Mathematical Approaches to Evaluation of Old Maps Contents and Accuracy

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Old maps and stages of their development. What is it an old map? Geographical grids on old maps, drawing, measurement and accuracy. Various Prime Meridians and their history. Geographical coordinates. Modern methods of cartometric analysis. Old Czech maps. Application of Misysview, Infomapa, and MATKART software. Results of evaluation.

Key words*: cartometry, cartometric analysis, old maps, geographical grid, coordinate systems, Helmert's transformation, statistical tests.*

Introduction

The definition of an „old map" is quite problematic. The period after the Discovery of America by Christopher Columbus in 1492 is most commonly considered for a golden era of Cartography. This era was followed by an approximately 30 years' long period of Ocean voyage discoveries of the known World, with the exception of Polar Regions and of Australia. This was an important impulse to development of Cartography. At the beginning the new information was entered into existing maps. The first drawings were only very approximate and positionally inaccurate. The maps were drawn on very expensive handmade parchment and existed only in one original. Later on these maps were engraved in copper plates and their black and white prints were colored by hand and published in tens of copies. Series of maps were assembled in atlases that belong at present to treasures of the most famous world galleries as they represent cultural heritage of humanity.

There is a question until what time the maps should be designated as the old once and since when the maps are considered as new once, i.e. modern once. The authors of this contribution decided to consider for „old maps" all cartographic products drawn up mostly by individual persons by hand on paper or engraved in metal. As long as these maps are produced in a cartographic projection (usually conic or azimuthal projections) and contain plot of geographic network or graphic length scale it is relatively easy to determine their positional accuracy. But it is necessary to point out that the positional accuracy is not constant on the entire area of the map and that it varies in different parts of the map. These variations can be represented graphically by means of quasideformation lines, i.e. lines of equal distortion of a feature investigated by cartometric method on the examined map.

The era of old maps defined in the way described in this presentation ends approximately in the 18th century. Later the maps were already produced by skilled teams of state employees and experts. These maps served mainly for land taxation – cadastral maps - and for military use – topographic surveys. It is necessary to remind, that the accurate geodetic control was built up for the Cadastral Map at scale 1:2880 as well as for Military Survey at scale 1: 28 880. Both of these cartographic products are still in use. In the Czech Lands and also in Slovakia these maps were represented by so called Maps of the Stable Cadastre and by maps of the 2nd Military Survey. In Czech Lands these surveys were carried out in the period 1807-1869 based on decision of Emperor Francis I. Originals of these maps are deposited in the War Archives in Vienna. But we are not going to investigate these maps which even from present point of view are modern once, but they may be consider as a milestone between historical and modern cartography.

Geographic Networks on Old Maps

It is necessary to bear in mind that in the time of the creation of old maps existed practically no accurate methods for determination of geographic coordinates and there was not a common opinion on the position of the Prime Meridian as it was the time before invention of the first precision clock which has been the most important tool for determination of longitude. We do not even know if a cartographic projection was used for construction of these maps. There are almost no publications dealing with an application of Prime Meridian by ancient cartographers. This applies namely to regional maps that were mentioning a Prime Meridian only exceptionally. Without knowing the assumptions of ancient cartographers and astronomers on the position

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of the Prime Meridian for calculation of longitude in the eastern part of the Mediterranean it is not possible to determine the position of the Prime Meridian used for regional maps in Central Europe. This situation has been briefly described by e.g. Peter Meurer by following words: „ Special literature dealing with analysis of longitude practically does not exist".

Historical Review of Geographic Coordinates

Let us briefly remind their definitions:

Latitude φ: Angle between the normal line in point P and the plane of equator. <0, 90°> for the northern hemisphere and <0, -90°> for the southern one.

Longitude λ: Angle between the plane of the local meridian and the plane of the Prime Meridian. <0°, 180°> for the eastern hemisphere and <0°, -180°> for the western one.

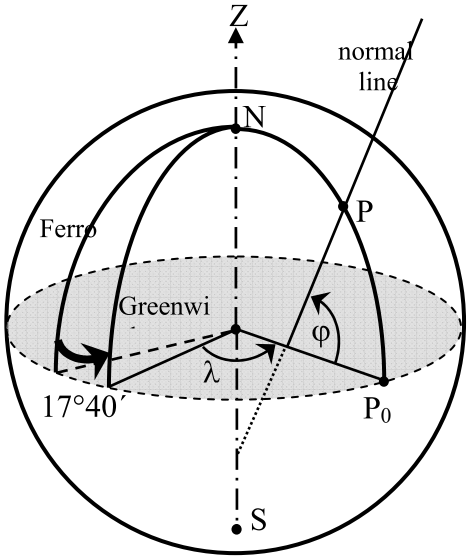

Fig. 1. Geographic coordinates.

Determination of latitude on the northern hemisphere is very simple. It is the vertical angle of the Polar Star above the horizon. This is because the Globe rotation axis intersects the skies just in the position of this star. In other words – all the stars of the northern hemisphere circulate around the Polar Star.

Determination of the longitude is a much more complicated than determination of the latitude. It is because in the past there was no uniquely determined Prime (Zero) Meridian and so different countries were using different Prime Meridians. The most ancient Prime Meridian, used already in the antiquity was the Ferro Meridian. This Meridian traverses lighthouse Faro de Orchilla el Hierro which is the most western place of Europe situated on Canary Islands belonging to Spain.

Difference in longitude between Ferro Meridian and Greenwich Meridian is 17º 40´. Nevertheless it is necessary to add that the value applied in the lands of former Austro-Hungarian Monarchy was more or less a convention than an accurate value, since the Cape of Orchilla lies 18º 09´ to the west from Greenwich as it is possible to find out e.g. by Google Earth.

Fig. 2. Stone cross marking the position of the Ferro Prime Meridian (Wikipedia).

Fig. 3. Lighthouse Faro de Orchilla (Wikipedia).

Also the French had their own Prime Meridian. Jean-Mathieu de Chazelles (1657 – 1710) determined on demand of the Science Academy in Paris longitudinal distances from the Paris Prime Meridian on several places in the region of East Mediterranean. Another French scientist De L´Isle (1675 – 1726) built cartography on a new basis. In 1724 he defined the Ferro Prime Meridian as a meridian with longitudinal distance exactly 20° to the West from the Observatory of Paris. This enabled a more realistic cartographic presentation of continents.

A Prime Meridian by itself is not sufficient for longitudinal determination of observer's position, e.g. of a ship on the Sea. It is necessary to know the time difference between the time valid at the Prime Meridian and the time at the local meridian, i.e. the position of the observer. In order to determine this difference an accurate clock is needed – a chronometer – and astronomic tables – the Star Almanac, with times of selected stars culminations at the Prime Meridian.

The first known proposal of absolute time measurement comes from the famous astronomer Galileo who already in 1616 proposed a method of time measurement by movements of the moons of Jupiter. The correspondence lasted 16 long years before it was interrupted by order of the Saint Inquisition without any result (except for the home arrest for Galileo). On October 22, 1707 a great naval disaster happened at the south east coast of England near Scilly Islands. Four of five British war vessels were shipwrecked on their home night voyage after a 12 days long navigation in foggy weather by. The catastrophe caused death of many men and resulted also in announcement of a reward for invention of a method for determination of longitude. Many scientists and savants were trying to solve this problem, e.g. Galileo Galilei, Leonard Euler, Edmond Halley, Robert Hooke, etc.

In 1714 the British Parliament adopted Edict on Longitude showing thus the priority in discovering a method for Longitude determination. The breakthrough had been invention of a precision clock by John Harrison in 1715 who constructed a precision marine chronometer. The weight of his first clock H1 was 250 kg and it was not very suitable instrument to use for navigation. Nevertheless his fourth model H4 from 1759 represented already a usable clock (diameter 12 cm). It is considered to be the most famous clock in the world (at least for navigation on Sea). The construction of this clock took to Harrison his entire life and the construction was completed only by his son William. The clock was tested already in 1784 when it had been known that its construction corresponds to the reward of 20 000 £ promised by the British Admiralty for solving the problem of Longitude determination. During 5 months the clock was losing merely 15 seconds. This enabled full development of Sea navigation using Sun, Moon, and 57 navigational stars for position determination.

The invention of John Harrison's (1693 – 1776) chronometer and Lunar Tables by Johann Tobias Mayers (1723 – 1762) solved the problem of Longitude determination even on Sea.

Fig. 4. Chronometer – model H4 1759 (Wikipedia).

The role of the Prime Meridian plays at present the Greenwich Meridian passing through the Greenwich Observatory in London. This meridian was adopted for the Prime one at a special conference (International Meridian Conference) held in October 1884 in Washington, hosting 41 delegates from 25 countries.

The Conference adopted following principles:

- Of a unique Prime Meridian replacing all the previous once.
- The meridian passing through the passage instrument of the Greenwich Observatory should be accepted as the Prime Meridian.
- All longitudes up to 180° to the East and West should be calculated starting from this Meridian.
- All countries should accept the Universal Day.

- As the Universal Day should be accepted the Mean Solar Day starting at midnight of the Mean Solar Time in Greenwich and lasting 24 hours.
- The Nautical and Astronomical Days should start everywhere by the Mean Midnight.
- All technical studies regulating and contributing to adoption of decimal systems of time and space division should be promoted.

The longitude of 180º represents also the calendar date.

The Prime Meridian was approved by 22 against 1 vote. Haiti voted against. France and Brasilia abstained. Abstention of France was an expression of political and scientific rivalry with England. The same motive was behind the English refusal to accept the decimal system of weights and measures as they stick until now by their miles, yards, feet, and pounds.

Fig. 5. The Prime Meridian - Old Royal Observatory Greenwich (Wikipedia).

By solving the problem of determination of geographic coordinates and by their international standardization the cartographic nets on maps were also unified and are used in this way until nowadays. When studying positional accuracy of old maps we have to carefully investigate what Prime Meridian was used.

Research of the Jean Baptist Homann's Map of Moravia from 1726

An example of a possible cartometric research will be presented on Homann's Map of Moravia. Outlines of the technological procedure are taken over from [1].

John Baptist Homann born in 1644 in Oberkammlach was a cartographer, geographer and publisher in Nürmberg. Among cartographers of his time he belonged to the most recognized representatives of the cartographic profession. Very important year for him was 1766 when he was appointed to Imperial Cartographer. During his life he published round 200 titles, 20 of them were taken over from Netherlands and French models. Very often he used coloring on his maps.

The map shows the territory of Moravia with small overlaps to Bohemia and Silesia. The map is colored. All the settlements are represented by graphic symbols classifying the settlements according their size into: „Urbes praecipuae", „Urbes minores", „Oppida", „Pagi" and „Arces" (in down size order). The maps show also waters, some main roads, ranges of mountains and forests. The frame of the map bears marks of latitudes and longitudes every 2'(graphic) and latitudes every 10' (numeric). Geographic net is drawn in 0.5° intervals. Each „geographic field" is delimited by two parallels and two meridians and marked on the frame by small characters. The Prime Meridian is not mentioned. In the left lower corner is drawn the scale rule. Maps are decorated in the upper and lower parts by allegoric scenes.

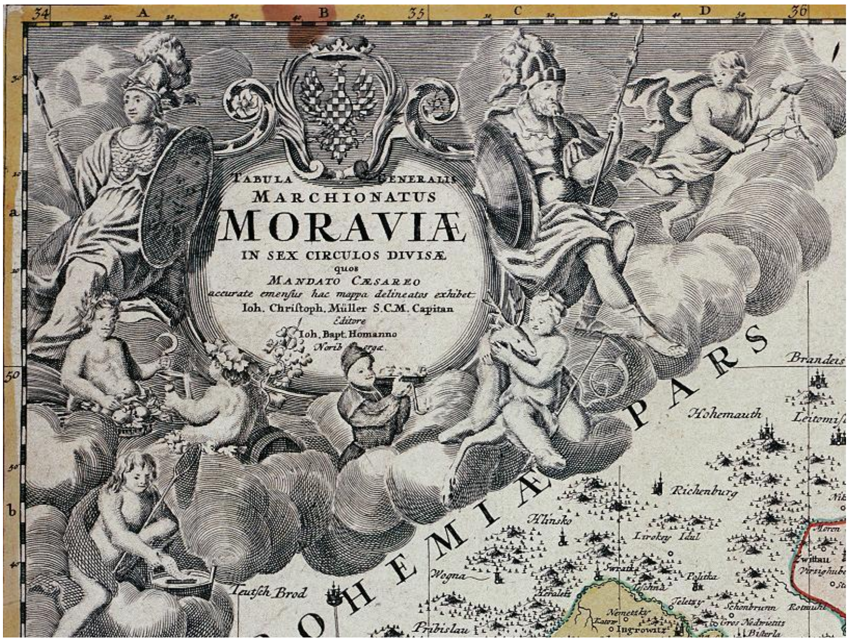

Fig. 6. Homann's map of Moravia from 1726, cutting (Source: The Map Collection of Charles University in Prague).

Selection of Nodal Points

Homann's map includes geographic net with 30' interval in latitudes as well as in longitudes. Intersections of all parallels and meridians form nodal points the coordinates of which were determined by program Misys. In all 46 nodal points were selected. Their geographic coordinates were rounded to minutes and graphical coordinates to full pixels. Graphical coordinates are coordinates of the raster in x and y axes. Origin of the system is situated in the upper right corner of the map.

Selection of Settlements

Names of settlements are written in German. Settlements (towns) are not drawn as built-up areas but only with circular graphic symbols. 64 settlements on the map were selected for determination of graphical and of the S-JTSK coordinates. S-JTSK coordinates were taken in Program Infomapa 14 on the spots that might have been considered as former town centers.

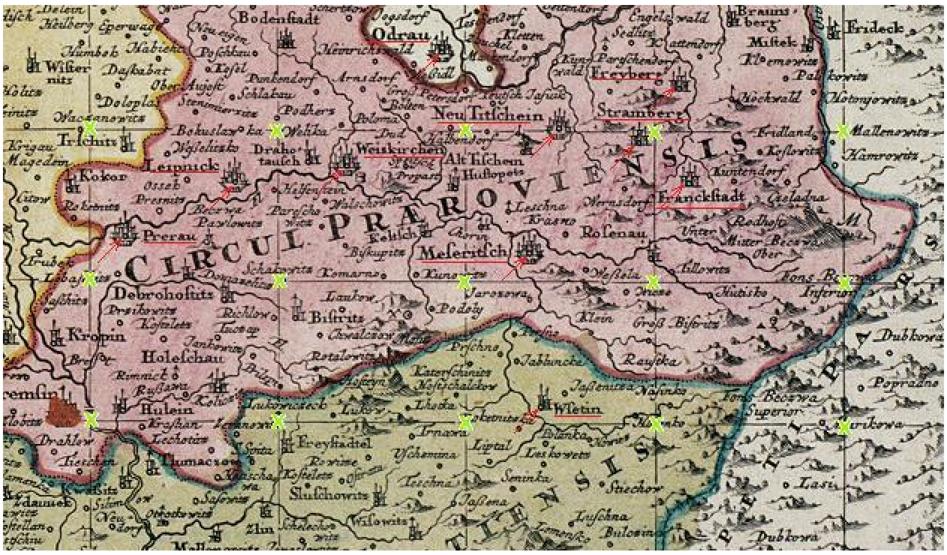

Fig. 7. Map window showing selection of points [1].

Five suitably situated settlements were selected. Graphical coordinates were read on the old map and geographical coordinates determined in Program Infomapa 14. By comparison of these coordinates the mutual shift of these maps was estimated. These points were used for calculation of the average shift, by which was used for correction of all nodal points coordinates. Corrected coordinates were used for calculation of geographic (plane) coordinates in a system that is near to the S-JTSK (System of the Uniform Trigonometric Cadastral Network), (hereafter marked as S-JTSK‘).

Following points were selected:

Tab. 1. Selection of points for shift determination.

Point	Name (old)	Name (new)
S – southern point	Landshut	Lanžhot
N – northern point	Altsstadt	Staré Město
W – western point	Pilgram	Pelhřimov
E – eastern point	Fridek	Frýdek-Místek
M – center	Konitz	Konice

Their coordinates in the map of J.B. Homann:

Tab. 2. Homann: Calculation of shift.

bod	Homann(φ)	Homann(λ)	Infomapa(φ)	Infomapa(λ)	$\Delta\varphi_H$	$\Delta\lambda_H$
S	48 ° 17 ′	36 ° 41 ′	48 ° 43 ′	16 ° 58 ′	0 ° 26 ′	-20 ° 17 ′
N	50 ° 21 ′	36 ° 48 ′	50 ° 9 ′	16 ° 56 ′	0 ° -12 ′	-20 ° 8 ′
W	49 ° 20 ′	34 ° 11 ′	49 ° 26 ′	15 ° 13 ′	0 ° 6 ′	-19 ° 2 ′
E	49 ° 26 ′	38 ° 40 ′	49 ° 41 ′	18 ° 21 ′	0 ° 15 ′	-20 ° -19 ′
M	49 ° 24 ′	36 ° 37 ′	49 ° 36 ′	16 ° 53 ′	0 ° 12 ′	-20 ° 16 ′
				Average: :	**0 ° 9 ′**	**-20 ° 5 ′**

Tab. 3. Resulting shift.

Δφ	Δλ
0 ° 10 ′	**-20 ° 5 ′**

Helmert‘s Transformation

After the calculation of Helmert’s transformation key between system of plane coordinates of nodal points in raster (outgoing system) and the S-JTSK‘ (destination system). The complete calculation was carried out in MATKART VB800 Program.

Program Matkart Version VB800 by prof. Ing. Bohuslav Veverka, Dr.Sc. and Mgr. Monika Čechurová, Ph.D., has been written for calculations on old maps. The dialogue window is divided into two parts that can be used independently. The upper window is for calculation of the transformation key of coordinates of nodal points of the network (intersections of parallels and meridians). Lower window is for calculation of transformation key for selected settlements and water branching. Coordinates of the points are entered by a text file.

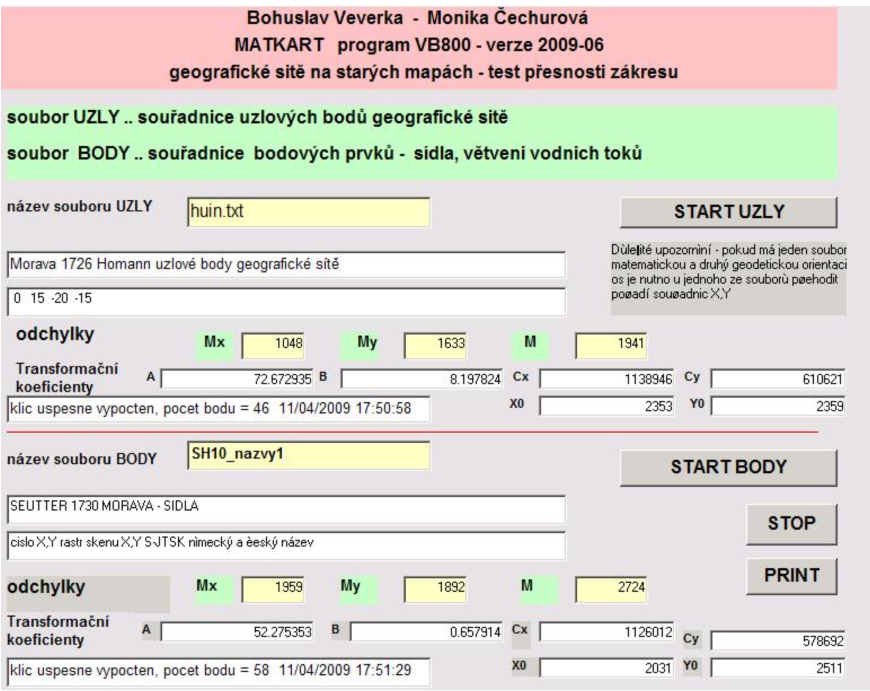

Fig. 8. Example of a Matkart program VB800 dialogue window.

The transformation key was calculated by SW VB800 using coordinates of identical points by Adjustment Method of Least Squares. The transformation coefficients are solved on condition of a minimum sum of squares of distances between the new and the original positions. As identical points are selected nodal points and points of settlements.

Processing of Measured Data

Due to some methodical errors in the selection of points for calculation of the shift (inaccurate registration of coordinates from the raster of Infomapa or identification of incorrect points taken for centers of settlement, etc.) some testing of transformation key sensitivity on change in shift size was carried out. The coordinates were changed by half a minute in both coordinates and in both directions. The changes were registered in a table. As it meant also mutual comparison of both maps, the shifts were changed equally.
Following values of the transformation key were determined for the selected shift:

Tab. 4. Homann: values of transformation key.

id	φ	λ	mx [mm]	my [mm]	m [mm]
0	0 ° 10 ′	-20 ° 5 ′	1101	1668	1999

The changes in the transformation key were as follows:

Tab. 5. Homann – testing of the transformation key.

id	φ	λ	mx	my	m	v_{mx} [mm]	v_{my} [mm]	v_m [mm]
1	-2 ° 10 ′	-22 ° 5 ′	3021	3202	4402	-1920	-1534	-2403
2	-1 ° 40 ′	-21 ° 35 ′	2589	2825	3831	-1488	-1157	-1832
3	-1 ° 10 ′	-21 ° 5 ′	2160	2463	3276	-1059	-795	-1277
4	0 ° -40 ′	-20 ° 35 ′	1739	2127	2748	-638	-459	-749
5	0 ° 40 ′	-19 ° 35 ′	832	1498	1713	269	170	286
6	1 ° 10 ′	-19 ° 5 ′	782	1446	1644	319	222	355
7	1 ° 40 ′	-18 ° 35 ′	992	1528	1822	109	140	177
8	2 ° 10 ′	-18 ° 5 ′	1351	1726	2192	-250	-58	-193

Evaluation of Nodal Points

The transformation key between the plane raster coordinates and the plane coordinates in the S-JTSK‘ (S-JTSK with a false origin) was calculated for a set of 800 nodal points by means of Matkart V800. Errors in the nodal points are between zero up to 6km (see point 37).

There is no coherence in rise and decrease of errors in this set. Higher deviations can be due to a damage of the map at the spots of registration of graphic coordinates, non-marked intersections of parallels and meridians.

Tab. 6. Homann – transformation key of nodal points.

Homann

č.b.	Mx	My	M	j_graf	λ_graf
1	2419	-420	2455	50 ° 30 ′	36 ° 30 ′
2	2005	1366	2426	50 ° 30 ′	37 ° 0 ′
3	1675	1594	2312	50 ° 30 ′	37 ° 30 ′
4	778	2133	2271	50 ° 30 ′	38 ° 0 ′
5	1642	-621	1756	50 ° 0 ′	36 ° 0 ′
6	1716	-419	1766	50 ° 0 ′	36 ° 30 ′
7	1653	214	1667	50 ° 0 ′	37 ° 0 ′
8	1053	863	1362	50 ° 0 ′	37 ° 30 ′
9	471	1217	1305	50 ° 0 ′	38 ° 0 ′
10	-361	1696	1734	50 ° 0 ′	38 ° 30 ′
11	-935	-908	1304	49 ° 30 ′	34 ° 30 ′
12	-71	-607	611	49 ° 30 ′	35 ° 0 ′
13	530	-116	543	49 ° 30 ′	35 ° 30 ′
14	860	-31	861	49 ° 30 ′	36 ° 0 ′
15	1001	-417	1084	49 ° 30 ′	36 ° 30 ′
16	1127	-133	1135	49 ° 30 ′	37 ° 0 ′
17	646	85	652	49 ° 30 ′	37 ° 30 ′
18	158	230	279	49 ° 30 ′	38 ° 0 ′
19	-714	336	789	49 ° 30 ′	38 ° 30 ′
20	-1745	805	1922	49 ° 0 ′	34 ° 0 ′
21	-1182	707	1377	49 ° 0 ′	34 ° 30 ′
22	-557	535	772	49 ° 0 ′	35 ° 0 ′
23	99	586	594	49 ° 0 ′	35 ° 30 ′
24	449	526	692	49 ° 0 ′	36 ° 0 ′
25	713	-292	771	49 ° 0 ′	36 ° 30 ′
26	657	-398	768	49 ° 0 ′	37 ° 0 ′
27	354	-455	576	49 ° 0 ′	37 ° 30 ′
28	-149	-905	917	49 ° 0 ′	38 ° 0 ′
29	-767	-1139	1373	49 ° 0 ′	38 ° 30 ′
30	-3106	2967	4295	48 ° 30 ′	34 ° 0 ′
31	-1895	2493	3131	48 ° 30 ′	34 ° 30 ′
32	-1214	1880	2238	48 ° 30 ′	35 ° 0 ′
33	-571	1409	1521	48 ° 30 ′	35 ° 30 ′
34	-252	975	1007	48 ° 30 ′	36 ° 0 ′
35	119	-126	173	48 ° 30 ′	36 ° 30 ′
36	-30	-761	762	48 ° 30 ′	37 ° 0 ′
37	202	-5756	5760	48 ° 30 ′	37 ° 30 ′
38	-479	-1891	1951	48 ° 30 ′	38 ° 0 ′
39	-844	-2465	2606	48 ° 30 ′	38 ° 30 ′
40	-1370	2067	2480	48 ° 0 ′	35 ° 30 ′
41	-943	1349	1646	48 ° 0 ′	36 ° 0 ′
42	-766	2021	2162	48 ° 0 ′	36 ° 30 ′
43	-497	-1102	1208	48 ° 0 ′	37 ° 0 ′
44	-515	-1791	1863	48 ° 0 ′	37 ° 30 ′
45	-647	-3011	3079	48 ° 0 ′	38 ° 0 ′

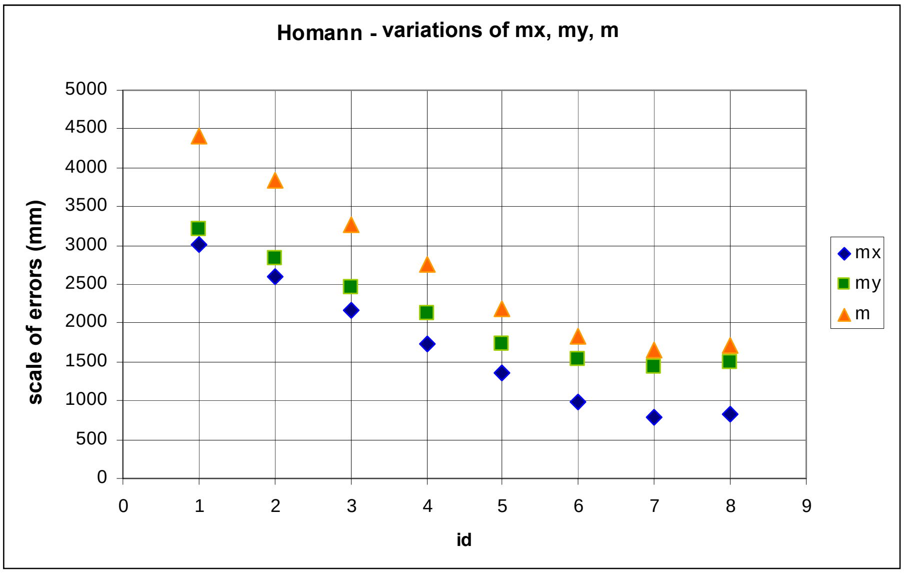

Fig. 9. Homann – Comparison of RMSE by empirical tuning of an optimal shift.

Conclusion

The purpose of this research consisted in finding an optimal transformation key between the representations on an old map and on a modern one. Input data for the calculations are raster values of coordinates of settlements on old map and their coordinates in the S-JTSK system. Helmert's transformation was selected because the number of identical points (settlements, branching of rivers, intersections of geographic network, etc.) is not limited. Another advantage of this transformation method is that it is a residual transformation providing us with calculation protocols with residual errors at identical points that enable us to trace an eventual error of point positioning.

Results presented in [1] a [2] prove that the scale of Homann's map varies about 1:635 000 and the accuracy of Helmert's transformation reaches about two kilometers which was more than expected. However this high accuracy may be due to random errors in maps. It will be necessary to carry out a number of further time consuming observations and calculations for its verification. Nevertheless it is possible to declare that old maps are not only beautiful but – considering the date of their origin – also relatively very accurate from the cartographic and geodetic point of view.

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References

[1] Kratochvílová A.: Accuracy analysis of geographic network on selected old maps (In Czech: Analýza přesnosti zákresu geografické sítě na vybraných starých mapách) [Diploma thesis at the Department of Mapping and Cartography of Faculty of Civil Engineering of the Czech Technical University in Prague, under supervision of prof. Bohuslav Veverka] *Prague 2009*.

[2] Vaněk, J: Cartometric analysis of selected historical maps of Moravia (In Czech: Kartometrická analýza vybraných historických map Moravy. [Diploma Thesis at the Department of Mapping and Cartography of Faculty of Civil Engineering of the Czech Technical University in Prague, under supervision of prof. Bohuslav Veverka] *Prague 2009*.

[3] Veverka, B., Zimová, R.: Topographic and Thematic Cartography (In Czech: Topografická a tematická kartografie), *CTU. Prague 2008*.