Open Charm and Beauty Chiral Multiplets in QCD

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We study the dynamics of the the spin zero open charm and beauty mesons using QCD spectral sum rules (QSSR), where we observe the important rôle of the chiral condensate $\langle\bar\psi\psi\rangle$ in the mass-splittings between the scalar-pseudoscalar mesons. Fixing the sum rule parameters for reproducing the well-known $D(0^-)$ and $D_s(0^-)$ masses, we re-obtain the running charm quark mass: $\bar m_c(m_c) = 1.13^{+0.08}_{-0.04}$ GeV, which confirms our recent estimate from this channel [1]. Therefore, using sum rules *with no-free parameters*, we deduce $M_{D_s(0^+)} \simeq (2297 \pm 113)$ MeV, which is consistent with the observed $D_s(2317)$ meson, while a small $SU(3)$ breaking of about 25 MeV for the $D_s(0^+)-D(0^+)$ mass-difference has been obtained. We extend our analysis to the B-system and find $M_{B(0^+)} - M_{B(0^-)} \simeq (422 \pm 196)$ MeV confirming our old result from moment sum rules [2]. Assuming an approximate (heavy and light) flavour and spin symmetries of the mass-splittings as indicated by our results, we also deduce $M_{D_s^*(1^+)} \simeq (2440 \pm 113)$ MeV in agreement with the observed $D_{sJ}(2457)$. We also get: $f_{D(0^+)} \simeq (217 \pm 25)$ MeV much bigger than f_π=130.6 MeV, while the size of the $SU(3)$ breaking ratio $f_{D_s(0^+)}/f_{D(0^+)} \simeq 0.93 \pm 0.02$ is opposite to the one of the 0^- channel of about 1.14.

1 Introduction

The recent BABAR, BELLE and CLEO observations of two new states $D_{sJ}(2317)$ and $D_{sJ}(2457)$ [3] in the $D_s\pi$, $D_s\gamma$ and $D_s\pi\gamma$ final states have stimulated a renewed interest in the spectroscopy of open charm states which one can notice from different recent theoretical attempts to identify their nature [3]. In this paper, we will try to provide the answer to this question from QCD spectral sum rules à la Shifman-Vainshtein-Zakharov [4]. In fact, we have already addressed a similar question in the past [2], where we have predicted using QSSR the mass splitting of the $0^+ - 0^-$ and $1^- - 1^+$ $\bar bu$ mesons using double ratio of moments sum rules based on an expansion in the inverse of the b quark mass. We found that the value of the mass-splittings between the chiral multiplets were about the same and approximately independent of the spin of these mesons signaling an heavy quark-type approximate symmetry:

$$M_{B(0^+)} - M_{B(0^-)} \approx M_{B^*(1^+)} - M_{B^*(1^-)} \approx (417 \pm 212)\text{ MeV} . \quad (1)$$

The effect and errors on the mass-splittings are mainly due to the chiral condensate $\langle\bar\psi\psi\rangle$ and to the value of the b quark mass. In this paper, we shall use an analogous approach to the open charm states. However, the same method in terms of the $1/m_c$ expansion and some other nonrelativistic sum rules will be dangerous here due to the relatively light value of the charm quark mass. Instead, we shall work with relativistic exponential sum rules used successfully in the light quark channels for predicting the meson masses and QCD parameters [5] and in the D and B channels for predicting the (famous) decay constants $f_{D,B}$ [1, 5, 6] and the charm and bottom quark masses [1, 5, 6, 7, 8].

2 The QCD spectral sum rules

We shall work here with the (pseudo)scalar two-point correlators:

$$\psi_{P/S}(q^2) \equiv i \int d^4x\ e^{iqx}\ \langle 0|\mathcal{T} J_{P/S}(x) J^\dagger_{P/S}(0)|0\rangle, \quad (2)$$

built from the (pseudo)scalar and (axial)-vector heavy-light quark currents:

$$J_{P/S}(x) = (m_Q \pm m_q)\bar Q(i\gamma_5)q, \qquad J^\mu_{V/A} = \bar Q\gamma^\mu(\gamma_5)q\ . \quad (3)$$

If we fix $Q \equiv c$ and $q \equiv s$, the corresponding mesons have the quantum numbers of the $D_s(0^-)$, $D_s(0^+)$ mesons. m_Q and m_s are the running quark masses. In the (pseudo)scalar channels, the relevant sum rules for our problem are the Laplace transform sum rules:

$$\mathcal{L}^H_{P/S}(\tau) = \int_{t_\leq}^{\infty} dt \, \mathrm{e}^{-t\tau} \, \frac{1}{\pi} \, \mathrm{Im}\psi^H_{P/S}(t), \quad \text{and} \quad \mathcal{R}^H_{P/S}(\tau) \equiv -\frac{d}{d\tau} \log \mathcal{L}^H_{P/S}(\tau), \tag{4}$$

where $t_\leq$ is the hadronic threshold, and H denotes the corresponding meson. The latter sum rule, or its slight modification, is useful, as it is equal to the resonance mass squared, in the simple duality ansatz parametrization of the spectral function:

$$\frac{1}{\pi} \, \mathrm{Im}\psi^H_P(t) \simeq f_D^2 M_D^4 \delta(t - M_D^2) \; + \; \text{“QCD continuum”}\Theta(t - t_c), \tag{5}$$

where the “QCD continuum comes from the discontinuity of the QCD diagrams, which is expected to give a good smearing of the different radial excitations [1]. The decay constant f_D is analogous to $f_\pi = 130.6$ MeV; t_c is the QCD continuum threshold, which is, like the sum rule variable τ, an (a priori) arbitrary parameter. In this paper, we shall impose the τ- and t_c-stability criteria for extracting our optimal results. The corresponding t_c value also agrees with the FESR duality constraints [9, 5] and very roughly indicates the position of the next radial excitations. However, in order to have a conservative result, we take a largest range of t_c from the beginning of τ- to the one of t_c-stabilities.
The QCD expression of the correlator is well-known to two-loop accuracy (see e.g. [5] and the explicit expressions given in [1, 6]), in terms of the perturbative pole mass M_Q, and including the non-perturbative condensates of dimensions less than or equal to six [2]. For a pedagocial presentation, we write the sum rule in the chiral limit ($m_s = 0$), where the expression is more compact. In this way, one can understand qualitatively the source of the mass splittings. The sum rule reads to order α_s:

$$\begin{aligned} \mathcal{L}^H_{P/S}(\tau) \;\; = \;\; & M_Q^2 \Bigg\{ \int_{M_Q^2}^{\infty} dt \, \mathrm{e}^{-t\tau} \, \frac{1}{8\pi^2} \left[3t(1-x)^2 \left[1 + \frac{4}{3} \left(\frac{\bar{\alpha}_s}{\pi} \right) f(x) \right] + \mathcal{O}(\alpha_s^2) \right] \\ & + C_4 \langle O_4 \rangle_{P/S} + \tau C_6 \langle O_6 \rangle_{P/S} \, \mathrm{e}^{-M_Q^2 \tau} \Bigg\} \, , \end{aligned} \tag{6}$$

The different terms are [3]:

$$\begin{aligned} x &\equiv M_Q^2/t, \\ f(x) &= \frac{9}{4} + 2\mathrm{Li}_2(\mathrm{x}) + \log \mathrm{x} \log(1 - \mathrm{x}) - \frac{3}{2} \log(1/\mathrm{x} - 1) \\ & \quad - \log(1-x) + x \log(1/x - 1) - (x/(1-x)) \log x, \\ C_4 \langle O_4 \rangle_{P/S} &= \mp M_Q \langle \bar{d} d \rangle \, \mathrm{e}^{-M_Q^2 \tau} + \langle \alpha_s G^2 \rangle \left(\frac{3}{2} - M_Q^2 \tau \right) /12\pi \\ C_6 \langle O_6 \rangle_{P/S} &= \mp \frac{M_Q}{2} \left(1 - \frac{M_Q^2 \tau}{2} \right) g \langle \bar{d} \sigma_{\mu\nu} \frac{\lambda_a}{2} G_a^{\mu\nu} d \rangle \qquad (7) \\ & \quad - \left(\frac{8\pi}{27} \right) \left(2 - \frac{M_Q^2 \tau}{2} - \frac{M_Q^4 \tau^2}{6} \right) \rho \alpha_s \langle \bar{\psi} \psi \rangle^2 \, , \qquad (8) \end{aligned}$$

where we have used the contribution of the gluon condensate given in [11], which is IR finite when letting $m_q \to 0$ [4]. The previous sum rules can be expressed in terms of the running mass $\bar{m}_Q(\nu)$ through the perturbative two-loop relation [13]:

$$M_Q \;\; = \;\; \bar{m}_Q(p^2) \left[1 + \left(\frac{4}{3} + \ln \frac{p^2}{M_Q^2} \right) \left(\frac{\bar{\alpha}_s}{\pi} \right) + \mathcal{O}(\alpha_s^2) \right] \, , \tag{9}$$

where M_Q is the pole mass. Throughout this paper we shall use the values of the QCD parameters given in Table 1.

[1] At the optimization scale, its effect is negligible, such that a more involved parametrization is not necessary.

[2] We shall include the negligible contribution from the dimension six four-quark condensates, while we shall neglect an eventual tachyonic gluon mass correction term found to be negligible in some other channels [10].

[3] Notice a missprint in the expression given in [2, 1] which does not affect the result obtained there.

[4] The numerical change is negligible compared with the original expression obtained in [12].

Table 1: QCD input parameters used in the analysis.

Parameters	References
$\Lambda_4 = (325 \pm 43)$ MeV	[5]
$\Lambda_5 = (225 \pm 30)$ MeV	[5]
$\bar{m}_b(m_b) = (4.24 \pm 0.06)$ GeV	[5, 8, 1, 14]
$\bar{m}_s(2\ \text{GeV}) = (111 \pm 22)$ MeV	[5, 8, 15, 16]
$\langle \bar{d}d\rangle^{1/3}(2\ \text{GeV}) = -(243 \pm 14)$ MeV	[5, 8, 17]
$\langle \bar{s}s\rangle / \langle \bar{d}d\rangle = 0.8 \pm 0.1$	[5, 18]
$\langle \alpha_s G^2 \rangle = (0.07 \pm 0.01)\ \text{GeV}^4$	[5, 19]
$M_0^2 = (0.8 \pm 0.1)\ \text{GeV}^2$	[5, 2]
$\alpha_s \langle \bar{\psi}\psi\rangle^2 = (5.8 \pm 2.4) \times 10^{-4}\ \text{GeV}^6$	[5, 19, 20]

We have used for the mixed condensate the parametrization:

$$g\langle \bar{d}\sigma_{\mu\nu}\frac{\lambda_a}{2}G_a^{\mu\nu}d\rangle \quad = \quad M_0^2\langle \bar{d}d\rangle, \tag{10}$$

and deduced the value of the QCD scale Λ from the value of $\alpha_s(M_Z) = (0.1184 \pm 0.031)$ [21, 22]. We have taken the mean value of m_s from recent reviews [5, 8, 16].

3 Calibration of the sum rule from the $D(0^-)$ and $D_s(0^-)$ masses and re-estimate of $\bar{m}_c(m_c)$

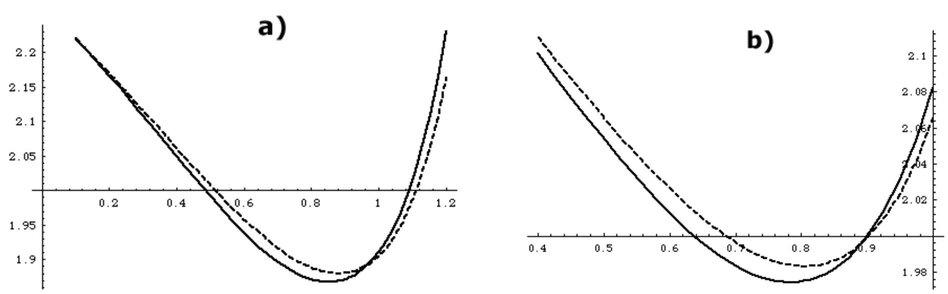

Figure 1: τ in GeV^{-2}-dependence of the a) $M_D(0^-)$ in GeV for $\bar{m}_c(m_c) = 1.11$ GeV and b) $M_{D_s(0^-)}$ in GeV for $\bar{m}_c(m_c) = 1.15$ GeV at a given value of $t_c = 7.5$ GeV^2. The dashed line is the result including the leading $\langle\bar{\psi}\psi\rangle$ contribution. The full line is the one including non-perturbative effects up to dimension-six.

This analysis has been already done in previous papers to order α_s and α_s^2 and has served to fix the running charm quark mass. We repeat this analysis here to order α_s for a pedagogical purpose. We show in Fig. 1a), the τ-dependence of the $D(0^-)$ and in Fig 1b) the one of the $D_s(0^-)$ masses for a given value of t_c, which is the central value of the range:

$$t_c = (7.5 \pm 1.5)\ \text{GeV}^2\ , \tag{11}$$

where the lowest value corresponds to the beginning of τ-stablity and the highest one to the beginning of t_c stability obtained by [1, 6, 5] in the analysis of f_D and f_{D_s}. This range of t_c-values covers the different choices of t_c used in the sum rule literature. As mentioned previously, the one of the beginning of t_c-stability coïncides, in general, with the value obtained from FESR local duality constraints [9, 5]. Using the input values of QCD parameters in Table 1, the best fits of $D(0^-)$ (resp. $D_s(0^-)$) masses for a given value of $t_c = 7.5$ GeV^2 correspond to a value of $\bar{m}_c(m_c)$ of 1.11 (resp. 1.15) GeV. Taking the mean value as an estimate, one can deduce:

$$\bar{m}_c(m_c^2) = (1.13^{+0.07}_{-0.02} \pm 0.02 \pm 0.02 \pm 0.02) \text{ GeV} , \tag{12}$$

where the errors come respectively from t_c, $\langle\bar{\psi}\psi\rangle$, Λ and the mean value of m_c required from fitting the $D(0^-)$ and $D_s(0^-)$ masses. This value is perfectly consistent with the one obtained in [1, 6] obtained to the same order and to order α_s^2, indicating that, though the α_s^2 corrections are both large in the two-point function and m_c [23], it does not affect much the final result from the sum rule analysis. In fact, higher corrections tend mainly to shift the position of the stability regions but affect slightly the output value of m_c. This value of m_c is in the range of the current average value (1.23 ± 0.05) GeV reviewed in [5, 8, 21]. However, it does not favour higher values of m_c allowed in some other channels and by some non relativistic sum rules and approaches. However, these non relativistic approaches might be quite inaccurate due to the relative smallness of the charm quark mass. Higher values of m_c would lead to an overestimate of the $D(0^-)$ and $D_s(0^-)$ masses. In the following analysis, we shall use the central value $\bar{m}_c(m_c) = 1.11$ (resp. 1.15) GeV for the non-strange (resp. strange) meson channels.

4 The $0^+ - 0^-$ meson mass-splittings

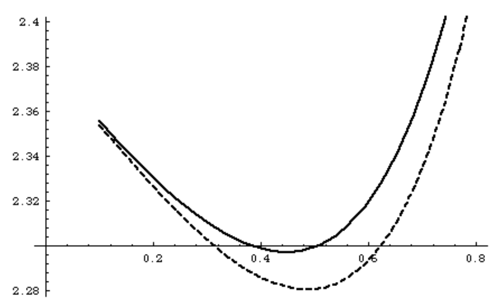

Figure 2: Similar to Fig. 1 but τ-behaviour of $M_{D_s(0^+)}$ for given values of $t_c = 7.5$ GeV^2 and $m_c(m_c) = 1.15$ GeV.

- We study in Fig. 2), the τ-dependence of the $D_s(0^+)$ mass at the values of t_c and m_c obtained previously. In this way, we obtain:

$$M_{D_s(0^+)} \simeq (2297^{+81+63}_{-98-70} \pm 11 \pm 2 \pm 11) \text{ MeV} \quad \Longrightarrow \quad M_{D_s(0^+)} - M_{D_s(0^-)} = (328 \pm 113) \text{ MeV} , \tag{13}$$

  where the errors come respectively from t_c, m_c, $\langle\bar{\psi}\psi\rangle$, m_s, and Λ. We have used the experimental value of $M_{D_s(0^-)}$. The reduction of the theoretical error needs precise values of the continuum threshold [5] and of the charm quark mass which are not within the present reach of the estimate of these quantities. Further discoveries of the continuum states will reduce the present error in the splitting. One should also notice that in the ratio of sum rules with which we are working, we expect that perturbative radiative corrections are minimized though individually large in the expression of the correlator and of the quark mass.

- The value of the mass-splittings obtained previously is comparable with the one of the $B(0^+)$-$B(0^-)$ given in Eq. (1), and suggests an approximate heavy-flavour symmetry of this observable.

- We also derive the result in the limit of $SU(3)_F$ symmetry where the strange quark mass is put to zero, and where the $\langle\bar{\psi}\psi\rangle$ condensate is chirally symmetric ($\langle\bar{s}s\rangle = \langle\bar{d}d\rangle$). In this case, one can predict an approximate degenerate mass within the errors:

$$M_{D_s(0^+)} - M_{D(0^+)} \simeq 25 \text{ MeV} , \tag{14}$$

[5]The range of t_c values 6-9 GeV^2 obtained previously for the $D(0^-)$ mesons coïncides a posteriori with the corresponding range for the $D(0^+)$ meson if one assumes that the splitting between the radial excitations is the same as the one between the ground states, i.e about 300 MeV. We have cheked during the analysis that this effect is unimportant and is inside the large error induced by the range of t_c used.

which indicates that the mass-splitting between the strange and non-strange 0^+ open charm mesons is almost not affected by $SU(3)$ breakings, contrary to the case of the 0^- mesons with a splitting of about 100 MeV.

- We extend the analysis to the case of the $B(0^+)$ meson. Here, it is more informative to predict the ratio of the 0^+ over the 0^- masses as the prediction on the absolute values though presenting stability in τ tend to overestimate the value of M_B. We obtain:

$$\frac{M_{B(0^+)}}{M_{B(0^-)}} \simeq (1.08 \pm 0.03 \pm 0.03 \pm 0.02 \pm 0.02) \quad \Longrightarrow \quad M_{B(0^+)} - M_{B(0^-)} \simeq (422 \pm 196) \text{ MeV} , \quad (15)$$

where the errors come respectively from t_c taken in the range $43 - 60$ GeV^2, m_b, $\langle\bar{\psi}\psi\rangle$, and τ. It agrees with the result in Eq. (1) obtained from moment sum rules [2]. We have used the value of $\bar{m}_b(m_b)$ given in Table 1.

5 The $1^+ - 1^-$ meson mass-splitting

Our previous results in Eqs. (1), (13) to (15) suggest that the mass-splittings are approximately (heavy and light) flavour and spin independent. Therefore, one can write to a good approximation the empirical relation:

$$M_{D_s(0^+)} - M_{D_s(0^-)} \approx M_{D(0^+)} - M_{D(0^-)} \approx M_{B(0^+)} - M_{B(0^-)} \approx M_{B(1^+)} - M_{B(1^-)} \approx M_{D_s^*(1^+)} - M_{D_s^*(1^-)} . \quad (16)$$

Using the most precise number given in Eq. (13), one can deduce:

$$M_{D_s^*(1^+)} - M_{D_s^*(1^-)} \simeq (328 \pm 113) \text{ MeV} \quad \Longrightarrow \quad M_{D_s^*(1^+)} \simeq (2440 \pm 113) \text{ MeV} . \quad (17)$$

This result is consistent with the 1^+ assignement of the $\bar{c}s$ meson $D_{sJ}(2457)$ discovered recently [3]. In a future work, we plan to study in details this spin one channel using QSSR.

6 The $D(0^+)$ and $D_s(0^+)$ decay constants

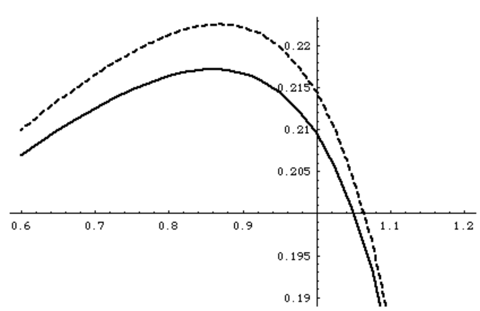

Figure 3: Similar to Fig. 1 but τ-behaviour of $f_D(0^+)$ for given values of $t_c = 7.5$ GeV^2 and $\bar{m}_c(m_c) = 1.11$ GeV.

For completing our analysis, we estimate the decay constant $f_{D(0^+)}$ analogue to $f_\pi = 130.6$ MeV. We show the behaviour of $f_{D(0^+)}$ versus τ, where a goood stablity is obtained. Adopting the range of t_c-values obtained previously and using $\bar{m}_c(m_c) = 1.11^{+0.08}_{-0.04}$ GeV required for a best fit of the non strange $D(0^+)$ meson mass, we deduce to two-loop accuracy:

$$f_{D(0^+)} = (217^{+5+15}_{-15-19} \pm 10 \pm 10) \text{ MeV} , \quad (18)$$

where the errors come respectively from the values of t_c, m_c, $\langle\bar{\psi}\psi\rangle$ and Λ. We have fixed $M_{D(0^+)}$ to be about 2272 MeV from our previous fit. It is informative to compare this result with the one of $f_D = (205 \pm 20)$ MeV, where the main difference can be attributed by the sign flip of the quark condensate contribution in the QCD expression of the corresponding correlators. A numerical study of the $SU(3)$ breaking effect leads to:

$$r_s \equiv \frac{f_{D_s(0^+)}}{f_{D(0^+)}} \simeq 0.93 \pm 0.02 \; , \qquad (19)$$

which is reverse to the analogous ratio in the pseudoscalar channel $f_{D_s}/f_D \simeq 1.14 \pm 0.04$ given semi-analytically in [24]. In order to understand this result, we give a semi-analytic parametrization of this $SU(3)$ breaking ratio. Keeping the leading term in m_s and $\langle\bar\psi\psi\rangle$, one obtains:

$$r_s \simeq \left(1 - \frac{m_s}{m_c}\right)\left[1 - 7.5\langle\bar s s - \bar d d\rangle\right]^{1/2}\left(\frac{M_{D_s(0^+)}}{M_{D(0^+)}}\right)^2 \simeq 0.9 \; , \qquad (20)$$

where the main effect comes from the negative sign of the m_s contribution in the overall normalization of the scalar current, while the meson mass ratio does not compensate this effect because of the almost equal mass of $D_s(0^+)$ and $D(0^+)$ obtained in previous analysis. This feature is opposite to the case of $f_{D(0^-)}$.

7 Summary and conclusions

Due to the experimental recent discovery of the $D_{sJ}(2317)$ and $D^*_{sJ}(2457)$, we have analyzed using QSSR the dynamics of the $0^\pm$ and $1^\pm$ open charm and beauty meson channels:

- We have re-estimated the running charm quark mass from the D and D_s mesons. The result in Eq. (12) confirms earlier results obtained to two- and three-loop accuracies [6, 1].
- We have studied the mass-splittings of the 0^+-0^- in the D systems using QSSR. Our result in the (0^+) channel given in Eq. (13) agrees with the recent experimental findings of the $D_{sJ}(2317)$ suggesting that this state is a good candidate for being a $\bar c s$ 0^+ meson.
- We also found, in Eq. (14), that the $SU(3)$ breaking responsible of the mass-splitting between the $D_s(0^+)$ and $D(0^-)$ is small of about 25 MeV contrary to the case of the pseudoscalar D_s-D mesons of about 100 MeV.
- We have extended our analysis to the B-system. Our results in Eqs. (13), (15) and (1) suggest an approximate (light and heavy) flavour and spin symmetries of the meson mass-splittings. We use this result to get the mass of the $\bar c s$ $D^*_s(1^+)$ meson in Eq. (17), which is in (surprising) good agreement with the observed $D^*_{sJ}(2457)$.
- Using QSSR, we have also determined the decay constants of the 0^+ mesons and compare them with the ones of the 0^- states. The result in Eq. (18), which is similar to the pseudoscalar decay constant $f_D \simeq 205$ MeV, suggests a huge violation of the heavy quark symmetry $1/\sqrt{M_D}$ scaling law. Finally, our results in Eqs. (19) and (20) indicate that the $SU(3)$ breaking act in an opposite way compared to the case of the 0^- channels.

Experimental or/and lattice measurements of the previous predictions are useful for testing the validity of the results obtained to two-loop accuracy in this paper from QCD spectral sum rules. However, a complete confirmation of the nature of these new states needs a detail study of their production and decays. We plan to come back to these points in a future work.

During the editorial preparation of this paper, there appears in the literature a paper using sum rules method to the same channel [25]. Instead of our result in Eq. (13): $M_{D_s(0+)} = (2297 \pm 113)$ MeV, the authors obtain $M_{D_s(0+)} = (2480 \pm 30)$ MeV which is higher than the BABAR result $D_s(2317)$ by about 160 MeV. Considering this deviation as significant, the authors conclude that the experimental candidate cannot be a $\bar c s$ state contrary to the conclusion reached in the present paper. However, by scrutinizing the analysis of Ref. [25], we find that the true errors of the analysis have been underestimated:
• As one can see from their figures, the quoted error of 30 MeV only takes into account the one due to the choice of the QCD contiuum threshold taken in a smaller range 8.1-9.3 GeV^2, than the more conservative value 6-9 GeV^2 used in the present paper, which induces a larger error of 80-98 MeV.
• The high central value obtained in [25] is related to a higher choice of the (ill-defined) charm quark pole mass of 1.46 GeV compared to the value 1.3 GeV which would have been deduced from its relation with the running charm quark mass 1.15 GeV in Eq. (9) used in this paper to reproduce the $D_s(0^-)$ mass. As the analysis is very sensitive to m_c, its induced error should be included in the final number. With m_c in Eq. (12), this uncertainty is about 70 MeV, and might be bigger if one considers all range of m_c given in the

literature. In this paper, we have used M_D andM_{D_s} as alternative channels for extracting m_c.
• Taking properly the different sources of errors including the running of condensate and the effects of some other anomalous dimensions, one then leads to a consistency within the errors of both theoretical estimates with the present data.

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