On The Economics Of Energy Allocation

MERLIN M. HACKBART, CHARLES W. HULTMAN,
AND JAMES R. RAMSEY*

The Arab petroleum embargo in late 1973, the gasoline shortage and rising fuel prices experienced by consumers in the last few years, and the difficulties encountered by some sectors in securing certain industrial materials focused attention in recent years on the possibility of a serious energy shortage and consequent dislocations in the U.S. Economy. Furthermore, the "energy crisis" revealed the lack of coordinated state and federal policies for dealing with shortages and energy allocations. Market restrictions, such as the OPEC cartel which brought on the energy crisis, are possible in the future. Hence, policies and procedures for allocating fuel during energy cut-offs and shortfalls must be developed by national and state governments if impacts are to be minimized. During the 1974 petroleum shortfall, the federal government allocated a quantity of fuel to state energy offices for further distribution to industries within each state. In general, states did not have analytical procedures or formal policies for allocating the additional fuel.

Purpose

A difficulty in analyzing the impact of a fuel shortage on the state economy and establishing allocative procedures is the large amount of data which must be reduced to management proportions. One approach to this data management problem is the use of an input-output model. Such a model can be used to provide a substantial amount of information about the economic impacts, both direct and indirect, of a fuel shortage on a state economy. In this study, the impact of a *petroleum* shortage on state industrial output and employment are estimated using such a model.

This paper includes a discussion of: (1) the procedures employed, and (2) the empirical results obtained from the application of the Kentucky input-output model in combination with linear programming techniques to establish a petroleum allocation system which would minimize the impact of petroleum cutbacks for Kentucky. The procedures employed in the Kentucky study could be applied in other states and for other energy shortages, such as natural gas, if a sufficiently disaggregated input-output model existed.

---

*Associate Professor and Professor of Economics, University of Kentucky; and Assistant Professor of Economics, Middle Tennessee State University, respectively.

Nature of the Model[1]

The Kentucky input-output table displays an array of interindustry flows of goods and services among sectors in the Kentucky economy. The flows depict industry or sector sales to all other industries or to final users (i.e., government, consumption, investment, and inventories). Consequently, all sector outputs are accounted for by the I-0 table. For this study, the Kentucky I-0 table was converted to a model via the behavioral assumptions of production function stability, constant returns to scale, and no substitution of production factors to carry out the petroleum supply cutback and allocation analysis.[2] The 52-sector Kentucky model was aggregated to a model of 43 sectors for this analysis. The aggregation was necessitated due to the smallness of certain sectors and the lack of sector specific wage data required for certain calculations described in this paper.

Formally, the aggregated 43-sector Kentucky model[3] can be defined as follows:

X_i = Total production in industry i
a_{ij} = Ratio of dollar of input used by industry j and produced by industry i
Y_i = Final demand in industry i

Production in each Kentucky industry is either sold to final consumers or bought by other industries. Given this, an expression for industry i relating final demand to production is:

$$X_i - \sum_{j=1}^{43} a_{ij}X_j = Y_i. \tag{1}$$

Collecting the X_is to form a vector X and Y_is to give a vector Y and a_{ij}s to form a coefficient matrix A allows (1) to be written:

$$X - AX = Y, \tag{2}$$

which can be further simplified to:

$$(I - A)X = Y. \tag{3}$$

This expression gives all final demands Y, and the production necessary to deliver these Ys can be expressed:

$$X = (I - A)^{-1}Y. \tag{4}$$

Methodology

The year 1974 was selected as a base year for analyzing the impact of a cutback in petroleum supplies on the Kentucky economy. By projecting an array of state final demands (Y) for 1974, total production (X) was computed by substitution into (4) above. Similarly, given (X), intersectoral flows were estimated by pre-multiplying the X vector times the coefficient matrix, A. The resulting matrix represented Kentucky economy final demand and production transactions for 1974. In the sections which follow, the respective methodologies utilized for estimating the employment and output impacts of a petroleum cutback as a guide for petroleum allocation policy utilizing linear programming techniques in combination with I-0 are reviewed.

Output impact. In the Kentucky Input-Output Model, row 22 of the A matrix represents the petroleum sector. Consequently, each element of this vector represents a petroleum utilization coefficient as it represents the direct petroleum input per dollar output by sector. Multiplying these coefficients, a_{ij}s, times the respective sector output levels, the X_js, yielded a value C which can be represented as in (5):

$$a_{22,1}X_1 + a_{22,2}X_2 + \ldots a_{22,43}X_{43} = C. \quad (5)$$

C, then, represents the total amount of petroleum (in dollars) directly required to produce the 1974 sector output totals. By changing the equality sign of (5) to a less-than-or-equal-to sign, the sum of oil usage was constrained to a level no greater than the total amount available. For analyzing the impact of a petroleum availability cutback on output, C was then assumed to be reduced to reflect the petroleum cutback. Notationally, the reduced petroleum availability can be represented as C′ and (5) becomes (6) reflecting the petroleum cutback.

$$a_{22,1}X_1 + a_{22,2}X_2 + \ldots a_{22,43}X_{43} = C' \quad (6)$$

A constraint was then imposed on each sector independently so that constraint (6) was expanded into a series of one-element relations written as follows:

$$\begin{aligned} a_{22,1}X_1 &= C_1' \\ a_{22,2}X_2 &= C_2' \\ &\ \ \vdots \\ a_{22,43}X_{43} &= C_{43}' \end{aligned} \quad (7)$$

From (3) it can be shown that given a final demand Y there is only one X which would solve the system. The linear programming problem was initially written as follows:

$$\text{maximize } Z = \sum_{i=1}^{43} X_i$$

$$\text{subject to: } (I - A) \quad X = Y \quad \text{(i)}$$

$$BX \leqq C' \quad \text{(ii)}$$

$$\text{and } X = 0, \quad \text{(iii)} \qquad (8)$$

where B is row 22 of A as described in (6). Here total production in the state (sum of the X_i) is maximized subject to the constraint of the input-output relations and the added constraint on oil supplies.

In order to achieve a linear programming solution, it was also necessary to specify an upper limit on projected final demands. FD_u represented the vector of 43 sectoral final demands.

The new problem was to:

$$\text{maximize } Z = \sum_{i=1}^{43} X_i$$

$$\text{subject to: } (I - A)X - Y = 0 \quad \text{(i)}$$

$$\hat{Y} \leqq FD_u \quad \text{(ii)}$$

$$BX \leqq C' \quad \text{(iii)}$$

$$X \geqq 0 \quad \text{(iv)} \qquad (9)$$

where $\hat{Y}$ = the *amount of final demand actually delivered.*

Solving the linear programming system (9), total production for the state, for each industry, and final demand delivered could have been estimated. However, a possible consequence of the absence of an allocation scheme for distributing oil, given a petroleum cutback and that total state production was to be maximized, was that entire industries could have been eliminated in the solution as far as supplying final demand. This situation was economically untenable but was possible with the final demand constraint stated simply as:

$$\hat{Y} \leqq FD_u \qquad (10)$$

Therefore, an alternative was to place a lower limit on the above constraint, such as:

$$FD_L \leqq \hat{Y} = FD_u \qquad (11)$$

where FD_L was an assumed lower limit on final demands and FD_u represents the previously specified upper limits.

The final step for estimating output impacts involved the development of dual activity variables or shadow prices. A dual variable shows the amount by which the objective function will change for a unit change in the appropriate resource constraint. The original solution variables were production figures, X, and final demand figures, Y. These numbers generally refer to the "columns" of the linear programming arrangement whereas the dual variables indicate the importance of the rows or constraints. In the construction of:

$$\text{maximize } Z = \sum_{i=1}^{43} X_i$$

$$\text{subject to: } (I - A)X - Y = 0 \qquad \text{(i)}$$

$$\hat{Y} \leqq FD_u \qquad \text{(ii)}$$

$$BX \leqq C' \qquad \text{(iii)}$$

$$\text{and } X, Y \geqq 0 \qquad \text{(iv)} \qquad (12)$$

the dual variables are associated with the 43 rows of line i, 43 rows of line ii, and 1 row of line iii of (11). Specifying $X, Y \geqq 0$ simply states that only zero or positive values are allowed in the solution. The concern of this study was with impacts of changes in oil supplies, therefore, of concern was the dual variables of line iii.

It is important to note the difference between the system (12) with a constraint such as that in line iii and a similar system with the following constraints:

$$a_{22,1}X_1 = C_1'$$

$$a_{22,2}X_2 = C_2'$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$a_{22,43}X_{43} \leqq C_{43}'. \qquad (13)$$

Each component of the summation of line iii in (12) is now a separate constraint. Given the values for C_1', C_2', ... C_{43}', a modified system using all 43 rows of (13) was simultaneously solved. If the C_i's are binding then the dual variables define the expected change in each sector's output attributable to a marginal increase in the quantity of oil allocated to each sector (i.e., the marginal value of an additional unit of oil to each sector). More specifically, each element of the resulting vector of dual variables, Θ, represents the expected change in each sector's output given a \$1.00 reduction in the petroleum available to that sector or industry. Vector Θ is of dimensions 1×43. The dual variables were used in the empirical results which are summarized in a following section.

Employment impact. In addition to output impacts as indicated by the dual variables, the input-output model was used to estimate the employment impacts of reductions in petroleum supplies. Using the previously defined notation, now let

V = A value-added coefficient vector for Kentucky industries
V^1 = A labor coefficient row vector
V^{11} = Residual value-added coefficient matrix.

The value-added coefficients in the V vector represented the total value added (labor, capital, land, and entrepreneurship) by industry per \$1.00 output. The vector was disaggregated into a V^1 (labor) vector and a V^{11} (residual) vector.

To disaggregate V, it was necessary to determine labor's share of value added.[4] To find the appropriate labor share of value added in Kentucky, average factor shares for each aggregate sector (manufacturing, non-manufacturing, and utilities) were utilized.[5] Factor shares for these major sectors had been estimated in an earlier study and are summarized in Table 1. In disaggregating the V vector into V^1, the appropriate labor share presented in Table 1 was multiplied by the V vector of value-added coefficients.

The columns of the $(I - A)^{-1}$ matrix contain coefficients showing the direct and indirect requirements by industry for an expansion of final demand by \$1.00 for each sector.

TABLE 1

Allocation of Factor Shares By Industry

	Capital	Labor
Manufacturing	0.50	0.50
Non-Manufacturing	0.25	0.75
Utilities	0.50	0.50

Source: Harold K. Charlesworth and William G. Herzel, *Kentucky Gross State Product, 1969*, (Lexington, Kentucky: Office of Business Development and Government Services, 1972), p. 13.

Pre-multiplying $(I - A)^{-1}$ by the V^1 vector will yield a row vector of labor value-added multipliers; or

$$V^1 (I - A)^{-1} = L \quad (14)$$

where:

L = Direct and indirect labor requirements per dollar of output by sector

Finally, the multiplication of each element in Θ, the expected changes in each sector's output, given a reduction in petroleum supplies, times the corresponding sector element in the vector L, the direct and indirect labor requirements, will yield the change in each sector's output resulting from an oil cutback which can be attributed to labor. That is,

$$\Theta_j l_j = Q \quad (j = 1 \ldots n) \quad (15)$$

where:

Θ_j = jth elements of vector Θ
l_j = jth elements of the L vector
Q = change in each sector's labor requirement resulting from a petroleum cutback

Q can be transformed into an employment impact estimate in terms of numbers of workers by division by the mean wages for each sector for the given period or time. Or,

$$N = \frac{Q}{\overline{W}} \quad (16)$$

where:

$\overline{W}$ = Average wages for each sector
N = Employment impact

Results

The above-outlined procedures, equations 12 through 16, were used to estimate both the output and employment impacts of a given petroleum cutback. To derive these estimates, an array of final demands $(\hat{Y})$ were projected for the state for 1974. The projected final demand was $14.322 billion and total production was computed (by [4]) to be $18.081 billion. As previously indicated, these figures describe the situation in the state economy given no constraints. It should also be noted that the original cutback in oil supplies was arbitrarily assumed to be 12 percent. Given this constraint, total state production declines to $17.454 billion and final demand declines to $13.880 billion. Finally, note that the calculated dual variables show the cost to the state in terms of output of a $1.00 reduction in petroleum to each industry. To avoid infinitesimal employment impact figures, the dual variable was arbitrarily converted to show the impacts of $1,000.00 reduction in petroleum supplies.

The estimated output and employment impacts are presented in Tables 2 and 3, respectively. For example, as seen in Table 2, if the petroleum available to the cigarette and cigar industry (SICs 211 and 212) is reduced by $1,000.00 then the total output for the state will be reduced by $10,105,090.

A rational allocative technique could be developed based upon the impact the reduction in petroleum has on industrial output. Hence, given a reduction in petroleum supplies allocative efforts should be made to guarantee fuel supplies for those industries near and at the top of Table 2. It is these industries for which the loss in state output is the greatest for fuel cutbacks.

At the same time, however, it must be remembered that the figures in Table 2 provide only a first general guide to the cost in terms of output of reducing the availability of petroleum to each of the state's industries. The possibility of other offsetting factors must be recognized. For example, some industries might be able to absorb a small reduction in petroleum availability by using existing supplies more efficiently. Second, some industries initially affected adversely by the indirect impact of petroleum shortages might be able to secure essential input from outside the state.

As shown in Table 3, the reduction in state employment resulting from a $1,000.00 cutback in petroleum supplies to the cigarette and cigar industry would be 234.74 workers. It can be concluded that the industry rankings presented in Table 3 offer a logical distribution procedure for allocating oil cutbacks. It would follow that an effort should be made to guarantee fuel supplies for those industries near or at the top of Table 3 since it is for these industries that the loss in employment is greatest if petroleum supplies are reduced.

Conclusions

The industry rankings presented in Tables 2 and 3 are, as would be expected, very close. In fact, the computation of a rank-order correlation coefficient shows a +0.91 relationship between the two different rankings of impacts from the reduction in petroleum supplies.[6] A perfect correlation between the rankings would be indicated by +1.00.

The results presented in Tables 2 and 3 must be considered tentative and interpreted with caution. First, the findings are subject to the data limitations of input-output analysis.[7] Second, due to the openness of a state economy, it must be assumed that the interregional trade coefficients are stable.[8] Any change in trade patterns to supplement energy resources would, of course, mitigate projected impacts.

TABLE 2

Changes in State Output Given a $1,000 Change
in Petroleum to Each Industry

SIC	Industry	Output Change Per Year
211, 212	Cigarette and Cigar Manufacturing	$10,105,090.00
214	Tobacco Stemming and Redrying	7,580,350.00
79	Amusements	1,559,960.00
48	Communications	1,309,310.00
80, 82	Medical and Educational Services	1,092,260.00
73	Business Services	863,250.00
63	Insurance	862,050.00
24	Lumber and Wood	844,050.00
27	Printing and Publishing	694,860.00
23	Apparel	670,710.00
60-62	Finance	648,030.00
52-59	Retail Trade	615,970.00
33	Primary Metals	592,660.00
65	Real Estate	587,380.00
70	Hotels and Lodging	493,750.00
22	Textile Mill Products	483,440.00
91-97	Government Enterprises	423,420.00
49	Electric, Gas, Water, and Sanitation	443,040.00
34	Fabricated Metals	319,150.00
72	Personal Services	234,670.00
50, 51	Wholesale Trade	220,540.00
32	Stone, Clay, Glass	196,130.00
12	Coal Mining	183,140.00
372-379	Other Transportation Equipment	180,900.00
10, 13, 14	Mining, Other	145,410.00
36	Electrical Machinery	130,310.00
15-17	Construction	130,270.00
26	Paper and Allied Products	122,720.00
35	Machinery Except Electrical	117,990.00
75	Automotive Repairs	116,980.00
0132	Tobacco	116,050.00
371	Motor Vehicle and Equipment	108,490.00
01, 02	Agriculture, Other	103,780.00
20	Food and Kindred Products	100,670.00
38	Professional and Scientific Instruments	97,200.00
42	Motor Freight Transportation	87,950.00
285	Paints and Allied Products	63,830.00
39	Miscellaneous Manufacturing	59,200.00
30	Plastics	56,910.00
25	Furniture and Fixtures	53,176.96
28	Chemicals	39,690.00
29	Petroleum Products	29,460.00
31	Leather Products	27,510.00

Source: Charles Hultman, Tom Patrick, and James Watts, "Energy in the Kentucky Economy: A Preliminary Report," 1974, (an unpublished paper), p. 46.

TABLE 3

Change in State Employment Resulting from a $1,000 Change in Petroleum to Each Industry

SIC	Industry	Change in Employment Per Year
211, 212	Cirgarette and Cigar Manufacturing	234.74
214	Tobacco Stemming and Redrying	229.39
79	Amusements	136.40
65	Real Estate	110.55
73	Business Services	109.45
60-62	Finance	99.98
52-59	Retail Trade	98.52
80, 82	Medical and Educational Services	93.40
48	Communications	80.31
63	Insurance	69.47
70	Hotels and Lodging	44.44
91-97	Government Enterprises	40.38
24	Lumber and Wood	39.86
23	Apparel and Other	33.51
27	Printing and Publishing	24.48
50, 51	Wholesale	20.45
22	Textile Mill Products	21.42
72	Personal Services	19.44
34	Fabricated Metals	14.25
49	Electric, Gas, Water, and Sanitation	14.19
01, 02	Agriculture, Other	14.10
0132	Tobacco	13.40
33	Primary Metals	12.84
75	Auto Repairs	10.46
12	Coal Mining	9.10
36	Electrical Machinery	8.20
32	Stone, Clay, Glass	6.96
10, 13, 14	Mining, Other	6.38
42	Motor Freight and Warehousing	5.93
15-17	Construction	5.06
38	Professional and Scientific Instruments	4.40
26	Paper and Allied Products	3.62
35	Machinery Except Electrical	3.57
285	Paints and Allied Products	3.53
20	Food and Kindred Products	3.12
372-379	Other Transportation Equipment	2.73
25	Furniture and Fixtures	2.15
30	Plastics	2.01
39	Miscellaneous Manufacturing	2.00
371	Motor Vehicle and Equipment	1.64
28	Chemicals	1.58
29	Petroleum Products	.84
31	Leather Products	.09

Given these cautions, the results presented offer two strategies for the allocation of petroleum: (1) allocation so as to maximize state industrial output, and (2) allocation so as to maximize state employment. Allocation based on either strategy has much merit.

Yet while it may seem initially that there could be no objections to allocations based on maximizing either employment or output, in fact, other priorities could be established. For example, an allocative priority system might be based on the nature of the output of the respective industries. Thus, it could probably be argued that a higher priority should be given to medical and educational services (SIC 80 and 82) than to the amusement industry (SIC 79) regardless of the impact on employment or output.

The allocative scheme described, then, does not offer value judgments regarding which objective should be achieved in coping with an energy shortage. It simply provides a rational priority system to achieve an objective which has already been specified. In the end, the planning agencies must combine their perception of social needs and desires with such tools of analysis as input-output tables and linear programming in order to implement an acceptable allocative scheme.

FOOTNOTES

[1]Since the cost of constructing an input-output table employing primary data is generally prohibitive for a state, the present model is based on secondary data. The 1963 U.S. Input-Output table is the source of the basic data. The data have been modified by use of survey and interviews to reflect the methods of production for Kentucky industries which vary considerably from those used in the U.S. Input-Output Model.

[2]See W. Leontief, *Input-Output Economics* New York: Oxford University Press, 1966); and William H. Miernyk, *The Elements of Input-Output Analysis* (New York: Random House, 1965), for further discussion of these behavioral assumptions of input-output modelling.

[3]The Kentucky model uses 1969 data for gross outlays (obtained from state corporate income tax returns, the Internal Revenue Service, and the U.S. Bureau of Census). For the non-manufacturing sectors, two assumptions were made: (1) production data are not significantly affected by inventory variation except in the trade sectors, and (2) receipts were considered to be production. Finally, it is noted that the model has been adjusted to account for price changes from 1963 to 1969.

[4]Previously, economic research has established different weights for labor's share of value added in the production process. For example, Kendrick and Nelson in separate works assign 0.25 as the weight for capital's share and 0.75 as the weight for labor's share. However, Solow's research reveals a weight of 0.51 for capital and 0.49 for labor. See John Kendrick, *Productivity Trends in the United States* (Princeton: Princeton University Press, 1961); Richard R. Nelson, "Aggregate Functions and Medium-Range Growth Projections," *American Economic Review,* LIV, No. 4, (September 1964), p. 578; and Robert Solow, "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics,* XXXIX, No. 3, (August 1957), pp. 312-320. (The Solow figures of 0.51 and 0.49 are taken from the Cheneny, Hollis, Arrow, Minhas, and Solow article in same journal, August 1961. However, this figure is based upon Solow's work which we are citing.)

[5]Harold K. Charlesworth and William G. Herzel, *Kentucky Gross State Product, 1969* (Lexington, Kentucky: Office of Business Development and Government Services, 1972), pp. 12-13.

[6]Let,

H_0: There is no relationship between the ranking of results presented in Tables 2 and 3.

H_1: Reject the null hypothesis.

For this test of significance ($n \geq 10$), the rank correlation coefficient can be tested using the t distribution. For a 5 percent level of significance, the critical t is approximately 1.684 and the computed t is 14.05; hence, the null hypothesis is rejected.

[7]See note 1 above.

[8]For a more detailed discussion of the stability of trade coefficients, see M. Jarvin Emerson, F. Charles Lamphear, and Leonard D. Atencio, "Toward a Dynamic Regional Export Model," *The Annals of Regional Science* (December 1969), pp. 127-138.