *is strictly decreasing in her level of sophistication.*[13]

The previous result is pretty intuitive. Consider the AB auction: denote by $A2_j$ and $A1_j$ the random variables corresponding to the averages $A2$ (the winning threshold) and $A1$, conditional on the fact that firms' levels range from 0 to j (and let $\overline{A2}_j$ and $\overline{A1}_j$ be their expectations). A level-k firm believes that the other firms range from level 0 to $(k-1)$; hence, to formulate her optimal bid δ_k, she computes the probability distribution of $A2_{k-1}$ (which, in turn, depends on $A1_{k-1}$): the intuition suggests that, typically, δ_k will be "close" (from below) to $\overline{A2}_{k-1}$. Now, if δ_k is indeed close to $\overline{A2}_{k-1}$ and given that, by construction, $\overline{A2}_{k-1}$ is larger than $\overline{A1}_{k-1}$, then $A2_k$ and $A1_k$, that incorporate also level-k firms' bids δ_k, will be larger than $A2_{k-1}$ and $A1_{k-1}$: thus, δ_{k+1}, which is close to $\overline{A2}_k$, will be larger than δ_k. And so on. Hence, firms' bids will be strictly increasing in their level of sophistication; or, equivalently, the distance from $\overline{A2}$ – the expected value of the *true* winning threshold, i.e. the one computed on the basis of the *true* distribution of levels in the population of firms[14] –

[11] Chang et al. (2015), who experimentally studied a simpler average bid auction, focused on the case of three bidders, because "the explicit formulation of a bidder's winning probability for a general n-bidder game is difficult, if not impossible, to obtain for $n \geq 4$"(Chang et al., 2015, page 1241).

[12] For the proofs, see Appendix B. Notice that, apart from the assumption of full support for G_0, we do not make any hypothesis on the shape of P and of G_0.

[13] The second statement holds only under a very mild assumption on the distribution of level-0 firms' bids. In some (exceptional) cases, it is possible that the (expected) distance of a firm's bid from $A3$ is constant in her level of sophistication.

[14] Thus, $\overline{A2}$ is nothing but $\overline{A2}_{\overline{k}+1}$, where $\overline{k}$ is the maximum sophistication level in the population of firms.

will be strictly decreasing. Proposition 2 states that the intuition described above is correct asymptotically. In fact, when n grows to infinity: *(i)* the impact of any firm's bid on $A2$ is negligible; *(ii)* the probability distribution of $A2_k$ converges to its expected value $\overline{A2}_k$; *(iii)* there is a very high probability that at least one level-0 firm bids close to $\overline{A2}_k$. Hence, the optimal bid of a level-k firm converges to $\overline{A2}_k$. However, we believe that this result holds also for generic values of n, at least under standard assumptions on G_0.[15]

The same intuition applies also to the ABL auction: here a level-k firm, upon choosing her optimal bid, must compute the probability distribution of $A3_{k-1}$ and $A2_{k-1}$, i.e. the lower and upper bounds of the interval from which the winning threshold will randomly be drawn. Given that any number in this interval has the same probability to be drawn, we expect a level-k firm to make a bid closer to the expected value (from her viewpoint) of the lower bound: $\overline{A3}_{k-1}$. Then, we can apply the same reasoning described above. In this case, however, depending on the distribution of level-0 firms' bids, bids can be strictly increasing (this occurs when $\overline{A3}_0 > \overline{A1}_0$) or strictly decreasing (this occurs when $\overline{A3}_0 < \overline{A1}_0$) in the level of sophistication of firms. We expect the latter to be the canonical case: for example, if G_0 is symmetric, then $\overline{A3}_0$ is necessarily lower than $\overline{A1}_0$. In both cases, anyway, the distance from the *true* expected value of $A3$ is strictly decreasing.

4 Empirical analysis

The previous section has shown that, in our context, the CH model implies that, if firms have different sophistication levels, this should reflect in different bids by them. An heterogeneity in bidding behavior is indeed apparent in our data (see Figure 2); however, deeper statistical analysis is needed to asses whether such heterogeneity is related to firms' sophistication in the direction prescribed by the CH model, namely that more sophisticated firms bid closer to the (expected) value of $A2$ in AB, of $A3$ in ABL (Proposition 2). However, to empirically test Proposition 2, we first need to measure firms' sophistication level.

4.1 A measure of firms' sophistication

In accordance with the fundamental idea of the CH model, a measure of firms' (i.e. managers') sophistication should capture their ability of thinking strategically in interactive situations. Needless to say, measuring this ability is a complicated task. One possibility would be to rely on some instruments, like some measure of ability, education or professional achievements of firms' managers.[16] We refrain from following this strategy for two reasons. First, we lack information on firms' managers or other firms' characteristics that may proxy strategic ability. Second, and most importantly, although innate and/or previously acquired skills certainly matter, the intuition and the literature suggest that individuals can learn to think strategically

[15]We performed a series of numerical simulations for different values of n. The results, reported in Appendix C, are consistent with the asymptotic result of Proposition 2.

[16]In experimental beauty-contest games, Burnham et al. (2009) and Brañas-Garza et al. (2012) showed that subjects who obtained higher scores in a psychometric test of cognitive ability performed better, while Chen et al. (2014) showed that subjects' working memory capacity is positively related to their CH level. Goldfarb and Xiao (2011), who fitted a CH model to the entry decisions by managers in the US local telephone markets, uncovered a significant positive relationship between managers' strategic ability on the one hand, their education and experience as CEOs on the other.

in games as they play over and over again.[17] Hence, in a context like ours in which we observe the same firms bidding repeatedly, an out-of-sample static measure of sophistication would miss this learning component. Instead, we need a measure of sophistication that can change dynamically within the sample. To this end, we follow a completely different approach: for each auction in our sample, we measure a firm's sophistication by the relative distance of that firm's bids from $A2$ in AB, from $A3$ in ABL, in the preceding auctions of that format which she participated in. The idea is that, if the CH model is indeed a good model of firms' bidding behavior, then we can "invert" the prediction of Proposition 2 and take the distance from $A2$ or $A3$ as an outcome-based measure of her capacity of thinking strategically.[18]

Specifically, the sophistication index of firm i at the moment in which she participates in the AB auction j is computed as:

$$BidderSoph_{ij}^{\mathrm{AB}} = \sum_{k \,\in\, \mathrm{AB}_{ij}} \left(1 - \frac{\Delta_{ik} - \Delta_k^{min}}{\Delta_k^{max} - \Delta_k^{min}} \right) \tag{1}$$

where AB_{ij} is the set of past AB auctions that took place before auction j which firm i participated in, Δ_{ik} is the distance of firm i's bid from the realized value of $A2$ in auction k and Δ_k^{min} and Δ_k^{max} are the distances from $A2$ of the closest and furthest bid submitted in auction k. Notice that each term in the summation in (1) is between 0 and 1 and takes value 0 (1) if firm i's bid was the furthest from (closest to) $A2$ in that auction. $BidderSoph_{ij}^{\mathrm{ABL}}$ – the sophistication index of firm i at the moment in which she participates in the ABL auction j – is defined similarly, with the caveat that, in this case, the Δ's are distances from $A3$.

The sophistication index (1) is clearly dynamic, as it changes from one auction to the next depending on the outcome of the last auction. Hence, it allows a firm's level of sophistication to increase or decrease relative to the others. The idea is that firms may learn to think strategically as they gain experience in the auction mechanism. Similarly, a firm may lose positions in the sophistication ranking if she does not take into account that other firms may become better strategic thinkers through learning. Notice that our sophistication index is auction format-specific, in the sense that participations to AB do not contribute to the firm's sophistication index when she bids in ABL. The idea is that what matters is not experience per se, but experience in that particular strategic situations.[19]

Figure 3 shows the distributions of bids in AB (left panel) and ABL (right panel) for highly and lowly sophisticates firms (i.e. firms with sophistication index above the 90th percentile vs. those with sophistication index below the 10th percentile). These graphs point out an heterogeneity in bidding behavior which goes in the direction suggested by Proposition 2. In particular: *(i)* bids by highly sophisticated firms are more concentrated than those by lowly sophisticated ones; *(ii)* highly sophisticated firms' bids are concentrated in the right tail of

[17] Gill and Prowse (2014), studying how cognitive ability (and character skills) influence learning to play equilibrium in a repeated p-beauty contest game, find that more cognitively able subjects make choice closer to equilibrium, converge more frequently to equilibrium play and earn more even as behavior approaches the equilibrium prediction.

[18] In some sense, we are adopting an approach similar to revealed preference: we derive the determinants of behavior by induction from the behavior itself.

[19] The use of a measure of a firm's sophistication that weighs all previous participations not only allows to take into account that a firm may learn to think strategically through experience, but also gives more robustness to the results: in a single auction, a firm may bid close to optimality by chance; in a series of auctions, a firm systematically bids close to optimality only if she is a good strategic thinker. By using a cumulative measure, the impact of lucky bids results downsized.

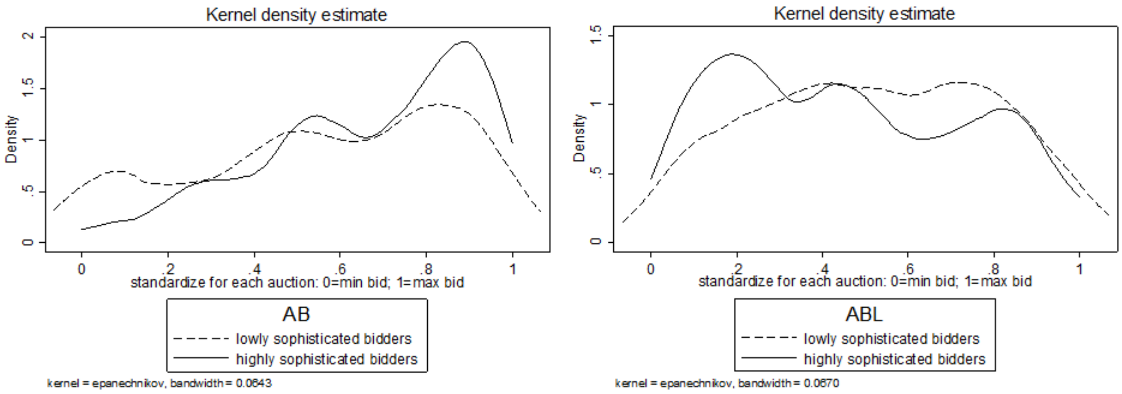

Figure 3 – Discounts in AB (left panel) and ABL (right panel): Kernel density estimation by highly and lowly sophisticated firms.

the distribution of bids in AB and in the left tail in ABL.[20]

Given that our sample contains (by far) more AB than ABL auctions, the average value of the sophistication index is, by construction, larger in AB. Looking at the whole sample, the frequency distribution of the index displays a higher concentration on low values and fewer observations on high values in both formats, indicating that there are few highly sophisticated (firms that make good bids in a large number of auctions). However, looking only at late years, the distribution tends to be smoother, suggesting that, after some time, more and more firms reach relatively high levels of their sophistication.[21]

4.2 Estimated equation

Given the measure of firms' sophistication just illustrated, we can now introduce and estimate a reduced form model to test Proposition 2. The model is the following (we omit the dependence on the auction format, but it is intended that there is one such equation for each format):

$$\log |Distance_{ij}| = \alpha + \beta \log(BidderSoph_{ij}) + \gamma F_i + \sigma FP_{ij} + \theta P_j + \epsilon_{ij}. \qquad (2)$$

In (2), the dependent variable, $\log |Distance_{ij}|$, is the logarithm of the difference (in absolute value) between firm i's bid in auction j and the realized value of $A2$ [$A3$] in that AB [ABL] auction. $BidderSoph_{ij}$ is firm i's sophistication index at the moment of participation in auction j, as defined by (1).[22] F_i represents a set of characteristics of firm i that do not vary

[20] A two-sample Kolmogorov-Smirnov test confirms that the distribution of bids by highly and lowly sophisticated firms are statistically different in both auction formats.

[21] See Figures D1 and D2 in Appendix D. See also Figure D3 in Appendix D, which shows the distribution of the sophistication index in AB auctions by firm size and period. Interestingly, distributions have similar shapes across firm sizes, although small firms reach lower levels of sophistication by the end of the period.

[22] We adopt a log-log model for two reasons: first, the theoretical analysis suggests a non-linear relationship between sophistication and distance from $A2$ or $A3$; second, a log-log model allows to interpret the coefficient attached to the sophistication index as an elasticity. Notice that observations with a sophistication index equal to zero are excluded as we require firms to show their bidding ability at least once before entering in the analysis. Our main results do not change when we include firms with a sophistication index equal to zero

over time, including proxies for size and location.[23] FP_{ij} is a set of firms' characteristics that can vary for each auction. This set includes the backlog of works (i.e. the number of pending public procurement projects a firm has at the moment she bids in auction j; it is a proxy for capacity constraints) and the subcontracting position. P_j is a set of variables to control for the characteristics of the auction, such as dimension and complexity of the auctioned work,[24] and the timing of the auction (i.e. year dummy variables to adjust for temporal shocks to the firms and the CA).

To reduce concerns about omitted variable problems affecting the relationship between the sophistication index (which is built upon firms' past behavior in auctions of that format) and firms' current behavior (e.g., some factors that may influence both the past and current performance of firms are not controlled for), in some specifications we also included firm-fixed effects to adjust for firm-specific characteristics. This enables us to focus on the within firm variation in the sophistication status. However, some relevant characteristics of the firms could vary over the time horizon of our analysis. For this reason, in some specifications we also add firm-year-fixed effects, thus controlling for those characteristics which can vary over time, like, for instance, productivity, financial position, and management skills. This set of fixed effects represents a suitable substitute for the inclusion of firm-year control variables that can be recalled from balance-sheets.[25]

4.3 Description of the sample

Table 1 shows summary statistics of the sample used in our estimations, broken down by auction format.[26] The sample of 232 AB auctions includes 8,927 bids made by 514 different firms; the sample of 28 ABL auctions includes 1,501 bids made by 319 different firms.[27]

(using log(1 + *BidderSoph*) as regressor) or when we adopt a log-linear specification (see Appendix D, Table D1).

[23]Because we do not have data on firms' employees or total assets, we construct proxies for firms' size based on the type of business entity: *Small* = one-man businesses, limited and ordinary partnerships; *Medium* = limited liability companies; *Large + cooperatives* = public corporations and cooperatives. The use of these proxies is motivated by the evidence of a positive correlation between the type of business entity and the size of Italian firms (see Moretti and Valbonesi, 2015, and Coviello et al., 2016). To proxy firms' location, we take the geographical distance between Aosta (i.e. the seat of the CA) and the chief town of the province where the firm has her headquarter (we assign a distance of 30 kilometers to firms located in Valle d'Aosta, see Moretti and Valbonesi, 2015). This variable is also a proxy for firms' costs (at least for the roadwork contracts, which represent the majority of our auctions). We thank an anonymous referee for this comment.

[24]In the procurement literature, the complexity of a project is usually proxied by the its value or the auction's reserve price, the expected contractual duration of works, dummies for the categories of works included in the project. We use all these proxies in our estimation. Notice that the contract value is determined by an engineer employed by the CA, according to a price list that enumerates the standardized costs for each type of work (see Decarolis, 2014, and Coviello and Marinello, 2014, for details on how the CA determines this price). Similarly, the expected duration of the work is computed by a CA's engineer and is stated in the call for tender.

[25]The presence of small and micro firms in our dataset makes it impossible for us to use balance-sheet information to construct additional controls or instrumental variables as these firms are typically under-represented in firm-level balance-sheet-based databases. In fact, a large number of firms is unmatched when we try to merge our dataset with the Bureau Van Dijk's Aida database of Italian firms.

[26]These descriptive statistics refer to the sample used for the empirical analysis proposed in this section. The original sample was slightly larger (267 auctions). The sophistication index is computed on this larger sample to avoid being influenced by partial observations. However, due to missing values in some control variables, our regression analyses are based on the restricted sample.

[27]We thus rely on an unbalanced panel of firms. In the AB sample, on average, a firm participated in 18.4 auctions: 17.32% of the firms participated in 2 auctions, 9.92% in 3 auctions, 6.42% in 4 auctions, 26.45% in

Table 1 – Estimated sample.

	AB			ABL		
	Obs.	Mean	SD	Obs.	Mean	SD
Firm-auction level:						
$\lvert Distance \rvert$	8927	1.555	2.437	1501	1.4333	1.990
$BidderSoph$	8927	24.789	24.699	1501	4.192	3.954
Backlog	8927	2.857	7.143	1501	2.005	4.963
Optional Subcontracting	8927	0.871	0.336	1501	0.817	0.387
Auction level:						
Reserve price (euro)	232	1,120,365	895,493.5	28	1,109,662	681,532.5
Expected duration (days)	232	301.431	166.172	28	402.857	177.353
No. Bidders	232	53.216	28.613	28	82.857	41.662
Building construction	232	0.134	0.341	28	0.107	0.315
Road works	232	0.388	0.488	28	0.286	0.460
Hydraulic works	232	0.306	0.462	28	0.321	0.476
Firm level:						
Small size	514	0.158	0.365	319	0.160	0.367
Medium size	514	0.589	0.492	319	0.624	0.485
Large size	514	0.253	0.435	319	0.216	0.412
Distance firm-CA (km)	514	449.463	448.476	319	344.765	391.891

The average auction's reserve price is around 1.1 million euros in both types of auctions and the average number of participating firms per auction is about 53 in AB and 83 in ABL. Most of the auctions concern road works (38.8% of the AB auctions; 28.6% of the ABL auctions), hydraulic works (30.6% of the AB auctions; 32.1% of the ABL auctions) and building construction (13.4% of the AB auctions; 10.7% of the ABL auctions). The two samples are pretty homogeneous also looking at firms' other characteristics, such as size (about 84% are medium or large firms), backlog (upon bidding, firms have, on average, between 2 and 3 pending public procurement projects) and subcontracting position (on average, more than 80% of the firms have the option to subcontract part of the work).

4.4 Main estimation result

In Table 2, columns (1)-(3), we present our estimation results for the sample of AB auctions. In all specifications, the negative and statistically significant coefficient of $\log(BidderSoph)$ shows that firms with a higher sophistication index tend to bid closer to $A2$, thus supporting the prediction contained in Proposition 2. This result is robust to the inclusion of covariates at auction-, firm- and firm-auction-level (column (1)), firm-fixed effects (column (2)), and firm-year-fixed effects (column (3)). The inclusion of fixed effects allows us to explore the within firm (or firm-year) variability and to reduce selection-bias and omitted variable problems: in particular, firm-fixed effects can capture the role of any idyosincratic (either innate or previously acquired) component of sophistication peculiar to that firm/manager, while firm-year-fixed effects can capture this same component also for firms whose management changed

5-10 auctions, 28.74% in 11-50 auctions, 9.85% in 50-100 auctions, and 1.14% in more than 100 auctions. In the ABL sample, on average, a firm participated in 5.7 auctions: 28.53% of the firms participated in 2 auctions, 19.44% in 3 auctions, 10.66% in 4 auctions, 26.65% in 5-10 auctions, and 14.74% in more than 10 auctions.

during the sample period.[28]

Table 2 – Main results.

Dependent variable	log $\lvert Distance \rvert$					
Auction format	AB	AB	AB	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)
log($BidderSoph$)	-0.171***	-0.170***	-0.243***	-0.386***	-0.468***	-0.522***
	(0.022)	(0.038)	(0.041)	(0.042)	(0.063)	(0.073)
Auction/project controls	YES	YES	YES	YES	YES	YES
Firm controls	YES	NO	NO	YES	NO	NO
Firm-FE	NO	YES	NO	NO	YES	NO
Firm-year-FE	NO	NO	YES	NO	NO	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES
Observations	8,927	8,838	8,573	1,501	1,410	1,266
R-squared	0.192	0.266	0.352	0.279	0.459	0.498

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table 2 also reports the results of the regressions for the sample of ABL auctions. Looking at this sample is illuminating because it allows us not only to test the role of firms' sophistication in a different average bid format, but also to address potential measurement error problems. In fact, while the AB format has long and widely been adopted in Italy to award public works, the ABL format was introduced in 2006 and only in Valle d'Aosta. Hence, while the sophistication index computed for the AB sample does not take into account that firms may have gained experience (and thus sophistication) by participating in AB auctions issued by other Italian CAs and/or in the past[29], these measurement error concerns are absent in the ABL case. Now, also the results for the ABL sample are consistent with Proposition 2: the relationship between sophistication index and the distance from $A3$ is highly significant and negative. This is true in all the specifications, without fixed effects (column (4)) and with firm- or firm-year-fixed effects (columns (5) and (6)). Interestingly, the estimated coefficient for the sophistication index in the ABL auctions indicate a larger negative effect than in AB. A plausible explanation for this could be related to the measurement error problems just discussed. In fact, in AB auctions we might be underestimating the true level of sophistication of firms, especially the most sophisticated ones, which are also those that are most likely to bid also outside Valle d'Aosta.[30]

[28]In Appendix D, Tables D1 and D2, we show that our main result is confirmed also when: *(i)* we replace auction covariates with auction-fixed effects; *(ii)* we add the number of bidders as a proxy for the auction's competitive pressure; *(iii)* we estimate an Heckman selection model; *(iv)* the sophistication index is not only auction format- but also work category-specific (i.e. the sophistication acquired in one type of work is irrelevant when that firm bids in an auction for a different type of work).

[29]Anyhow, we believe that the impact of the experience gained outside Valle d'Aosta should be limited because the knowledge of the specificity of each market (first of all, its players) is extremely important. Moreover, the sophistication accumulated in the past (i.e. before year 2000) should be captured by the fixed effects.

[30]We thank an anonymous referee for suggesting this interpretation.

4.5 Learning dynamics

The previous analysis showed that, in line with the prediction obtained from a CH model, there is a stable negative relationship between firms' sophistication index and the distance of their bids from $A2$ or $A3$. However, that analysis does not say much about the dynamics behind this relationship. In particular, do firms learn to think and bid strategically as they participate in more and more auctions? And, if so, what are the determinants and the characteristics of this learning process? Our starting point is the evidence suggested from the kernel density distribution of bids in AB auctions issued during the first (2000) and last (2005) year covered by our dataset. Figure 4 shows that, compared to year 2000, bids in 2005 are generally much more concentrated on the right side of the distribution, thus suggesting that a learning process is most likely to be taking place.

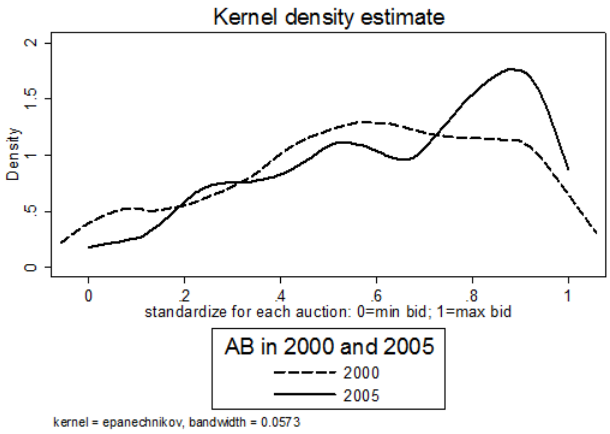

Figure 4 – Discounts in year 2000 and 2005: Kernel density estimation.

To investigate such process more in depth, we decompose firm i's sophistication index at auction j into two components. The first component is simply the number of past participations by firm i in auctions of the same format as j and is meant to capture the pure role of experience; we denote this variable by $PastPart$. The second component is the average performance of firm i in all previous auctions of the same format as j, measured as the average distance of her bid from $A2$ or $A3$. This variable, denoted by $PastPerf$, is intended to proxy the degree at which a firm learns to think and bid strategically from her past performance. Furthermore, through the fixed effects at the firm-year level, we implicitly take into account also a third component: the stock of strategic skills of a firm/manager in any given year, both innate and acquired in the past.

Focusing on the sample of AB auctions, Table 3, column (1), shows the results obtained by estimating a regression model like (2) with firm-year-fixed effects, where the regressor $\log(BidderSoph)$ is replaced by $\log(PastPart)$. The latter has a negative and statistically significant coefficient, showing that experience does play a role: everything else equal, firms that participate more bid better (i.e. closer to $A2$) than those that participate less. In column (2), we add $\log(1 + PastPerf)$ to the model specification: interestingly, its coefficient is positive and significant, while the sign of $\log(PastPart)$ remains negative and statistically significant. This evidence indicates that the main driver of a firm's learning over time seems to be the number of past bids and that, at least in AB auctions, a firm tends to learn more from a poor than from a good past performance, as if a poor performance acted as a stimulus for the firm to improve her strategic reasoning in the future.

To get deeper evidence on the learning process we investigate whether the fact of winning an auction plays a role. To this end, we introduce the number of past wins ($\log(1+PastWins)$) in our model specification. Results for the AB auctions indicate that, similarly to what happens with $PastPerf$, a firm seems to learn more from her failures than from her successes (see Table 3, columns (3) and (4)).[31]

The results for the ABL auctions[32] confirm that the number of past participations positively affect future bidding performance (the coefficient of $\log(PastPart)$ is always negative and significant). However, differently from the AB sample, $\log(1 + PastPerf)$ has a negative sign (indicating that firms learn from their good past performances), which turns out to be nonsignificant when firm-year fixed effects are included. Interestingly, $\log(1 + PastWins)$ in ABL auctions is not statistically significant. This is probably not surprising, as the winner in an ABL auction is determined randomly: hence, the fact of winning produces less information than in the AB format.

To sum up, the learning process seems to be mainly driven by pure experience: firms improve their capacity of bidding strategically as they participate to more and more auctions. Instead, the performance in past auctions has little explanatory power: once firms' idiosyncratic skills are controlled for (through the fixed effects), a better average performance or a larger number of wins in the past either has no impact on bidding behavior in future auctions (in ABL) or worsens the capacity of bidding optimally (AB), as if it acted as a negative stimulus to improve strategic thinking. This analysis also allows us to exclude that the relationship we uncovered between the sophistication index and bidding performance is simply due to inertia in bidding over time: firms learn how to bid strategically through experience, so those firms that participate more can eventually become more sophisticated than those that participate less, even if the latter were better strategic thinkers initially.

Given the peculiarity of our dataset characterized by a change in the auction format, and given the results about the learning dynamics just illustrated, it is interesting to understand whether firms in ABL auctions drew lessons from what they learned in the AB auctions (in our sample, 240 firms participated both in AB and ABL auctions). Recall that our sophistication index was, by construction, auction format-specific, in the sense that participations to AB do not contribute to firms' sophistication when they bid in ABL. Hence, answering this question is an indirect way to test how restrictive this assumption is. To this end, we focus on the sample of ABL auctions and introduce in our model (2) an additional variable, $BidderSophAB$, representing, for each firm, the level of the sophistication index at the end of the period of AB auctions. Table 3, column (5), shows that a higher sophistication index achieved in the AB period is associated with a lower distance from $A3$ in ABL. However, when we re-introduce (in column (6)) the firm's sophistication associated to the ABL auctions ($BidderSoph$), the coefficient of the former index is not statistically different from zero, while the auction-specific one is still negative and statistically significant. This result suggests that what really matters

[31] Notice that, in a specification without firm-year- (or firm-) fixed effects, the interpretation of the coefficient is different: negative coefficients of both $\log(1 + PastPerf)$ and $\log(1 + PastWins)$ indicate that, between firms, those with better past performance tend to consistently bid closer to the reference point than those with worse past performance. See Appendix D, Table D3.

[32] These results are reported Appendix D, Table D4.

Table 3 – Learning dynamics.

Dependent variable	log $\lvert Distance \rvert$							
Auction format	AB	AB	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log($PastPart$)	-0.366***	-0.442***	-0.505***					
	(0.044)	(0.048)	(0.054)					
log(1+$PastPerf$)		5.018***	4.937***					
		(0.732)	(0.729)					
log(1+$PastWins$)			0.624***	0.555***				
			(0.173)	(0.164)				
log($BidderSoph$)				-0.293***		-0.412***		-0.409***
				(0.046)		(0.045)		(0.047)
log($BidderSophAB$)					-0.088**	-0.008		
					(0.039)	(0.042)		
log($PastPartAB$)							-0.063	0.013
							(0.041)	(0.046)
log(1+$PastPerfAB$)							-1.741**	-1.297
							(0.836)	(0.811)
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	NO	NO	NO	NO	YES	YES	YES	YES
Firm-year-FE	YES	YES	YES	YES	NO	NO	NO	NO
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,573	8,573	8,573	8,573	1,356	1,356	1,356	1,356
R-squared	0.354	0.362	0.364	0.353	0.231	0.284	0.234	0.286

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

is the firm's strategic ability acquired in that specific type of auction.

Similar results are obtained when we introduce in the model specification the number of participations ($PastPartAB$) and the average past performance ($PastPerfAB$) in AB auctions (columns (7) and (8)). The coefficients of these two variables are not significant, once we control for the ability acquired by the firm in the ABL auctions.

5 Discussion

The analysis presented in the previous section provides evidence that supports, at least qualitatively, a non-equilibrium model of bidding behavior by firms in average bid auctions: observed deviations from the optimal bid are related to a measure of firms' capacity of bidding strategically, the sophistication index; this relation goes in the direction predicted by a CH model. Therefore, our (continuous) sophistication index proxies the (discrete) CH-level of sophistication by firms. The analysis showed that the relation between sophistication index and bidding behavior is robust to a number of determinants, including auction's, firm's and firm-auction's specific characteristics. Most importantly, the relation holds also when we analyze the ABL format, which is new to the firms and characterized by a stochastic component that makes it more complicated for firms to formulate their bidding strategies.

In this section, we show that our explanation is robust to a number of issues that, potentially, may undermine it. In particular, we discuss: *(i)* whether our sophistication index, which we interpret as a measure of strategic thinking ability, may be actually capturing other firms' structural factors, in particular their competitiveness; *(ii)* the role of potential cartels jointly bidding in the auctions; *(iii)* an instrumental variable approach to deal with endogeneity problems; *(iv)* additional empirical evidence to rule out alternative explanations.

5.1 Strategic ability vs. competitiveness

In standard procurement auctions, the main determinant of a firm's bid is her production cost: more competitive firms can and do make more aggressive offers, outbidding less competitive ones. A natural question that arises in our context is whether our sophistication index, which, in our view, proxies strategic ability, actually captures some structural competitive feature of the firm: productivity, proximity, or, more generally, any element that translates into a cost advantage. Now, notice first that, in our main empirical exercise, we alternatively control for firm's characteristics (such as size and distance between the firm headquarter and the CA), firm-fixed, or firm-year-fixed effects, as well as for the firm's subcontracting position (whether the firm must subcontract part of the work or not) and for the number of pending projects the firm is involved in at the time of bidding (which captures her productive capacity).[33] Most factors that may generate a competitive advantage/disadvantage of one firm over the others are likely to be accounted for by these variables.

However, one might argue that we do not fully control for factors that, similarly to the sophistication index, can vary for each firm within a year or for different types of project. To shed light on these questions, we first estimated a model which includes firm-year-category of work-fixed effects and one with firm-semester-fixed effects: in both models, the coefficient of $\log(BidderSoph)$ in AB auctions is very much in line with that of our main model specification (see Table 4, columns (1) and (2).[34]

[33]Jofre-Bonet and Pesendorfer (2003), using a dataset of a sequence of first-price procurement auctions for highway construction in California, showed that the outcome of one auction may affect bids in successive ones because the winner of a previous auction, having committed some or all of her capacity, may have larger costs when participating in the next auctions (because, for example, it will have to rent additional equipment). In our context, this dynamic component seems to be absent, as the variable *Backlog* – the number of pending public procurement projects that a firm has at the moment of bidding – is never significant. Our intuition is that, even though it is plausible that having committed capacity can affect firms' costs, costs simply do not matter much for (optimal) bidding in average bid auctions.

[34]Our result is robust also to the inclusion of firm-semester-category of work-fixed effects. Due to the smaller sample dimension, results are slightly weaker for the ABL format (see Appendix D, Table D5).

Table 4 – Identification issues.

Auction format	AB	AB	AB	AB	AB+ABL	AB+ABL	AB+ABL	AB+ABL
Dependent variable	$\log \lvert Distance \rvert$	$\log \lvert Distance \rvert$	*Discount*	$\log \lvert Distance \rvert$	$\log(BidderSoph)$	$\log(BidderSoph)$	$\log \lvert Distance \rvert$	$\log \lvert Distance \rvert$
Quantile (th)			90					
					I-stage	I-stage	II-stage	II-stage
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log(BidderSoph)$	-0.243***	-0.137***	-0.003***	-0.228***			-0.196***	-0.237***
	(0.045)	(0.047)	(0.001)	(0.060)			(0.023)	(0.024)
ABL					-0.534**	-0.527**		
					(0.226)	(0.213)		
$\log(BidderSoph00-04)$					0.786***			
					(0.038)			
ABL*$\log(BidderSoph00-04)$					-0.688***	-0.705***		
					(0.062)	(0.060)		
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	NO	NO	YES	NO	YES	NO	YES	NO
Firm-FE	NO	NO	NO	NO	NO	YES	NO	YES
Firm-year-category of work-FE	YES	NO	NO	NO	NO	NO	NO	NO
Firm-semester-FE	NO	YES	NO	NO	NO	NO	NO	NO
Firm-year-FE	NO	NO	NO	YES	NO	NO	NO	NO
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,068	8,208	8,927	3,594	1,888	1,888	1,888	1,888
R-squared	0.413	0.408	0.189	0.377	0.888	0.882	0.102	0.106
Hansen J (p-value)						0.256	0.285	

OLS estimations in columns (1), (2) and (4). Quantile regression in column (3). 2SLS in columns (5)-(8).
Robust standard errors clustered at firm-level in parentheses. Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction (in columns (1)-(4) only). *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Second, we performed an analysis on the *level* of bids. We start from the conjecture that, if our sophistication index were actually capturing some cost advantage, we would expect to observe either a positive or a null relation between the sophistication index and the level of a firm's bid. In fact, irrespective of the auction format and of the submitted bid, if a firm's costs are binding (in the sense that, had her costs been lower, she would have made a different bid), a cost reduction would necessarily result in a higher bid; if, instead, her costs are not binding, a further cost reduction should have no impact on her bid. On the other hand, if our sophistication index captures strategic ability, as we claim, we would expect to observe a negative [positive] relationship between the sophistication index and the bid level for firms making relatively high [low] bids. In fact, in average bid auctions, an improvement in strategic ability would lead firms that bid too high [low] to reduce [increase] their bids towards the optimal bid, which is necessarily somewhere in the interior of the range of submitted bids. Now, focusing on the AB sample, our evidence is indeed in line with the latter interpretation and inconsistent with the one based on cost advantage: the effect of the sophistication index on the level of bids, though nonsignificant overall, is negative and significant for those firms bidding the highest (see the result of the regression at the 90-th quantile in Table 4, column (3)). [35]

5.2 Potential collusion

A very interesting aspect that is worth addressing is related to possible collusive behaviors by firms. In a recent paper, Conley and Decarolis (2016), using a different dataset of AB auctions, argue that this format can be characterized by the presence of colluding firms which drive the winning threshold to let one member of the cartel win. Hence, the evidence on AB auctions would be the result of a cooperative behavior by groups of firms. Instead, our approach is totally different: we cannot exclude the presence of colluding firms, but we provide some evidence that also a fully non-cooperative non-equilibrium behavior might be at work. In this sense, our work suggests a complementary explanation. Nevertheless, we can provide some arguments supporting the robustness of our findings to the presence of collusion. First, it seems reasonable to assert that, if collusion is at work, it is less likely to be present in ABL than in AB auctions: given the inherent uncertainty in the determination of the winning firm, in ABL a successful collusive strategy is much more complicated to be implemented. In this sense, the fact that the coefficient of the sophistication index is significant also in ABL and larger than in AB (see Table 2) is reassuring for our explanation. Second, without any intention to provide evidence on the presence of cartels in the auctions issued by the Regional Government of Valle d'Aosta (note that, unlike in Conley and Decarolis, in our sample no cartels have been detected and sanctioned by the court; this makes more difficult to study possible collusive behavior in our setting), we tried to isolate the influence of *potential* collusive groups. To this end, we identified potential cartels following Conley and Decarolis (2016). In particular, using information on objective links among firms (e.g., firms sharing the same managers, the owners, the location, subcontracting relationship, joint bidding, etc.), the Conley and Decarolis' algorithm indicates that, in our sample, 172 potential groups of firms are present. Once detected these groups, we included in our baseline model specification two

[35] See Appendix D, Tables D6 and D7, for evidence on ABL auctions and further evidence at different quantiles of the distribution of bids. Notice that, in both formats, the coefficients of the sophistication index in the quantile regressions exactly match the pattern predicted by a CH model: they are negative [positive] and statistically significant for high [low] bid levels.

variables measuring, for each firm and each auction: *(i)* the number of (potentially) associated firms bidding in that auction; *(ii)* this number over the total number of firms belonging to that group. Our main results continue to hold after the inclusion of these controls.[36] It must anyway be observed that data display a (weak) positive correlation between our index of sophistication and the number of (potentially) associated firms bidding in that auction, both in AB (0.144) and in ABL (0.188) auctions. Moreover, independent firms – those with no potential associate in the auction – show, on average, lower sophistication than associated firms (the difference is about 35% in AB and 45% in ABL): these numbers may suggest that, if cartels are present, their members are typically more sophisticated than independent firms. Now, to better isolate the role of potential collusion from that of strategic ability, we estimated our baseline model on a restricted sample including only firms with no connection with any other firm participating in that auction (Table 4, column (4)). Again, our main result is confirmed, thus supporting the idea that our explanation actually captures bidding behavior by firms, at least for those that act non-cooperatively.

5.3 Instrumental variables

The results presented so far are robust to the inclusion of rich sets of fixed effects and control variables in the estimated equation. However, to further address potential endogeneity concerns, we also consider an instrumental variable (IV) approach, aimed at capturing some exogenous components behind the sophistication index. Given that information on several relevant firms' characteristics (such as information coming from balance-sheets or managerial and organizational structure of the firm) is not available, we choose an approach that jointly combines out-of-sample information and the exogenous change of format from AB to ABL.

In particular, our identification strategy works as follows. First, we limit our analysis to year 2005 and 2006, the last year of adoption of the AB format and the first year of adoption of the ABL format, respectively. Focusing on such a short period of time, we can reduce omitted variable problems and exploit the 2006 variation in the auction format, which, it is worth recalling, took place only in Valle d'Aosta. Second, we consider three instrumental variables for the sophistication index, namely: *(i)* the dummy variable *ABL* (which takes value equal 1 for ABL and 0 for AB) to capture the average overall change in the firms' sophistication level determined by the change of format; *(ii)* for each firm, the value of her sophistication index at the end of 2004 (*BidderSoph00-04*), a proxy for the firm-fixed level of sophistication (relative to the others) acquired out-of-sample; *(iii)* the interaction between the previous two variables, which accounts for possibly differential effects that the new ABL format had on firms' current sophistication across firms with different levels of past (and out-of-sample) sophistication.

First-stage estimation results, reported in Table 4 column (5), show that the coefficients of the three instruments are statistically different from zero (the Hansen J-test reported in column (8) confirms the validity of the instruments). The signs of the coefficients indicate that: *(i)* the change of format reduced the overall level of bidders' sophistication (since the sophistication index is specific for each auction format); *(ii)* the higher the firm's past (and out-of-sample) sophistication in AB auctions, the higher her level of current sophistication in AB auctions (confirming that experience and past performance have an effect over time within the same auction format); *(iii)* the heterogeneity in sophistication across firms recorded at

[36] Estimation results for this analysis are available in Appendix D, Tables D8 and D9. We really thank Francesco Decarolis for providing us with his codes and data.

the end of 2004 is greatly reduced when the new ABL format has been adopted. The results of the second-stage estimation (column (7)) confirm a negative and significant relationship between the instrumented current level of sophistication of the firm and the distance of her bids from $A2$ in AB and $A3$ in ABL. Notice that these results are robust to the inclusion of firm-fixed effects (Table 4, columns (6) and (8)).[37] Notice also that we control for several auction's characteristics (such as typology, value and duration of the work), thus reducing the possibility that our instruments affect firms' bidding behavior through other channels than sophistication.

5.4 Further evidence

Beyond the main result that more sophisticated firms bid closer to $A2$ in AB, to $A3$ in ABL, there is additional evidence that supports our interpretation, in the sense that it can easily be reconciled within a CH model, much less so within an alternative explanation.[38]

First, notice that firms make relatively lower bids in ABL than in AB: this is clear from our data (see Figure 2); furthermore, if we run a regression on a sample of (min-max rescaled) bids offered both in AB and ABL auctions (taking all the covariates included in equation (2)), the coefficient for the ABL auction dummy is negative and statistically significant.[39] Now, if firms were making equilibrium bids, we should observe the opposite (see Proposition 1); instead, in a CH model, given the behavior of level-0 firms, bids made by firms of higher levels are larger in AB than in ABL, since, by definition, $A2_k$, the (asymptotically) optimal bid by a level-$(k+1)$ firm in AB, is larger than $A3_k$.

Second, data show that not only the *average* distance from $A2$ or $A3$ is decreasing in the firm's sophistication level, but also the *variance* of this distance follows the same pattern. In fact, running a regression where the dependent variable is the standard deviation of the distance of the auction's bids from $A2$ or $A3$, the coefficient of the auction's average sophistication index is negative and significant in both formats. Moreover, in a regression where the dependent variable is the rolling standard deviation of the distance of a firm's latest five bids from $A2$ or $A3$, the coefficient of the firm's rolling average sophistication index is negative and significant, even when firm-fixed effects are controlled for. Now, while there is no particular reason to expect a relationship between variance of bidding and other firm's characteristics (e.g. her production costs), a negative relationship between sophistication index and variance of bidding can be justified within a CH model with at least three arguments. First, in a CH model, level-0 firms bid randomly, while more sophisticated firms bid (essentially) deterministically. Second, if firms make some payoff-sensitive errors,[40] then more sophisticated firms will make more precise bids (with less variance). The intuition is simple: to compute her

[37] In Appendix D, Table D10, we show similar estimation results after enlarging the sample of auctions to the period 2004-2007.

[38] This evidence is collected in Appendix D, Table D11.

[39] Notice that, even if we cannot fully separate in a difference-in-differences style the trend forces from the effects related to the introduction of the (new) ABL auction format in Valle d'Aosta (as we do not observe AB auctions after 2005), evidence based on a larger sample of procurement auctions (data from Coviello et al., 2016) indicate that, for other Italian regions not adopting the ABL auction format, the average winning discount in 2006 is approximately at the same level as in 2005 or it has increased for some Northwestern regions. Given that, between 2005 and 2006, no major changes occurred in in Valle d'Aosta's economy conditions and in the procurement regulation (other than the introduction of ABL auction format), we can assume that, had Valle d'Aosta not introduced the ABL auction, the trend would have followed a nondecreasing path similar to the one observed in the other regions.

[40] See the simulation exercise in Appendix C.

optimal bid, a level-k firm estimates the distribution of the winning threshold on the basis of the behaviors of level-0 to level-$(k-1)$ firms. For higher level firms, this distribution has lower variance, being less and less affected by the random behavior of level-0 firms. Third, as it seems reasonable, more sophisticated firms may be less prone to errors than less sophisticated ones.

Third, data show that the higher the number of participating firms in an auction, the higher [the lower] the value of $A2$ [$A3$] in AB [ABL]. This can be seen by looking at descriptive statistics (the simple correlation between the number of participants and $A2$ [$A3$] in AB [ABL] is positive [negative]) and by estimating a regression with $A2$ or $A3$ as the dependent variable and the number of participants and other auction-level controls as regressors. This evidence is consistent with a CH model of bidding behavior: in the viewpoint of a sophisticated firm, who determines her bid on the basis of her own estimates of the distribution of $A2$ or $A3$, a lower number of participants increases the variance of this distribution. Since the winning bid is the one that gets closer to the winning threshold from *below* in AB and from *above* in ABL, a sophisticated firm will find it optimal to bid cautiously: in AB, a little below the expected value of $A2$, in ABL a little above the expected value of $A3$. As the number of participants increases, the variance of $A2$ or $A3$ will reduce, and sophisticated firms can be more confident in bidding very close to their expected values.

6 Conclusion

This paper studies bidding behavior by firms in two versions of average bid auctions adopted by a regional contracting authority in Italy for the procurement of public works. Our empirical evidence is inconsistent with Nash equilibrium behavior, a situation in which all firms are playing their best response to other firms' bids. We proposed an interpretation based on a non-equilibrium CH model of bidding behavior: more sophisticated firms, being better strategic thinkers, are able to get more accurate beliefs on the behavior of other firms and bid closer to optimality. Introducing a dynamic measure of sophistication which takes into account the goodness of a firm's bids in all past auctions of the same format in our sample, we showed that the main prediction of the CH model is consistent with our data. We also investigated whether and how firms learn to think and bid strategically through experience, showing that both the number of participations and the average past performance explain firms' performance in future auctions and that this learning process has a convergence path. We finally discussed endogeneity issues and alternative explanations for the observed behavior in the investigated setting (in particular, collusion) by providing some robustness checks on our empirical analysis.

The general lesson that can be learned from our work is twofold. First, our paper provides evidence that departures from equilibrium can be significant and persistent also in the field, and even when players are firms and stakes are large. This is likely to be the case especially in complex environments, for example when the profit of one firm depends in a non-obvious way on the decisions made by all other firms. In this case, the equilibrium requirement that all players (firms) have correct beliefs seems particularly, and maybe unrealistically, demanding.

Second, the CH model and, more generally, models that take into account that strategic thinking may be heterogeneously distributed across economic agents, represent a valid alternative to equilibrium models in complex games, even outside the lab. Moreover, while in the experimental literature on CH models, convergence to equilibrium is usually observed, so that CH models mainly capture initial responses to games, our evidence shows that, even if convergence is taking place, it does so at a slow rate. This is not surprising once one takes into account that, unlike in the lab, in the field the set of players can change continuously over time: new unexperienced firms may enter the stage, others may exit. Hence, even though experience may help firms become more sophisticated, the entry of unsophisticated firms may slow down the process of convergence.

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Appendix

A Proof of Proposition 1

Before proving Proposition 1, for convenience, we restate (more in detail) the assumptions of the model and we prove two lemmas that are used to prove the Proposition.

The model. A single contract is auctioned off through an AB or ABL auction. There are n risk neutral firms that participate in the auction. Firm i's cost of completing the job is given by $c_i(x_1, x_2, \ldots, x_n)$, where x_i is a cost signal privately observed by firm i (x_i is the *type* of firm i). We sometimes write compactly x_{-i} to denote the vector of signals of all firms other than i. We assume that firm i's cost is separable in her own and other firms' signals and linear in x_i, that $c_i(x_i, x_{-i}) = a_i x_i + \Gamma_i(x_{-i})$, with $a_i > 0$, $\partial\Gamma_i/\partial x_j \geq 0$, for all $i, j \neq i$. Firm i's signal is distributed according to a cumulative distribution function $F_i(x_i)$, with full support $[\underline{x}_i, \overline{x}_i]$ and density $f_i(x_i)$. Signals are independent. The cost functions as well as the signals' distributions are common knowledge. Firms submit sealed bids formulated in terms of percentage discounts over the reserve price R. We restrict our attention to situations in which all firms always participate in the auction, because they find it worthwhile to do so. Without this restriction, one should take into account the possibility that a firm may decide not to participate for some cost signal realizations: this would complicate the analysis but would not change the results qualitatively. Moreover, this restrictions rules out the possibility of non serious bids.

Now, let $d_i \in [0, 1]$ denote firm i's bid (discount). The expected profit of firm i, type x_i, when she participates and bids d_i and the other firms follow the strategies δ_{-i} is:

$$\pi_i(x_i, d_i, \delta_{-i}) = [(1 - d_i)R - C_i(x_i, d_i, \delta_{-i})] \ \mathrm{PW}_i(d_i, \delta_{-i}),$$

where $\mathrm{PW}_i(d_i, \delta_{-i})$ is the probability that firm i wins when she bids d_i and the other firms follow the strategies δ_{-i}, and $C_i(x_i, d_i, \delta_{-i})$ is the expected cost of firm i when her signal is x_i, she bids d_i and the other firms follow the strategies δ_{-i}, conditional on the fact that i wins the auction. In symbols,

$$C_i(x_i, d_i, \delta_{-i}) = a_i x_i + E_{-i}\left[\Gamma_i(x_{-i}) \,|\, i \text{ wins when strategies are } (d_i, \delta_{-i})\right].$$

In the AB auction, the winning bid is the bid closest *from below* to $A2$. In the ABL auction, the winning bid is the bid closest *from above* to W, provided that this bid does not exceed $A2$. If no bid satisfies this requirement, the winning bid will be the one equal, if there is one, or closest from below to W. In both auctions, if all firms submit the same bid, the contract is assigned randomly. Similarly, if two or more firms make the same winning bid, the winner is chosen randomly among them.

We first show that, for both auctions, whatever the strategies of the others, a firm has always the possibility of placing a bid that gives her a strictly positive probability of winning the auction. Since this result is pretty intuitive, we omit the proof.

LEMMA 1. *Consider firm i and denote by δ_{-i} the bidding strategies of the other firms. Then, for any δ_{-i}, there exists d_i such that* $\mathrm{PW}_i(d_i, \delta_{-i}) > 0$.

This result, together with the restriction of full participation, implies that, in equilibrium, all firms will have a strictly positive probability of winning the auction.

We now show that, for both auctions, equilibrium bids are monotone.

LEMMA 2. *Let $\delta = (\delta_1, \delta_2, \ldots, \delta_n)$ be a (Bayes-) Nash equilibrium of either auction formats. Then, for all i, $\delta_i(x_i)$ is weakly decreasing.*

Proof. Consider firm i, and let $d_i = \delta_i(x_i)$, $d_i' = \delta_i(x_i')$, be her equilibrium bids when her signals are x_i and x_i', respectively, with $x_i < x_i'$. Notice first that, in equilibrium, the probability of winning the auction must be weakly decreasing in types. In fact, since d_i and d_i' are equilibrium bids, it must be true that:

$$[(1-d_i)R - C_i(x_i, d_i, \delta_{-i})]\ \mathrm{PW}_i(d_i, \delta_{-i}) \geq \left[(1-d_i')R - C_i(x_i, d_i', \delta_{-i})\right]\ \mathrm{PW}_i(d_i', \delta_{-i}), \tag{3}$$

and that

$$\left[(1-d_i')R - C_i(x_i', d_i', \delta_{-i})\right]\ \mathrm{PW}_i(d_i', \delta_{-i}) \geq \left[(1-d_i)R - C_i(x_i', d_i, \delta_{-i})\right]\ \mathrm{PW}_i(d_i, \delta_{-i}). \tag{4}$$

Summing them up, we obtain

$$\left[C_i(x_i', d_i, \delta_{-i}) - C_i(x_i, d_i, \delta_{-i})\right] \mathrm{PW}_i(d_i, \delta_{-i}) \geq \left[C_i(x_i', d_i', \delta_{-i}) - C_i(x_i, d_i', \delta_{-i})\right] \mathrm{PW}_i(d_i', \delta_{-i}).$$

Notice that $C_i(x_i', d, \delta_{-i}) - C_i(x_i, d, \delta_{-i}) = a_i(x_i' - x_i) > 0$, for all d. Hence, we obtain

$$\mathrm{PW}_i(d_i, \delta_{-i}) \geq \mathrm{PW}_i(d_i', \delta_{-i}),$$

, in equilibrium, the probability of winning the auction is weakly decreasing in types.

We now show that the equilibrium bidding function $\delta_i(x_i)$ must be weakly decreasing. Now, suppose, by contradiction, that there exists x_i, x_i', with $x_i < x_i'$ and $d_i < d_i'$. Notice that, because, in equilibrium, $\mathrm{PW}_i(d_i, \delta_{-i}) > 0$ and $\mathrm{PW}_i(d_i', \delta_{-i}) > 0$, the LHS and the RHS of (3) are strictly positive and the LHS of (4) is weakly positive. Hence, multiplying (3) by (4), we get

$$\begin{gathered}
[(1-d_i)R - C_i(x_i, d_i, \delta_{-i})]\left[(1-d_i')R - C_i(x_i', d_i', \delta_{-i})\right] \geq \\
\left[(1-d_i')R - C_i(x_i, d_i', \delta_{-i})\right]\left[(1-d_i)R - C_i(x_i', d_i, \delta_{-i})\right],
\end{gathered}$$

and, after some manipulation,

$$\begin{gathered}
R\left[1 - d_i' + C_i(x_i', d_i', \delta_{-i})\right]\left[C_i(x_i', d_i, \delta_{-i}) - C_i(x_i, d_i, \delta_{-i})\right] \geq \\
R\left[1 - d_i + C_i(x_i', d_i, \delta_{-i})\right]\left[C_i(x_i', d_i', \delta_{-i}) - C_i(x_i, d_i', \delta_{-i})\right].
\end{gathered}$$

Now, since $C_i(x_i', d, \delta_{-i}) - C_i(x_i, d, \delta_{-i}) = a_i(x_i' - x_i) > 0$, for all d, the inequality above reduces to

$$C_i(x_i', d_i', \delta_{-i}) - C_i(x_i', d_i, \delta_{-i}) \geq d_i' - d_i > 0.$$

Hence, we get $C_i(x_i', d_i', \delta_{-i}) > C_i(x_i', d_i, \delta_{-i})$; but this implies

$$\left[(1-d_i')R - C_i(x_i', d_i', \delta_{-i})\right]\ \mathrm{PW}_i(d_i', \delta_{-i}) < \left[(1-d_i)R - C_i(x_i', d_i, \delta_{-i})\right]\ \mathrm{PW}_i(d_i, \delta_{-i}),$$

which contradicts (4).

From the monotonicity property above, we can derive more precise predictions on the (Bayes-) Nash equilibria in the two formats. Let's start with the AB auction.

Proposition 1-*(i). In the AB auction, there is a unique equilibrium in which all firms submit a* 0*-discount (irrespective of their signals), , for all* i*,* $\delta_i(x_i) = 0$*, for all* x_i*.*

Proof. The proof proceeds in three steps.

Step 1. In equilibrium, for all i, there must exist $\hat{x}_i > \underline{x}_i$ such that $\delta_i(x_i) = \bar{d}$, for all $x_i \in [\underline{x}_i, \hat{x}_i)$, there must be a strictly positive probability that all firms make the same discount $\bar{d}$ (where $\bar{d}$ is the largest conceivable discount in equilibrium, see Lemma 2). Suppose not. Let $\bar{d}_i = \max_{x_i} \delta_i(x_i)$ be the largest bid of firm i (from Lemma 2, we know that $\bar{d}_i = \delta_i(\underline{x}_i)$) and let $\bar{d} = \max_i \bar{d}_i$ be the maximum conceivable bid in equilibrium. Notice that a firm that bids $\bar{d}$ can win if and only if all other firms bid $\bar{d}$. However, under our hypothesis, there exists at least one firm that, with probability one, bids less than $\bar{d}$. Hence, at least one of the firms that bid $\bar{d}$ has a zero probability of winning the auction, but this cannot occur in equilibrium. Hence, we have reached a contradiction.

Step 2. $\bar{d} = 0$. To see this, notice that a firm bidding $\bar{d}$ wins if and only if all other firms bid $\bar{d}$ as well, and in this case every firm will win with probability $1/n$. However, a downward deviation would be profitable in any case: by making a lower bid, any firm will win with probability one when all other participating firms bid $\bar{d}$ (moreover, with a lower discount). The incentive to make a lower bid does not bite only when a lower bid is not allowed, only when $\bar{d} = 0$.

Step 3. For all i, $\hat{x}_i = \overline{x}_i$. This is an immediate consequence of the fact that equilibrium bidding functions are weakly decreasing.

Consider now the ABL auction. In this case there is a multiplicity of equilibria. This discrepancy with respect to the AB format is not much due to the different way in which the winning threshold is computed but rather to the fact that in ABL the winning bid is the one closest from above (rather than below) to the winning threshold, provided this bid does not exceed $A2$.

Proposition 1-*(ii). In the ABL auction, there exists a continuum of equilibria in which all firms make the same discount* d *(irrespective of their signals), for all* i*,* $\delta_i(x_i) = d$ *for all* x_i*, where* d *is such that* $\pi_i(x_i, d, \delta_{-i}) > 0$ *for all* i *and for all* x_i*.*

Proof. If all firms make the same bid d, whatever their signal is, every firm will have a $1/n$ chance of winning. If firm i (of any type) makes a bid larger than d, then $A2$ will necessarily be equal to d and firm i will have a zero probability of winning as her bid exceeds $A2$. If instead firm i (of any type) makes a bid below d, then W will necessarily be equal to d and the winner will be one of the other firms. Again, the probability of winning of firm i will fall to zero. Therefore, d is the only bid that guarantees a strictly positive probability of winning.

Beyond the flat equilibria described above, the ABL auction may possibly have other equilibria. In any case, these equilibria display a very large degree of pooling on the maximum discount. The next propositions formalizes this idea.

Proposition 1-*(iii)* - First statement. *Consider any equilibrium of the ABL auction: let* $\bar{d}$ *denote the highest conceivable bid in equilibrium,* $\bar{d} = \max_i \delta_i(\underline{x}_i)$*; let* K *be the set of firms that bid* $\bar{d}$ *with strictly positive probability and let* k *denote the cardinality of* K*. Then, in any equilibrium,* $k \geq n - \tilde{n}$*.*

Proof. The proof proceeds by showing that if $k < n - \tilde{n}$, any firm $\in K$ has a profitable (downward) deviation.

- $k \leq \tilde{n}$. In this case, any firm $i \in K$ that bids $\bar{d}$ would have a zero probability of winning the auction ($A2$ will necessarily be lower than $\bar{d}$, hence $\bar{d}$ cannot be a winning bid); but this cannot occur in equilibrium.

- $\tilde{n} < k < n - \tilde{n}$. Consider any firm $i \in K$ with signal $\underline{x}_i$. This firm bids $\bar{d}$ and can win the auction if and only if $A2 = \bar{d}$ and the winning threshold W is greater than or equal to the largest bid not equal to (lower than) $\bar{d}$. If this occurs, the winner will be chosen randomly from those firms that bid $\bar{d}$. Hence, the expected profit of firm i, type $\underline{x}_i$ is

$$\pi_i(\underline{x}_i, \bar{d}, \delta_{-i}) =$$

$$\sum_{j=\tilde{n}}^{k-1} \frac{(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)}{j+1} \Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j),$$

where J denotes the number of firms in K, beyond firm i, that do bid $\bar{d}$.

Consider now what happens when firm i, type $\underline{x}_i$ bids slightly less than $\bar{d}$. In this case, her expected profit would at least be

$$\pi_i(\underline{x}_i, \bar{d} - \varepsilon, \delta_{-i}) \geq$$

$$\sum_{j=\tilde{n}}^{k-1} [(1-\bar{d}+\varepsilon)R - C_i(\underline{x}_i, \bar{d}-\varepsilon, \delta_{-i}|J=j)]\Pr(\bar{d}-\varepsilon \text{ is winning bid}|J=j)\Pr(J=j).$$

In order for $\bar{d}$ to be the equilibrium bid of firm i, type $\underline{x}_i$, it must hold that $\pi_i(\underline{x}_i, \bar{d}, \delta_{-i}) \geq \pi_i(\underline{x}_i, \bar{d}-\varepsilon, \delta_{-i})$, for all $\varepsilon > 0$. In the limit,[41] this implies that

$$\sum_{j=\tilde{n}}^{k-1} -\frac{j}{j+1}[(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j) \geq 0. \quad (5)$$

Notice that $\Pr(\bar{d}$ is winning bid$|J=j)$ must be strictly greater than zero for at least some j (if not, firm i, type $\underline{x}_i$, would have a zero probability of winning and would rather deviate downward or not participate). Hence, if the term between square brackets in (5) is positive for all j (notice that individual rationality implies that at least one of these terms must be strictly positive), then the inequality above cannot be satisfied. However, consider the possibility that the term between square brackets in (5) is positive for some j and strictly negative for the others. Notice that, because all bidding functions are weakly decreasing, $C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)$ must be weakly decreasing in j. Hence, there must be some $\hat{n}$ such that the term between square brackets is strictly negative for $\tilde{n} \leq j \leq \hat{n}$, and positive for $\hat{n} < j \leq k-1$. In light of this, inequality (5) can be written as

$$\sum_{j=\tilde{n}}^{\hat{n}} \frac{j}{j+1}[C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j) - (1-\bar{d})R]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j) \geq$$

$$\sum_{j=\hat{n}+1}^{k-1} \frac{j}{j+1}[(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j).$$

Notice that the LHS of the inequality above (which now contains only strictly positive terms) is necessarily lower than

$$\sum_{j=\tilde{n}}^{\hat{n}} \frac{\hat{n}}{j+1}[C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j) - (1-\bar{d})R]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j),$$

[41] Notice that, for $j \leq n - \tilde{n} - 1$, when $\varepsilon \to 0$, $\Pr(\bar{d}-\varepsilon \text{ is winning bid}|J=j) \to \Pr(\bar{d} \text{ is winning bid}|J=j)$, and $C_i(\underline{x}_i, \bar{d}-\varepsilon, \delta_{-i}|J=j) \to C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)$.

and the RHS is necessarily strictly greater than

$$\sum_{j=\hat{n}+1}^{k-1} \frac{\hat{n}}{j+1}[(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j).$$

But this would imply that $\pi_i(\underline{x}_i, \bar{d}, \delta_{-i}) < 0$, which contradicts the fact that this is an equilibrium.

PROPOSITION 1-*(iii)* - SECOND STATEMENT. *In any equilibrium of the ABL auction in which there is at least one firm $i \in K$ such that $\mathrm{PW}_i(\bar{d}, \delta_{-i}) \geq \mathrm{PW}_i(\bar{d}-\varepsilon, \delta_{-i})$ for $\varepsilon \to 0^+$, the probability that at least $n - \tilde{n} - 1$ firms do bid $\bar{d}$ must be larger than $\sum_{j=n-\tilde{n}-1}^{k-1} r^j / \sum_{j=0}^{k-1} r^j$, where r solves $\sum_{j=1}^{k-(n-\tilde{n}-1)} r^j = T$, where $T = (n-\tilde{n})(n-\tilde{n}-2)/(n-\tilde{n}-1)$.*

Proof. Consider firm i, type $\underline{x}_i$. This firm bids $\bar{d}$ and wins the auction with probability

$$\mathrm{PW}_i(\bar{d}, \delta_{-i}) = \sum_{j=\tilde{n}}^{n-\tilde{n}-3} \frac{\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j)}{j+1} + \sum_{j=n-\tilde{n}-2}^{k-1} \frac{\Pr(J=j)}{j+1},$$

where J is the number of firms in K that do bid $\bar{d}$ (beyond firm i itself). Notice that, when $J \geq n - \tilde{n} - 2$, the winning threshold W is necessarily equal to $\bar{d}$. Suppose that firm i, type $\underline{x}_i$, bids slightly less than $\bar{d}$. Her probability of winning the auction would at least be

$$\mathrm{PW}_i(\bar{d}-\varepsilon, \delta_{-i}) \geq \sum_{j=\tilde{n}}^{n-\tilde{n}-3} \Pr(\bar{d}-\varepsilon \text{ is winning bid}|J=j)\Pr(J=j)$$

$$+\Pr(\bar{d}-\varepsilon \text{ is winning bid}|J=n-\tilde{n}-2)\Pr(J=n-\tilde{n}-2).$$

Notice that, when $J > n - \tilde{n} - 2$, W will be equal to $\bar{d}$ and $\bar{d} - \varepsilon$ cannot be a winning bid.

By assumption, for sufficiently small ε, it must be $\mathrm{PW}_i(\bar{d}, \delta_{-i}) \geq \mathrm{PW}_i(\bar{d}-\varepsilon, \delta_{-i})$. In the limit, this inequality becomes

$$\sum_{j=\tilde{n}}^{n-\tilde{n}-3} \frac{\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j)}{j+1} + \sum_{j=n-\tilde{n}-2}^{k-1} \frac{\Pr(J=j)}{j+1} \geq$$

$$\sum_{j=\tilde{n}}^{n-\tilde{n}-3} \Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j) + \Pr(J=n-\tilde{n}-2),$$

or, equivalently,

$$\sum_{j=n-\tilde{n}-1}^{k-1} \frac{\Pr(J=j)}{j+1} - \frac{n-\tilde{n}-2}{n-\tilde{n}-1}\Pr(J=n-\tilde{n}-2) \geq \sum_{j=\tilde{n}}^{n-\tilde{n}-3} \frac{j}{j+1}\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j). \tag{6}$$

Notice that the RHS of the (6) is positive. Hence, in order for (6) to be satisfied, it must necessarily hold that

$$\sum_{j=n-\tilde{n}-1}^{k-1} \frac{\Pr(J=j)}{j+1} \geq \frac{n-\tilde{n}-2}{n-\tilde{n}-1}\Pr(J=n-\tilde{n}-2). \tag{7}$$

Notice that the LHS of the (6) is lower than

$$\sum_{j=n-\tilde{n}-1}^{k-1} \frac{\Pr(J=j)}{n-\tilde{n}}.$$

Hence, in order for (7) to be satisfied, it must necessarily hold that

$$\sum_{j=n-\tilde{n}-1}^{k-1} \Pr(J=j) \geq \frac{(n-\tilde{n})(n-\tilde{n}-2)}{n-\tilde{n}-1} \Pr(J=n-\tilde{n}-2). \tag{8}$$

Our goal is to find a lower bound to $\sum_{j=n-\tilde{n}-1}^{k-1} \Pr(J=j)$ knowing that (8) must necessarily hold.

Notice that the number J of firms that bid $\bar{d}$ (beyond firm i) is the number of successes in $k-1$ independent trials, where the probability of success in the l-th trial is $p_l = F_l(\hat{x}_l)$; hence, J is a random variable with Poisson binomial distribution. Now, denote by r_j the ratio $\frac{\Pr(J=j)}{\Pr(J=j-1)}$. Inequality (8) can be rewritten as

$$r_{n-\tilde{n}-1} \geq \frac{T}{1+\sum_{j=n-\tilde{n}}^{k-1} \prod_{i=n-\tilde{n}}^{j} r_i}, \tag{9}$$

where $T = (n-\tilde{n})(n-\tilde{n}-2)/(n-\tilde{n}-1)$.

It can easily be shown that, if $r_j = t$, then $r_{j-1} > t$, r_j is increasing in j. Hence, we have the following constraints:

$$r_{k-1} > r_{k-2} > \ldots > r_1. \tag{10}$$

Finally, it must be that $\sum_{j=0}^{k-1} \Pr(J=j) = 1$, which can be rewritten as

$$\Pr(J=n-\tilde{n}-1) = \frac{\prod_{i=1}^{n-\tilde{n}-1} r_i}{1+\sum_{j=1}^{k-1} \prod_{i=1}^{j} r_i}. \tag{11}$$

Our objective is to find a lower bound to $\sum_{j=n-\tilde{n}-1}^{k-1} \Pr(J=j)$, we want to solve

$$\inf_{\{r_i\}} \quad \Pr(J=n-\tilde{n}-1) \left[1+\sum_{j=n-\tilde{n}}^{k-1} \prod_{i=n-\tilde{n}}^{j} r_i\right]$$

under the constraints (9), (10), (11).

The solution to the above problem is no greater than the solution to the problem

$$\inf_{\{r_i\}} \quad \Pr(J=n-\tilde{n}-1) \left[1+\sum_{j=n-\tilde{n}}^{k-1} \prod_{i=n-\tilde{n}}^{j} r_i\right]$$

under the constraints (9), (11) and under the constraint

$$r_{k-1} \geq r_{k-2} \geq \ldots \geq r_1. \tag{12}$$

(We replaced (10) with a looser constraint). It's easy to show, that, in the solution to the above problem all constraints (9) and (12) are binding. Hence, the objective function is minimized at

$$r_1 = r_2 = \ldots = r_k - 1 = r, \quad \text{with} \quad \sum_{j=1}^{k-(n-\tilde{n}-1)} r^j = T,$$

and the minimum is $\sum_{j=n-\tilde{n}-1}^{k-1} r^j / \sum_{j=0}^{k-1} r^j$.

B Proof of Proposition 2

Proposition 2 can be easily obtained as a corollary of the following two lemmas that precisely characterize the asymptotic (optimal) behavior of level-k firms, $k \geq 1$.

LEMMA 3

(i) Consider the AB auction. Let $\delta_k^{(n)}(x)$ be the bidding strategy of a level-k firm, type x, for $k \geq 1$, when there are n firms and the other firms' levels range from 0 to $k-1$ (and the proportion of level-j firms is $p_j / \sum_{i=0}^{k-1} p_i$). Then, as $n \to \infty$, $\delta_k^{(n)}(x) \to \overline{A2}_{k-1}$ for all x, where:

- $\overline{A2}_0 = \mathbb{E}[d_0 | \overline{A1}_0 < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A2}_j = (p_0 \mathbb{E}[d_0 | \overline{A1}_j < d_0 < d_{[90]}] + \sum_{i=1}^{j} p_i \overline{A2}_{i-1} \mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_j]}) / (p_0 + \sum_{i=1}^{j} p_i \mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_j]})$;
- $\overline{A1}_0 = \mathbb{E}[d_0 | d_{[10]} < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A1}_j = (p_0 \overline{A1}_0 + \sum_{i=1}^{j} p_i \overline{A2}_{i-1}) / (\sum_{i=0}^{j} p_i)$;
- $d_{[10]}$ and $d_{[90]}$ are the 10-th and 90-th percentile of $G_0(d)$, $G_0(d_{[10]}) = 0.1$ and $G_0(d_{[90]}) = 0.9$.

(ii) Consider the ABL auction. Let $\delta_k^{(n)}(x)$ be the bidding strategy of a level-k firm, for $k \geq 1$, when there are n firms and the other firms' levels range from 0 to $k-1$ (and the proportion of level-j firms is $p_j / \sum_{i=0}^{k-1} p_i$). Then, as $n \to \infty$, $\delta_k^{(n)}(x) \to \overline{A3}_{k-1}$ for all x, where:

- for $j \geq 1$, $\overline{A3}_j = (\overline{A2}_j + d_{[10]})/2$;
- $\overline{A2}_0 = \mathbb{E}[d_0 | \overline{A1}_0 < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A2}_j = (p_0 \mathbb{E}[d_0 | \overline{A1}_j < d_0 < d_{[90]}] + \sum_{i=1}^{j} p_i \overline{A3}_{i-1} \mathbb{1}_{[\overline{A3}_{i-1} > \overline{A1}_j]}) / (p_0 + \sum_{i=1}^{j} p_i \mathbb{1}_{[\overline{A3}_{i-1} > \overline{A1}_j]})$;
- $\overline{A1}_0 = \mathbb{E}[d_0 | d_{[10]} < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A1}_j = (p_0 \overline{A1}_0 + \sum_{i=1}^{j} p_i \overline{A3}_{i-1}) / (\sum_{i=0}^{j} p_i)$;
- $d_{[10]}$ and $d_{[90]}$ are the 10-th and 90-th percentile of $G_0(d)$, $G_0(d_{[10]}) = 0.1$ and $G_0(d_{[90]}) = 0.9$.

Proof.

(i) Consider the AB auction. Let $A1_{k-1}$ and $A2_{k-1}$ be the value of $A1$ and $A2$ when firms' levels range from 0 to $k-1$ (with frequencies $(p_0 / \sum_{i=0}^{k-1} p_i, \ldots, p_{k-1} / \sum_{i=0}^{k-1} p_i)$), level-0 firms bid according to $G_0(d)$ and level-j firms, $0 < j \leq k-1$ bid their best response to their own beliefs. Consider a level-1 firm first. In order to choose her optimal bid, a level-1 firm has to compute the probability distribution of the winning threshold $A2_0$, which in turn depends on $A1_0$. Now, $A1_0 = \sum_{j=\tilde{n}+1}^{n-\tilde{n}} d_0^{(j)} / (n - 2\tilde{n})$, where $d_0^{(j)}$ is the j-th lowest bid by the level-0 firms. Let Y_i, $i = 1, \ldots n$ be a sequence of i.i.d. random variables with distribution $G_Y(y) = G_0(y | d_{[10]} < d_0 < d_{[90]})$. The crucial thing to show

is that, when $n \to \infty$, $A1_0$ converges almost surely to $\overline{A1}_0 = \mathbb{E}[Y]$. To see this, notice first that, by the strong law of large numbers, $\sum_{j=1}^{n} d_0^{(j)}/n \xrightarrow{a.s.} \mathbb{E}[d_0]$, and, consequently, $d_0^{(\tilde{n})} \xrightarrow{a.s.} d_{[10]}$, $d_0^{(n-\tilde{n}+1)} \xrightarrow{a.s.} d_{[90]}$. Now, let $m_1 = \min\{m \in 1, \ldots, n : d_0^{(m)} > d_{[10]}\}$ and $m_n = \max\{m \in 1, \ldots, n : d_0^{(m)} < d_{[90]}\}$. Notice that $\sum_{j=m_1}^{m_2} d_0^{(j)}/(m_2-m_1+1)$ converges almost surely to $\mathbb{E}[Y]$ (because the random variables $d_0^{(l)}$, with $l \in [m_1, m_2]$ have the same distributions as the Y_i's). Given this, in order to show that $A1_0 \xrightarrow{a.s.} \mathbb{E}[Y]$, it is sufficient to show that the difference $A1_0 - \sum_{j=m_1}^{m_2} d_0^{(j)}/(m_2 - m_1 + 1)$ converges almost surely to 0. Now, this difference can be written as

$$A1_0 - \frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{n - 2\tilde{n}} + \frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{n - 2\tilde{n}} - \frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{m_2 - m_1 + 1} \tag{13}$$

Notice that, since $d_0 \in [\underline{d}, \overline{d}] \subseteq [0, 1]$, the first two addends in (13) are certainly no greater than

$$\frac{|m_1 - \tilde{n}| + |m_2 - (n - \tilde{n}) + 1|}{n - 2\tilde{n}},$$

and this term goes to 0 almost surely. The last two terms in (13) can be written as

$$\frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{m_2 - m_1 + 1} \left(\frac{m_2 - m_1 + 1}{n - 2\tilde{n}} - 1 \right).$$

Notice that the first fraction converges to $\mathbb{E}[Y]$, and that $(m_2 - m_1 + 1)/(n - 2\tilde{n})$ goes to 1. Hence, expression (13) converges to 0 almost surely.

In a similar way, one can show that $A2_0$ converges almost surely to $\overline{A2}_0 = \mathbb{E}[d_0|\overline{A1}_0 < d_0 < d_{[90]}]$. Moreover, notice that, because $G_0(d)$ has full support, when n grows to infinity, for all $\varepsilon > 0$, $\Pr(d_0 \in (\overline{A2}_0 - \varepsilon, \overline{A2}_0)) \to 1$. Hence, as n increases, to get a positive chance of winning, a level-1 has to make a bid which is closer and closer to the expected value (from her viewpoint) of the winning threshold $A2$, $\delta_1^{(n)}(x) \to \overline{A2}_0$, for all x.

Consider now a level-2 firm. From her point of view, the winning threshold is $A2_1$, which, in turn, depends on $A1_1$. Reasoning in the same way as before, and given that level-1 firms' bids tend to $\overline{A2}_0$, one show that $A1_1$ converges almost surely to $\overline{A1}_1 = (p_0\overline{A1}_0 + p_1\overline{A2}_0)/(p_1 + p_2)$, and $A2_1$ converges almost surely to $\overline{A2}_1 = (p_0\mathbb{E}[d_0|\overline{A1}_1 < d_0 < d_{[90]}] + p_1\overline{A2}_0)/(p_0 + p_1)$. As n increases, to get a positive chance of winning, a level-2 has to make a bid which is closer and closer to the expected value (from her viewpoint) of the winning threshold $A2$, $\delta_2^{(n)}(x) \to \overline{A2}_1$, for all x.

Proceeding recursively, it is easy to show that, for all $k \geq 1$, $A1_k$ converges almost surely to $\overline{A1}_k = (p_0\overline{A1}_0 + \sum_{i=1}^{k} p_i\overline{A2}_{i-1})/(\sum_{i=0}^{k} p_i)$, and $A2_k$ converges almost surely to

$$\overline{A2}_k = \frac{p_0\mathbb{E}[d_0|\overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=1}^{k} p_i\overline{A2}_{i-1}\mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_k]}}{p_0 + \sum_{i=1}^{k} p_i\mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_k]}}.$$

Hence, $\delta_k^{(n)}(x) \to \overline{A2}_{k-1}$ for all x.

(ii) Apart from minor differences, the proof is the same for the ABL auction. Just one point is worth mentioning: from the point of view of a level-k firm, when n grows to infinity, the interval from which the winning threshold is drawn, converges to $[\overline{A3}_{k-1}, \overline{A2}_{k-1}]$. Now, since every number in this interval has the same probability of being extracted, a level-k firm will bid closer and closer to the lowest value of this interval, $\delta_k^{(n)}(x) \to \overline{A3}_{k-1}$.

LEMMA 4

(i) *In the AB auction,* $\overline{A2}_{k-1} < \overline{A2}_k$*, for all* $k \geq 1$*.*

(ii) *In the ABL auction: if* $\overline{A1}_0 < (d_{[10]} + \overline{A2}_0)/2$*, then* $\overline{A2}_{k-1} < \overline{A2}_k$ *for all* $k \geq 1$*; if* $\overline{A1}_0 > (d_{[10]} + \overline{A2}_0)/2$*, then* $\overline{A2}_{k-1} > \overline{A2}_k$ *for all* $k \geq 1$*; if* $\overline{A1}_0 = (d_{[10]} + \overline{A2}_0)/2$*, then* $\overline{A2}_{k-1} = \overline{A2}_k$ *for all* $k \geq 1$*.*

Proof.

(i) Notice first that, by construction, for all k, $\overline{A1}_k < \overline{A2}_k$: in fact, $\overline{A2}_k$ is a weighted average of numbers that are strictly greater than $\overline{A2}_k$. Second, for all $k \geq 1$, $\overline{A1}_{k-1} < \overline{A1}_k < \overline{A2}_{k-1}$: for $k = 1$, this is fairly obvious; for $k > 1$, notice that, since $\overline{A1}_{k-1} = (p_0\overline{A1}_0 + \sum_{i=1}^{k-1} p_i\overline{A2}_{i-1})/\sum_{i=0}^{k-1} p_i$, we have that

$$\sum_{i=0}^{k-1} p_i\overline{A1}_{k-1} = p_0\overline{A1}_0 + \sum_{i=1}^{k-1} p_i\overline{A2}_{i-1}.$$

Using this and substituting into the expression for $\overline{A1}_k$, we get

$$\overline{A1}_k = \frac{\sum_{i=0}^{k-1} p_i\overline{A1}_{k-1} + p_k\overline{A2}_{k-1}}{\sum_{i=0}^{k} p_i}.$$

Hence, $\overline{A1}_k$ is a weighted average of $\overline{A1}_{k-1}$ and $\overline{A2}_{k-1}$, but since $\overline{A1}_{k-1} < \overline{A2}_{k-1}$, it must be $\overline{A1}_{k-1} < \overline{A1}_k < \overline{A2}_{k-1}$.

We now show, by induction, that, if $\overline{A2}_{j-1} < \overline{A2}_j$ for all $j \leq k$, then $\overline{A2}_k < \overline{A2}_{k+1}$. So, assume $\overline{A2}_{j-1} < \overline{A2}_j$ for all $j \leq k$, $k \geq 1$; let $s = \min j = 0, \ldots, k-1|\overline{A1}_k < \overline{A2}_j$ and let $t = \min j = 0, \ldots, k|\overline{A1}_{k+1} < \overline{A2}_j$. Notice that, necessarily, it must be $s \leq t$; when $s < t$, we have

$$\begin{aligned}
\overline{A2}_k =& \frac{p_0\mathbb{E}[d_0|\overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=s+1}^{k} p_i\overline{A2}_{i-1}}{p_0 + \sum_{i=s+1}^{k} p_i} \\
=& \frac{p_0\mathbb{E}[d_0|\overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=s+1}^{t} p_i\overline{A2}_{i-1} + \sum_{i=t+1}^{k} p_i\overline{A2}_{i-1}}{p_0 + \sum_{i=s+1}^{t} p_i + \sum_{i=t+1}^{k} p_i}.
\end{aligned}$$

Hence,

$$(p_0 + \sum_{i=s+1}^{t} p_i + \sum_{i=t+1}^{k} p_i)\overline{A2}_k - \sum_{i=s+1}^{t} p_i\overline{A2}_{i-1} = p_0\mathbb{E}[d_0|\overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=t+1}^{k} p_i\overline{A2}_{i-1}. \quad (14)$$

Now, notice that, since $\overline{A1}_{k+1} > \overline{A1}_k$, it must be

$$\begin{aligned}\overline{A2}_{k+1} =& \frac{p_0 \mathbb{E}[d_0 | \overline{A1}_{k+1} < d_0 < d_{[90]}] + \sum_{i=t+1}^{k+1} p_i \overline{A2}_{i-1}}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ >& \frac{p_0 \mathbb{E}[d_0 | \overline{A1}_{k} < d_0 < d_{[90]}] + \sum_{i=t+1}^{k+1} p_i \overline{A2}_{i-1}}{p_0 + \sum_{i=t+1}^{k+1} p_i}.\end{aligned}$$

Using (14), the last inequality becomes

$$\begin{aligned}\overline{A2}_{k+1} >& \frac{(p_0 + \sum_{i=s+1}^{t} p_i + \sum_{i=t+1}^{k} p_i)\overline{A2}_k - \sum_{i=s+1}^{t} p_i \overline{A2}_{i-1} + p_{k+1}\overline{A2}_k}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ =& \frac{(p_0 + \sum_{i=t+1}^{k} p_i + p_{k+1})\overline{A2}_k + \sum_{i=s+1}^{t} p_i (\overline{A2}_k \overline{A2}_{i-1})}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ =& \overline{A2}_k + \frac{\sum_{i=s+1}^{t} p_i (\overline{A2}_k \overline{A2}_{i-1})}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ \geq& \overline{A2}_k.\end{aligned}$$

When $s = t$, the whole derivation above goes through with the only difference that all terms involving $\sum_{i=s+1}^{t}$ are absent. To complete the proof, we have to show that $\overline{A2}_0 < \overline{A2}_1$, which is fairly obvious, since $\overline{A2}_1 = (p_0 \mathbb{E}[d_0 | \overline{A1}_1 < d_0 < d_{[90]}] + p_1 \overline{A2}_0)/(p_0 + p_1)$, is a wighted average of $\overline{A2}_0$ and a number ($\mathbb{E}[d_0 | \overline{A1}_1 < d_0 < d_{[90]}]$) strictly greater than $\overline{A2}_0$.

(ii) The proof for the ABL auction follows exactly the same line as the previous one with one caveat: if $\overline{A1}_0 < \overline{A3}_0$, we have that the sequence of $\overline{A3}_k$'s is strictly increasing; if, instead, $\overline{A1}_0 > \overline{A3}_0$, the sequence of $\overline{A3}_k$'s is strictly decreasing (in the proof, all inequalities are reversed); of course, it is in principle possible that $\overline{A1}_0 = \overline{A3}_0$, in which case the sequence of $\overline{A3}_k$'s is constant. (Typically, we expect $\overline{A1}_0 > \overline{A3}_0$: in fact, $\overline{A3}_0$ is the average between $d_{[10]}$ and $\overline{A2}_0$, and the latter is no greater than $d_{[90]}$; hence, if G_0 is symmetric, $\overline{A3}_0$ is necessarily below the mean of $G_0(d)$. To have $\overline{A1}_0 \leq \overline{A3}_0$, $G_0(d)$ must be heavily skewed.)

The previous result immediately implies Proposition 2, that, for convenience, is reported below.

Proposition 2. *In the AB auction, in the limit, the (expected) distance of a firm's bid from A2 is strictly decreasing in her level of sophistication. In the ABL auction, in the limit, the (expected) distance of a firm's bid from A3 is strictly decreasing in her level of sophistication.*[42]

Proof. Take the AB auction. If we denote by k^{max} the highest level of sophistication in the population of firms, then: if k^{max} is finite, the expected value of $A2$, when $n \to \infty$, is simply $\overline{A2}_{k^{max}+1}$; if not, the expected value of $A2$, when $n \to \infty$, is $\lim_{k \to \infty} \overline{A2}_k$. In any case, $\overline{A2}_k < \mathbb{E}[A2]$, for all k. This, together with the fact that the sequence of $\overline{A2}_k$'s is strictly

[42]To be precise, this proposition holds only when $\overline{A1}_0 \neq \overline{A3}_0$; when $\overline{A1}_0 \neq \overline{A3}_0$, the (expected) distance of a firm's bid from $A2$ is constant in her level of sophistication.

increasing, implies that the distance between $\overline{A2}_k$ (which is the optimal bid of a level-$k+1$ firm) and $\mathbb{E}[A2]$ is strictly decreasing in k.
For the ABL auction, the proof is analogous.

C Numerical simulations

In this section, we present the results of some simulation exercises from a CH model of bidding behavior in AB and ABL. The purpose of this exercise is twofold: on the one hand, it shows that the main prediction of the CH model – the distance of a firm's bid from $A2$ in AB, from $A3$ in ABL, is strictly decreasing in her level of sophistication – does not hold only asymptotically (as was proved in Proposition 2), but also for finite n; on the other hand, it provides support to the additional empirical evidence presented in Subsection 5.4.

The simulations are run under the following assumptions and parametrization: we fix the reserve price to 100 and assume that firms' costs are private and independently and identically distributed according to a uniform distribution on the interval $[\underline{c} = 50, \bar{c} = 70]$, with increments of 0.2. We assume that firms' levels of sophistication range from 0 to 2[43] and that they are distributed according to a truncated Poisson with parameter λ.[44] Level-0 firms are assumed to draw their bids from a uniform distribution over the interval $[0, 0.3]$. This assumption is roughly consistent with our evidence (the minimum and maximum discounts observed in our sample are 0 and 0.421 in AB and 0.016 and 0.317 in ABL) and ensures that level-0 firms will never play dominated strategies.[45] Level-1 firms choose their bids to maximize their expected payoffs under the belief that all other firms are level-0, while level-2 firms choose their bids to maximize their expected payoffs under the belief that other firms are a mixture of level-0 and level-1. Given the behavior of level-0, level-1 and level-2 firms, we compute the expected value of $A2$ (for the AB auction) or $A3$ (for the ABL auction), and, for each level, the expected value and the variance (in square brackets) of the distance between their bids and $A2$ or $A3$. Since our objective is to check the consistency of the results of the simulations with real data, we must allow for errors. Hence, the distance from $A2$ or $A3$ is computed supposing that level-1 and level-2 firms' bids are subject to logistic errors: every bid is played with positive probability but the probability that a level-l firm ($l = 1, 2$) with cost c bids $\hat{d}$ is $\exp(\eta\Pi_l(\hat{d}; c))/\sum_d \exp(\eta\Pi_l(d; c))$, where $\Pi_l(d; c)$ is the expected payoff of a level-l firm when her cost is c and she bids d, and where η denotes the error parameter (with $\eta = 0$ meaning random behavior and $\eta \to \infty$ meaning no errors). We also computed the truly optimal bid, , the bid that would maximize the expected payoff of a firm who has fully correct beliefs about the behavior of other firms. Proposition 2 showed that, when $n \to \infty$, this truly optimal bid converges to $A2$ in AB, to $A3$ in ABL, but for finite n, it may be different. Hence, it is important to verify whether $A2$ and $A3$ are indeed good proxies for the optimal bid. The results of the simulations are reported in Tables C1-C6, for different values of the parameter of the distribution of levels ($\lambda = 0.5, 1, 2$), of the number of firms ($n = 25, 50, 100$) and of the parameter of the error distribution ($\eta = 0.5, 1, 2$).

[43] We consider only level-1 and level-2 firms because experimental evidence has shown that the majority of subjects performs no more than 2 levels of iteration (see, e.g., Crawford et al. 2013).

[44] Hence, the probability that a firm's level of sophistication is l ($l = 0, 1, 2$) is equal to $\frac{e^{\lambda}\lambda^l/l}{\sum_{i=0}^{2} e^{\lambda}\lambda^i/i} = \frac{\lambda^l/l}{1+\lambda+\lambda^2/2}$. Hence, a higher λ means that firms are, on average, more sophisticated.

[45] In this sense, level-0 firms have at least a minimum degree of rationality. Their random behavior could be interpreted as the consequence of a total absence of any precise beliefs about the behavior of others. The assumption that level-0 players do not play dominated strategies represents a small departure from the standard CH-literature. However, we believe that this represents a reasonable assumption in real world applications: all firms, also the most naive ones, should easily realize that it is not a good idea to offer a discount that would not allow it to cover the cost of realizing the work. In a similar vain, Goldfarb and Xiao (2011), in their estimated CH-model of entry decisions by firms, endow level-0 firms with a minimum degree of rationality.

Table C1 – Simulation results for the AB auction with $\eta = 0.5$.

n	λ	*A2*	*distance from A2*			*opt.*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*	*bid*	*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	5.2 [0.9]	4.2 [0.6]	20.4	8.5 [2.4]	5.3 [0.9]	4.3 [0.6]
	1	20.1	8.4 [2.4]	5.2 [0.9]	4.0 [0.5]	19.5	8.2 [2.3]	5.1 [0.9]	3.6 [0.4]
	2	19.8	8.3 [2.3]	5.1 [0.9]	1.6 [0.1]	19.5	8.2 [2.3]	5.1 [0.9]	1.3 [0.1]
50	0.5	21.0	8.8 [2.6]	6.7 [1.5]	5.9 [1.2]	20.1-21.3	8.5 [2.4]	6.6 [1.4]	5.8 [1.1]
	1	20.7	8.6 [2.5]	6.6 [1.5]	5.9 [1.2]	20.4	8.5 [2.4]	6.6 [1.4]	5.8 [1.1]
	2	20.6	8.6 [2.5]	6.6 [1.5]	2.6 [0.2]	20.4	8.5 [2.4]	6.6 [1.4]	2.4 [0.2]
100	0.5	21.0	8.8 [2.6]	7.7 [2.0]	7.1 [1.7]	20.4	8.5 [2.4]	7.5 [1.9]	7.0 [1.6]
	1	20.7	8.6 [2.5]	7.6 [1.93]	7.5 [1.88]	20.4	8.5 [2.4]	7.5 [1.9]	7.4 [1.8]
	2	20.6	8.6 [2.5]	7.6 [1.9]	4.2 [0.6]	20.4	8.5 [2.4]	7.5 [1.9]	4.1 [0.5]

Table C2 – Simulation results for the AB auction with $\eta = 1$.

n	λ	*A2*	*distance from A2*			*opt.*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*	*bid*	*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	2.7 [0.2]	1.1 [0.0]	20.4	8.5 [2.4]	2.7 [0.2]	1.2 [0.0]
	1	20.1	8.4 [2.4]	2.6 [0.2]	1.5 [0.1]	19.5	8.2 [2.3]	2.6 [0.2]	0.9 [0.0]
	2	19.8	8.3 [2.3]	2.6 [0.2]	0.6 [0.0]	19.5	8.2 [2.3]	2.6 [0.2]	0.3 [0.0]
50	0.5	21.0	8.8 [2.6]	4.2 [0.6]	2.1 [0.1]	20.1-21.3	8.5 [2.4]	4.1 [0.6]	2.4 [0.2]
	1	20.7	8.6 [2.5]	4.1 [0.6]	2.4 [0.2]	20.4	8.5 [2.4]	4.0 [0.5]	2.2 [0.2]
	2	20.6	8.6 [2.5]	4.1 [0.6]	0.5 [0.0]	20.4	8.5 [2.4]	4.0 [0.5]	0.3 [0.0]
100	0.5	21.0	8.8 [2.6]	6.0 [1.2]	3.3 [0.4]	20.4	8.5 [2.4]	5.9 [1.2]	3.6 [0.4]
	1	20.7	8.6 [2.5]	6.0 [1.2]	4.7 [0.7]	20.4	8.5 [2.4]	5.9 [1.2]	4.5 [0.7]
	2	20.6	8.6 [2.5]	6.0 [1.2]	0.9 [0.0]	20.4	8.5 [2.4]	5.9 [1.2]	0.7 [0.0]

Table C3 – Simulation results for the AB auction with $\eta = 2$.

n	λ	*A2*	*distance from A2*			*opt.*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*	*bid*	*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	1.1 [0.03]	0.2 [0.00]	20.4	8.5 [2.4]	1.1 [0.04]	0.3 [0.00]
	1	20.1	8.4 [2.4]	1.0 [0.03]	0.8 [0.02]	19.5	8.2 [2.3]	1.0 [0.03]	0.3 [0.00]
	2	19.8	8.3 [2.3]	1.0 [0.03]	0.4 [0.00]	19.5	8.2 [2.3]	1.0 [0.03]	0.1 [0.00]
50	0.5	21.0	8.8 [2.6]	1.5 [0.08]	0.2 [0.00]	20.1-21.3	8.5 [2.4]	1.5 [0.07]	0.9 [0.03]
	1	20.7	8.6 [2.5]	1.4 [0.06]	0.5 [0.01]	20.4	8.5 [2.4]	1.4 [0.06]	0.3 [0.00]
	2	20.6	8.6 [2.5]	1.4 [0.06]	0.3 [0.00]	20.4	8.5 [2.4]	1.4 [0.06]	0.0 [0.00]
100	0.5	21.0	8.8 [2.6]	2.8 [0.26]	0.5 [0.01]	20.4	8.5 [2.4]	2.8 [0.26]	1.0 [0.03]
	1	20.7	8.6 [2.5]	2.7 [0.25]	1.2 [0.05]	20.4	8.5 [2.4]	2.8 [0.26]	1.0 [0.03]
	2	20.6	8.6 [2.5]	2.7 [0.25]	0.2 [0.00]	20.4	8.5 [2.4]	2.8 [0.26]	0.0 [0.00]

Looking at the results of these numerical simulations, we detect some regularities, that we summarize below.

(a) For all values of n, λ, and η, the optimal bid (, the bid that maximizes the expected payoff of a firm that has fully correct beliefs about the behavior of all other firms) is essentially unaffected by the private cost. In fact, of the 54 possible combinations of parameters considered, there are only two cases in which the optimal bid is not constant

Table C4 – Simulation results for the ABL auction with $\eta = 0.5$.

n	λ	*A3*	*distance from A3* *level 0*	*level 1*	*level 2*	*opt. bid*	*distance from opt. bid* *level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	5.6 [1.1]	4.5 [0.7]	15.0	7.6 [1.9]	5.0 [0.8]	4.1 [0.5]
	1	14.5	7.6 [1.9]	5.2 [0.9]	3.2 [0.4]	15.3	7.6 [1.9]	4.9 [0.8]	3.1 [0.3]
	2	15.6	7.6 [1.9]	4.9 [0.8]	2.1 [0.1]	15.3	7.6 [1.9]	4.9 [0.8]	2.0 [0.1]
50	0.5	12.7	7.8 [2.0]	6.8 [1.6]	6.0 [1.2]	13.5	7.6 [1.9]	6.6 [1.4]	5.7 [1.1]
	1	13.5	7.7 [2.0]	6.6 [1.4]	5.0 [0.8]	15.0	7.6 [1.9]	6.3 [1.3]	4.7 [0.7]
	2	15.3	7.6 [1.9]	6.2 [1.3]	3.3 [0.4]	15.3	7.6 [1.9]	6.2 [1.3]	3.3 [0.4]
100	0.5	12.7	7.8 [2.0]	7.3 [1.8]	6.9 [1.6]	15.6	7.6 [1.9]	6.9 [1.6]	6.6 [1.5]
	1	13.1	7.7 [2.0]	7.2 [1.7]	6.5 [1.4]	14.1	7.6 [1.9]	7.0 [1.6]	6.4 [1.4]
	2	14.3	7.6 [1.9]	7.0 [1.6]	5.4 [1.0]	14.1	7.6 [1.9]	7.0 [1.6]	5.4 [1.0]

Table C5 – Simulation results for the ABL auction with $\eta = 1$.

n	λ	*A3*	*distance from A3* *level 0*	*level 1*	*level 2*	*opt. bid*	*distance from opt. bid* *level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	4.2 [0.6]	2.3 [0.2]	15.0	7.6 [1.9]	3.1 [0.3]	1.5 [0.1]
	1	14.5	7.6 [1.9]	3.4 [0.4]	1.2 [0.0]	15.3	7.6 [1.9]	3.0 [0.3]	0.9 [0.0]
	2	15.6	7.6 [1.9]	2.8 [0.3]	0.5 [0.0]	15.3	7.6 [1.9]	3.0 [0.3]	0.5 [0.0]
50	0.5	12.7	7.8 [2.0]	5.8 [1.1]	4.1 [0.5]	13.5	7.6 [1.9]	5.4 [1.0]	3.6 [0.4]
	1	13.5	7.7 [2.0]	5.4 [1.0]	2.6 [0.2]	15.0	7.6 [1.9]	4.8 [0.8]	2.0 [0.1]
	2	15.3	7.6 [1.9]	4.8 [0.8]	0.9 [0.0]	15.3	7.6 [1.9]	4.8 [0.8]	0.9 [0.0]
100	0.5	12.7	7.8 [2.0]	6.8 [1.5]	6.0 [1.2]	15.6	7.6 [1.9]	6.2 [1.3]	5.5 [1.0]
	1	13.1	7.7 [2.0]	6.6 [1.4]	5.2 [0.9]	14.1	7.6 [1.9]	6.3 [1.3]	4.9 [0.8]
	2	14.3	7.6 [1.9]	6.3 [1.3]	3.0 [0.3]	14.1	7.6 [1.9]	6.3 [1.3]	3.1 [0.3]

Table C6 – Simulation results for the ABL auction with $\eta = 2$.

n	λ	*A3*	*distance from A3* *level 0*	*level 1*	*level 2*	*opt. bid*	*distance from opt. bid* *level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	3.3 [0.4]	1.7 [0.1]	15.0	7.6 [1.9]	1.9 [0.1]	0.5 [0.0]
	1	14.5	7.6 [1.9]	2.4 [0.2]	0.9 [0.0]	15.3	7.6 [1.9]	1.7 [0.1]	0.4 [0.0]
	2	15.6	7.6 [1.9]	1.5 [0.1]	0.2 [0.0]	15.3	7.6 [1.9]	1.7 [0.1]	0.3 [0.0]
50	0.5	12.7	7.8 [2.0]	4.5 [0.7]	2.3 [0.2]	13.5	7.6 [1.9]	3.9 [0.5]	1.7 [0.1]
	1	13.5	7.7 [2.0]	3.9 [0.5]	1.6 [0.1]	15.0	7.6 [1.9]	2.9 [0.3]	0.6 [0.0]
	2	15.3	7.6 [1.9]	2.8 [0.3]	0.3 [0.0]	15.3	7.6 [1.9]	2.7 [0.3]	0.3 [0.0]
100	0.5	12.7	7.8 [2.0]	5.7 [1.1]	4.2 [0.6]	15.6	7.6 [1.9]	4.6 [0.7]	3.4 [0.4]
	1	13.1	7.7 [2.0]	5.4 [1.0]	3.0 [0.3]	14.1	7.6 [1.9]	5.0 [0.8]	2.6 [0.2]
	2	14.3	7.6 [1.9]	4.9 [0.8]	0.9 [0.0]	14.1	7.6 [1.9]	5.0 [0.8]	1.0 [0.0]

in the private cost: in AB, with $\eta = 0.5$, $n = 50$, $\lambda = 0.5$ and in AB, with $\eta = 1$, $n = 50$, $\lambda = 0.5$. Moreover, in these two cases, the range of the optimal bidding function is pretty narrow (about 1%). This confirms the intuition that, in these auctions, costs do not matter much for bidding.

(b) For all values of n, λ, and η, the optimal bid is extremely close to the expected value of $A2$ in AB, of $A3$ in ABL. This supports the intuition that, in these auctions, $A2$ and $A3$ are good proxies for the optimal bid, even when n is finite.

(c) For all values of n, λ, and η, the distance of a firm's bid from the expected value of $A2$ in AB, of $A3$ in ABL, is decreasing in her level of sophistication. Hence, the main theoretical prediction of the asymptotic CH model (Proposition 2) seem to hold also when n is relatively small.

(d) For given n, λ, and η, level-1 and level-2 firms' bids are, on average, lower in ABL than in AB. This fact is consistent with the empirical evidence discussed in Subsection 5.4.

(e) In either auction, for all values of n, λ, and η, the variance of the distance from $A2$ or $A3$ is decreasing in the sophistication level of the firm. This fact is consistent with the empirical evidence discussed in Subsection 5.4.

(f) For given λ and η, the optimal bid and the expected value of $A2$ are increasing in n in AB, the optimal bid and the expected value of $A3$ are decreasing in n in ABL. This fact is consistent with the empirical evidence discussed in Subsection 5.4.

D Additional empirical evidence

This Section presents additional empirical evidence, both descriptive and inferential, recalled and commented in Sections 4 and 5 of the paper. In particular:

- Figures D1, D2 and D3 report descriptive evidence about the distribution of the sophistication index over time and by firm size;
- Table D1, columns (1) and (5), show that our main empirical result does not change when we amend our baseline model (2) including in the estimation those firms with a sophistication index equal to 0 (replacing $\log(BidderSoph)$ with $\log(1 + BidderSoph)$);
- Table D1, columns (2) and (6), show that our main empirical result does not change when we amend our baseline model (2) adopting a log-linear specification instead of a log-log one;
- Table D1, columns (3) and (7), show that our main empirical result does not change when we amend our baseline model (2) adding the number of bidders as a control variable;
- Table D1, columns (4) and (8), show that our main empirical result does not change when we amend our baseline model (2) replacing auction controls with auction-fixed effects;
- Table D2, columns (1)-(4), show, for the AB auctions, that our main empirical result does not change when we adopt a two-step Heckman model to control for selection bias problems;
- Table D2, columns (5)-(10), show, for the AB auctions, that our main empirical result does not change when the sophistication index is category-specific: when a firm participates in auction j, only her performances in past auctions of the same format and of the same category of work as j are considered in the computation of her sophistication level;
- Table D3, columns (1)-(4), show, for the AB sample, the estimation results when firm- and firm-year-fixed effects are not included in the model discussed in Subsection 4.5;
- Table D3, columns (5)-(8), show, for the AB sample, the estimation results when firm-year-fixed effects are replaced by firm-fixed effects in the model discussed in Subsection 4.5;
- Table D4, columns (9)-(12), show, for the ABL sample, the estimation results of the model discussed in Subsection 4.5;
- Table D4, columns (1)-(4), show, for the ABL sample, the estimation results when firm- and firm-year-fixed effects are not included in the model discussed in Subsection 4.5;
- Table D4, columns (5)-(8), show, for the ABL sample, the estimation results when firm-year-fixed effects are replaced by firm-fixed effects in the model discussed in Subsection 4.5;

- Table D5 shows that our main empirical result does not change when we amend our baseline model (2) replacing firm- or firm-year-fixed effects with firm-semester or firm-category of work- or firm-category of work-semester- or firm-category of work-year-fixed effects;
- Table D6 shows the estimation results of a model in which the dependent variable is the level of bids;
- Table D7 shows the estimation results of quantile regression models in which the dependent variable is the level of bids;
- Table D8 shows that our main empirical result does not change when we control for potential cartels in AB;
- Table D9 shows that our main empirical result does not change when we control for potential cartels in ABL;
- Table D10 shows that the results of our IV regressions are unchanged when we focus on the longer period 2004-2007;
- Table D11, columns (1) and (2), show that bids are, on average, lower in ABL than in AB;
- Table D11, column (3), shows that $A2$ in AB increases with the number of participating firms;
- Table D11, column (4), shows that $A3$ in ABL decreases with the number of participating firms;
- Table D11, columns (5)-(10), show that the standard deviation of the average distance between bids and $A2$ [$A3$] in an AB [ABL] auction is decreasing in the average sophistication level of the firms participating in that auction.

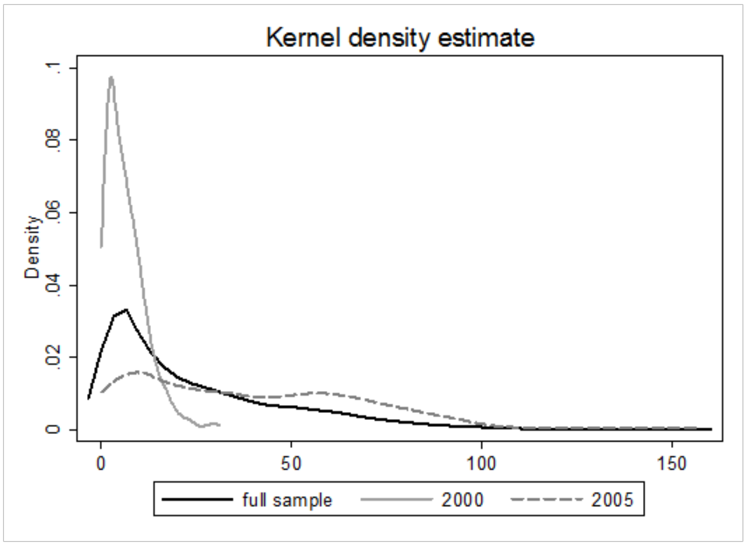

Figure D1 – Distribution of the sophistication index in AB.

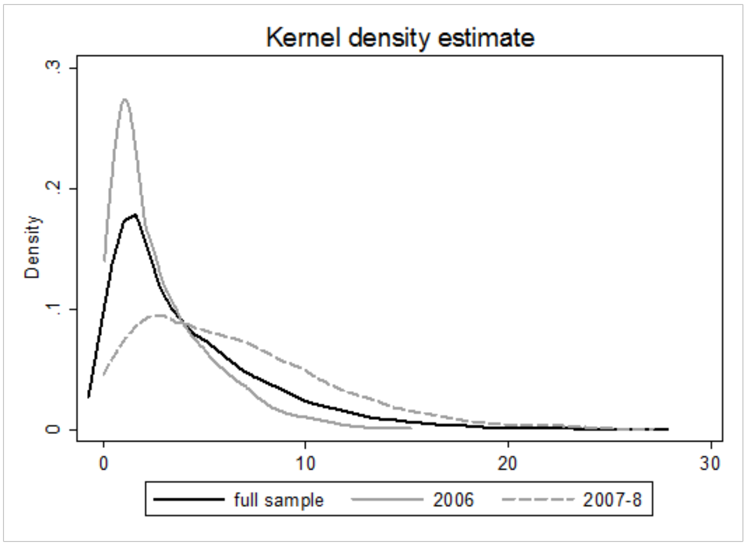

Figure D2 – Distribution of the sophistication index in ABL.

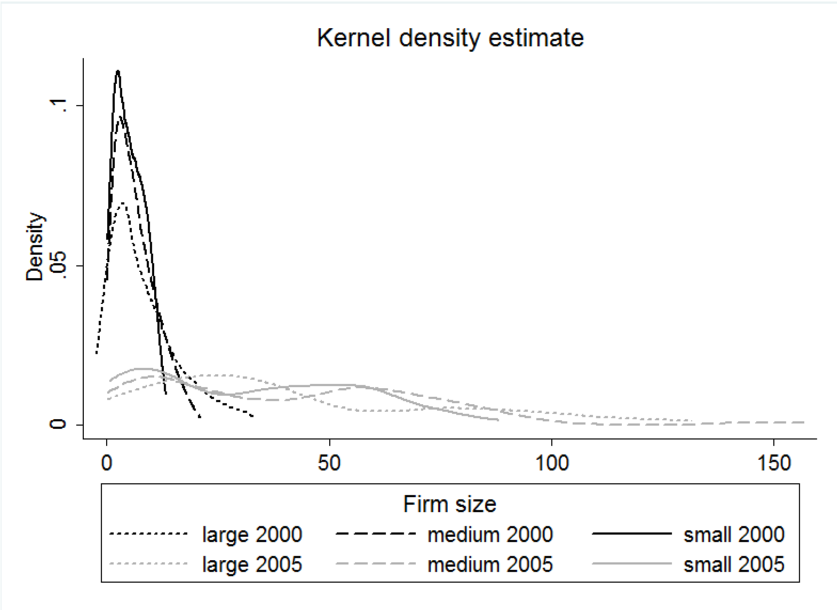

Figure D3 – Distribution of the sophistication index in AB by firm size.

Table D1 – Robustness checks on the baseline model specification.

Dependent variable:	log $\|Distance\|$							
Auction format	AB	AB	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(1+$BidderSoph$)	-0.240***				-0.568***			
	(0.048)				(0.083)			
$BidderSoph$		-0.020***				-0.113***		
		(0.003)				(0.021)		
log($BidderSoph$)			-0.241***	-0.140***			-0.440***	-0.293***
			(0.041)	(0.023)			(0.072)	(0.050)
Auction/project controls	YES	YES	YES	NO	YES	YES	YES	NO
Firm controls	NO	NO	NO	YES	NO	NO	NO	YES
Firm-year FE	YES	YES	YES	NO	YES	YES	YES	NO
Auction-FE	NO	NO	NO	YES	NO	NO	NO	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,965	8,965	8,573	8,924	1,591	1,591	1,266	1,501
R-squared	0.361	0.361	0.356	0.379	0.524	0.516	0.506	0.361

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction, and, in columns (3) and (7), the number of bidders. *Firm controls* include: dummy variables for firm size and for the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D2 – AB auctions: selection bias problems (two-step Heckman model) and category-specific sophistication index.

Dependent variable:	log $\|Distance\|$	Pr.Part.	log $\|Distance\|$	Pr.Part.	log $\|Distance\|$					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
log($BidderSoph$)	-0.158***		-0.522***	0.228***						
	(0.026)		(0.038)	(0.021)						
log($TimeToBid$)		0.034**		0.152***						
		(0.014)		(0.032)						
log($BidderSophInCat$)					-0.128***		-0.150***		-0.229***	
					(0.029)		(0.042)		(0.052)	
log($1 + BidderSophInCat$)						-0.179***		-0.254***		-0.388***
						(0.032)		(0.051)		(0.063)
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm-controls	YES	YES	YES	YES	YES	YES	NO	NO	NO	NO
Firm-FE	NO	NO	NO	NO	NO	NO	YES	YES	NO	NO
Firm-year FE	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES
Firm-auction controls	NO	NO	NO	NO	YES	YES	YES	YES	YES	YES
Observations	3,877	13,517	13,517	13,517	3,658	3,982	3,658	3,910	3,556	3,727
R-squared	0.190	0.047			0.190	0.199	0.267	0.277	0.372	0.378

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for firm's size and for the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
The analysis focuses on AB auctions for roadworks because they represent the largest share of projects in our data (87 auctions). OLS regression in column (1) shows the coefficient of $BidderSoph$ estimated on the subsample of roadworks. The potential market for roadworks is defined as those firms that, according to our dataset, bid at least once for roadworks in a given year. As an exogenous instrument that is related to the probability of firms' participation but has an influence only on the cost of participation, we use $TimeToBid$, which is the length of time between the date when the project is advertised and when the bid letting occurs (this instrument is also used by Gil and Marion 2013, and Moretti and Valbonesi 2015). The hypothesis is that the longer the time between the beginning of project's publicity and the deadline for bid's submission, the lower the cost borne by firms to prepare their bids. Our data show that there is variability in terms of auctions' advertise lead time, with an average of 28.6 days (and a standard deviation of 11.4 days). In columns (3) and (4), the second and first stage of a two-step Heckman selection model are reported.

Table D3 – Learning dynamics: further results for AB auctions.

Dependent variable	log $\|Distance\|$							
Auction format	AB							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log($PastPart$)	-0.139***	-0.120***	-0.113***		-0.212***	-0.225***	-0.243***	
	(0.024)	(0.021)	(0.023)		(0.041)	(0.042)	(0.043)	
log(1+$PastPerf$)		-2.825***	-2.811***			0.917*	0.860	
		(0.229)	(0.230)			(0.541)	(0.548)	
log(1+$PastWins$)			-0.032	-0.027			0.195**	0.192**
			(0.048)	(0.056)			(0.080)	(0.076)
log($BidderSoph$)				-0.166***				-0.186***
				(0.025)				(0.039)
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	YES	YES	NO	NO	NO	NO
Firm-FE	NO	NO	NO	NO	YES	YES	YES	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,927	8,927	8,927	8,927	8,838	8,838	8,838	8,838
R-squared	0.188	0.204	0.204	0.192	0.267	0.267	0.268	0.267

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D4 – Learning dynamics: results for ABL auctions.

Dependent variable	log $\|Distance\|$											
Auction format	ABL											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
log($PastPart$)	-0.272***	-0.260***	-0.271***		-0.558***	-0.528***	-0.538***		-0.601***	-0.564***	-0.575***	
	(0.058)	(0.049)	(0.050)		(0.073)	(0.077)	(0.077)		(0.075)	(0.079)	(0.079)	
log(1+$PastPerf$)		-2.873***	-2.912***			-0.735	-0.752			-0.870	-0.873	
		(0.326)	(0.327)			(0.690)	(0.688)			(0.707)	(0.703)	
log(1+$PastWins$)			0.350**	0.323**			0.353	0.347			0.374	0.358
			(0.150)	(0.153)			(0.246)	(0.241)			(0.285)	(0.275)
log($BidderSoph$)				-0.395***				-0.477***				-0.517***
				(0.043)				(0.064)				(0.067)
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES				
Firm controls	YES	YES	YES	YES	NO	NO	NO	NO	NO	NO	NO	NO
Firm-FE	NO	NO	NO	NO	YES	YES	YES	YES	NO	NO	NO	NO
Firm-year-FE	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,501	1,501	1,501	1,501	1,410	1,410	1,410	1,410	1,266	1,266	1,266	1,266
R-squared	0.244	0.296	0.298	0.280	0.459	0.459	0.460	0.460	0.450	0.450	0.451	0.451

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D5 – Further results: firm-category of work-fixed effects and firm-semester fixed effects.

Dependent variable	log $\|Distance\|$					
Auction format	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)
log($BidderSoph$)	-0.166***	-0.124**	-0.089	-0.476***	-0.507***	0.132
	(0.040)	(0.052)	(0.117)	(0.076)	(0.102)	(0.164)
Auction/project controls	YES	YES	YES	YES	YES	YES
Firm-semester-FE	NO	NO	YES	NO	NO	NO
Firm-category of work-FE	YES	NO	NO	YES	NO	NO
Firm-category of work-semester-FE	NO	YES	NO	NO	NO	YES
Firm-category of work-year-FE	NO	NO	NO	NO	YES	NO
Firm-auction controls	YES	YES	YES	YES	YES	YES
Observations	8,642	7,463	1,154	1,287	1,053	838
R-squared	0.303	0.478	0.580	0.528	0.584	0.691

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D6 – Sophistication and bid levels: overall sample.

Dependent variable	$\log(Discount)$	$Discount$	$\log(Discount)$	$Discount$	$\log(Discount)$	$Discount$	$\log(Discount)$	$Discount$
Auction format	AB	AB	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log(BidderSoph)$	0.006	0.083	-0.016	-0.247*	-0.014	-0.367	-0.015	-0.390
	(0.010)	(0.117)	(0.013)	(0.142)	(0.022)	(0.239)	(0.024)	(0.260)
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm-FE	YES	YES	NO	NO	YES	YES	NO	NO
Firm-year-FE	NO	NO	YES	YES	NO	NO	YES	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,838	8,838	8,573	8,573	1,410	1,410	1,266	1,266
R-squared	0.348	0.530	0.422	0.590	0.525	0.678	0.583	0.715

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D7 – Sophistication and bid levels: quantile regressions.

Quantile (th)	5	10	20	30	40	50	60	70	80	90	95
Auction format						AB					
Dependent						log($Discount$)					
log($BidderSoph$)	0.040***	0.028***	0.011***	0.007***	0.004***	0.002*	0.001	0.001	-0.000	-0.003***	-0.008***
	(0.008)	(0.006)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
Dependent						$Discount$					
log($BidderSoph$)	0.635***	0.421***	0.193***	0.125***	0.074***	0.036**	0.023	0.013	-0.004	-0.061***	-0.166***
	(0.110)	(0.090)	(0.047)	(0.027)	(0.023)	(0.018)	(0.017)	(0.015)	(0.016)	(0.017)	(0.037)
Observations	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927
Auction format						ABL					
Dependent						log($Discount$)					
log($BidderSoph$)	0.065***	0.054***	0.026***	0.013***	0.004	0.000	-0.008	-0.022***	-0.051***	-0.081***	-0.091***
	(0.018)	(0.016)	(0.008)	(0.005)	(0.004)	(0.004)	(0.005)	(0.008)	(0.013)	(0.005)	(0.011)
Dependent						$Discount$					
log($BidderSoph$)	0.599***	0.537***	0.295***	0.153***	0.053	0.002	-0.101	-0.297***	-0.764***	-1.330***	-1.615***
	(0.160)	(0.144)	(0.099)	(0.058)	(0.047)	(0.047)	(0.069)	(0.110)	(0.188)	(0.092)	(0.161)
Observations	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES

Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D8 – Controlling for potential cartels in AB auctions.

Dependent	log $|Distance|$							
Auction format	AB							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log($BidderSoph$)	-0.163***	-0.163***	-0.196***	-0.193***	-0.283***	-0.283***	-0.153***	-0.199***
	(0.023)	(0.023)	(0.037)	(0.037)	(0.044)	(0.044)	(0.031)	(0.054)
log(1+$Groupmembers$)	-0.169***	0.031	-0.229***	-0.182	-0.237***	-0.240*		
	(0.032)	(0.068)	(0.045)	(0.114)	(0.054)	(0.137)		
log(1+$Groupmembers$) x $ShareGroupmembers$		-0.416***		-0.729***		-0.465		
		(0.120)		(0.252)		(0.283)		
$ShareGroupmembers$		0.153		1.260**		0.897		
		(0.095)		(0.546)		(0.674)		
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	NO	NO	NO	NO	YES	NO
Firm-FE	NO	NO	YES	YES	NO	NO	NO	YES
Firm-year-FE	NO	NO	NO	NO	YES	YES	NO	NO
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,037	8,037	7,945	7,945	7,945	7,945	3,861	3,828
R-squared	0.202	0.203	0.266	0.267	0.368	0.369	0.181	0.302

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D9 – Controlling for potential cartels in ABL auctions.

Dependent					log $\|Distance\|$				
Auction format					ABL				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
log(*BidderSoph*)	-0.369***	-0.356***	-0.430***	-0.428***	-0.490***	-0.485***	-0.270***	-0.365***	-0.421***
	(0.049)	(0.051)	(0.067)	(0.067)	(0.079)	(0.080)	(0.071)	(0.104)	(0.125)
log(1+*Groupmembers*)	0.029	0.301**	-0.141	0.157	-0.170	0.105			
	(0.056)	(0.120)	(0.124)	(0.444)	(0.150)	(0.603)			
log(1+*Groupmembers*)									
x *ShareGroupmembers*		-0.528**		-0.458		-0.466			
		(0.243)		(0.453)		(0.549)			
ShareGroupmembers		0.315		0.100		0.157			
		(0.263)		(1.385)		(1.861)			
Auction/project controls	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	NO	NO	NO	NO	YES	NO	NO
Firm-FE	NO	NO	YES	YES	NO	NO	NO	YES	NO
Firm-year-FE	NO	NO	NO	NO	YES	YES	NO	NO	YES
Firm-auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,316	1,316	1,316	1,316	1,312	1,312	578	578	578
R-squared	0.264	0.268	0.459	0.461	0.569	0.570	0.240	0.598	0.716

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D10 – Further 2SLS estimations

Dependent variable	log $(BidderSoph)$	log $(BidderSoph)$	$\|Distance\|$	$\|Distance\|$
Auction format	AB+ABL 1'stage	AB+ABL 1'stage	AB+ABL 2'stage	AB+ABL 2'stage
	(1)	(2)	(3)	(4)
$\log(BidderSoph)$			-0.162***	-0.182***
			(0.024)	(0.029)
ABL	-0.535***	-0.612***		
	(0.165)	(0.154)		
$\log(BidderSoph00-03)$	0.705***			
	(0.041)			
$ABL * \log(BidderSoph00-03)$	-0.585***	-0.582***		
	(0.050)	(0.047)		
Auction/project controls	YES	YES	YES	YES
Firm controls	YES	NO	YES	NO
Firm-fixed effects	NO	YES	NO	YES
Firm-auction controls	YES	YES	YES	YES
Observations	2,954	2,954	2,954	2,954
R-squared	0.831	0.815	0.060	0.062
Hansen-J test			0.722	0.490

Robust standard errors clustered at firm-level in parentheses.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.

Table D11 – Additional empirical edvidence.

Dependent variable:	*Normalized Discount*	*Normalized Discount*	*A2*	*A3*	*Auction St.Dev.*	*Auction St.Dev.*	*Firm St.Dev.*	*Firm St.Dev.*	*Firm St.Dev.*	*Firm St.Dev.*
Auction format	AB+ABL	AB+ABL	AB	ABL	AB	ABL	AB	ABL	AB	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ABL	-0.366***	-0.420***								
	(0.022)	(0.018)								
log($No.Participants$)			0.463**	-0.712**						
			(0.221)	(0.284)						
log($MeanBidderSoph$)					-0.304*	-0.734**				
					(0.161)	(0.165)				
log($RollingMeanBidderSoph$)							-0.292***	-0.604***	-0.644***	-0.943***
							(0.047)	(0.168)	(0.113)	(0.225)
Auction/project controls	YES	YES	YES	YES	YES	YES	NO	NO	NO	NO
Firm controls	YES	NO	NO	NO	NO	NO	YES	YES	NO	NO
Firm-fixed effects	NO	YES	NO	NO	NO	NO	NO	NO	YES	YES
Firm-auction controls	YES	YES	NO	NO	NO	NO	NO	NO	NO	NO
Observations	10,428	9,647	232	28	228	26	7,273	656	7,273	656
R-squared	0.188	0.248	0.848	0.993	0.243	0.761	0.212	0.225	0.380	0.601

In columns (1)-(2) and (7)-(10): OLS estimations and robust standard errors clustered at firm-level in parentheses.
In columns (3)-(6), an IRLS estimator is used to account for the influence of outliers (given the small samples).
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.
In columns (1) and (2), the dependent variable is the (min-max rescaled) discount offered by firms. ABL is a dummy variable which takes value 1 (0) if the auction is ABL (AB). Though not reported, the sophistication index is included among the covariates.
In columns (3) and (4) the dependent variable is the (auction-specific) reference point.
In columns (5) and (6) the dependent variable $SDauction$ is the standard deviation of the (absolute value of the standardized) distance of bids from the reference point.
In columns (7) and (12) the dependent variable $RollingSDfirm$ is the rolling standard deviation of the (absolute value of the standardized) distance of bids from the reference point in the last five auctions.
$MeanBidderSoph$ is the average of the sophistication index across firms in the auction.
$RollingMeanBidderSoph$ is the rolling average of the sophistication index of the firm in the last five auctions.
Auction/project controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
In columns (7)-(10) time dummies taking the value 1 for the mean year of the bidders' last five auctions are included.