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ABSTRACT

Multi-criteria decision-making (MCDM) concerns selecting, ranking or sorting a set of alternatives which are evaluated with respect to a number of criteria. There are several MCDM methods, the two core elements of which are (i) evaluating the performance of the alternatives with respect to the criteria, (ii) finding the importance (weight) of the criteria. There are several methods to find the weights of the criteria, however, when it comes to the alternative measures with respect to the criteria, usually the existing MCDM methods use simple monotonic linear value functions. Usually an increasing or decreasing linear function is assumed between a criterion level (over its entire range) and its value. This assumption, however, might lead to improper results. This study proposes a family of piecewise value functions which can be used for different decision criteria for different decision problems. Several real-world examples from existing literature are provided to illustrate the applicability of the proposed value functions. A numerical example of supplier selection (including a comparison between simple monotonic linear value functions, piecewise linear value functions, and exponential value functions) shows how considering proper value functions could affect the final results of an MCDM problem.

1. Introduction

Decision theory is primarily concerned with identifying the best decision. In many real-world situations the decision is to select the best alternative(s) from among a set of alternatives considering a set of criteria. This subdivision of decision-making, which has gained enormous attention, due to its practical value, in the past recent is called multi-criteria decision-making (MCDM). More precisely, MCDM concerns problems in which the decision-maker faces m alternatives $(a_1, a_2, \ldots, a_m)$, which should be evaluated with respect to n criteria $(c_1, c_2, \ldots, c_n)$, in order to find the best alternative(s), rank or sort them. In most cases, an additive value function is used to find the overall value of alternative i, U_i, as follows:

$$U_i = \sum_{j=1}^{n} w_j u_{ij}, \tag{1}$$

where u_{ij} is the value of alternative i with respect to criterion j, and w_j shows the importance (weight) of criterion j. In some problems, the decision-maker is able to find u_{ij} from external sources as objective measures, in some other problems, u_{ij} reflects a qualitative evaluation provided by the decision-maker(s), experts or users as subjective measures. Price of a car is an objective criterion while comfort of a car is a subjective one. For objective criteria, we usually use physical quantities, for instance, 'International System of Units' (SI), while for subjective criteria, we do not have such standards, which is why we mostly use pairwise comparison, linguistic variables, or Likert scales in order to evaluate the alternatives with regard to such criteria. In order to find the weights, w_j, the decision-maker might use different tools and methods, from the simplest way, which is assigning weights to the criteria intuitively, to use simple methods like SMART (simple multi-attribute rating technique) (Edwards, 1977), to more structured methods like multiple attribute utility theory (MAUT) (Keeney & Raiffa, 1976), analytic hierarchy process (AHP) (Saaty, 1977), and best worst method (BWM) (Rezaei, 2015, 2016). While these methods are usually called 'multi attribute utility and value theories' (Carrico, Hogan, Dyson, & Athanassopoulos, 1997), there is another class of methods, called outranking methods, like ELECTRE (ELimination and Choice Expressing REality) family (Roy, 1968), PROMETHEE methods (Brans, Mareschal, & Vincke, 1984) which do not necessarily need the weights to select, rank or sort the alternatives. What, however, is in common in these methods is the way they consider the nature of the criteria. That is to say, in the current literature, one of the common assumptions about the criteria (most of the time it is not explicitly mentioned in the literature), is monotonicity.

Definition 1 (Keeney & Raiffa, 1976). Let u represents a value function for criterion X, then u is monotonically increasing if:

$$[x_1 > x_2] \Leftrightarrow [u(x_1) > u(x_2)]. \tag{2}$$

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Definition 2 (Keeney & Raiffa, 1976). Let u represents a value function for criterion X, then u is monotonically decreasing if:

$$[x_1 > x_2] \Leftrightarrow [u(x_1) < u(x_2)]. \quad (3)$$

A function which is not monotonic is called non-monotonic and may have different shapes. For instance, a value function with the first part increasing and the second part decreasing called non-monotonic, by splitting of which, we have two monotonic functions.

This assumption – monotonicity – however, is an oversimplification in some real-world decision-making problems. Another simplification is the use of simple linear functions over the entire range of a criterion. Considering the two assumptions (monotonicity, linearity), we usually see simple increasing and decreasing linear value functions for the decision criteria in MCDM problems. The literature is full of such applications. For instance, many of the studies reviewed in the following review papers implicitly adopt such assumptions: the MCDM applications in supplier selection (Ho, Xu, & Dey, 2010), in infrastructure management (Kabir, Sadiq, & Tesfamariam, 2014), in sustainable energy planning (Pohekar & Ramachandran, 2004), and in forest management and planning (Ananda & Herath, 2009). While in some studies the use of monotonic and/or linear value function might be logical, their use in some other applications might be unfitting. For instance, Alanne, Salo, Saari, and Gustafsson (2007), for evaluation of residential energy supply systems use monotonic-linear value functions for all the selected evaluation criteria including "global warming potential (kg CO_2 m^{-2} a^{-1})", and "acidification potential (kg SO_2 m^{-2} a^{-1})". Considering a monotonic-linear value function for such criteria implies that the decision-maker accepts any level of such harmful environmental criteria for an energy supply system. However, if the decision-maker does not accept some high levels of such criteria (which seems logical), a piecewise linear function might better represent the preferences of the decision-maker (see the decrease-level value function in the next section).

Some authors have discussed nonlinear monotonic value functions (e.g., exponential value functions by Kirkwood, 1997; Pratt, 1964). Others use qualitative scoring to address the non-monotonicity (Brugha, 2000; Kakeneno & Brugha, 2017; O'Brien & Brugha, 2010). We can also find some forms of eliciting piecewise linear value function in Jacquet-Lagreze and Siskos (2001), and Stewart and Janssen (2013). Some other value function construction or elicitation frameworks can be found in Herrera, Herrera-Viedma, and Verdegay (1996), Lahdelma and Salminen (2012), Mustajoki and Hämäläinen (2000), Stewart and Janssen (2013), and Yager (1988). Although in PROMETHEE we use different types of piecewise functions for pairwise comparisons (Brans, Mareschal, & Vincke, 1984), the functions are not used to evaluate the decision criteria. So, despite some efforts in literature, there is no a library of some standard piecewise linear value functions which can be used in different methods like AHP or BWM. It is also important to note that while in many studies value functions are elicited according to the preference data we have from the decision-maker(s), in MCDM, usually we use the value function as a subjective input. This implies that, in MCDM methods (except a few methods, such as UTA), the value function is not elicited, but an approximation is used. This also suggests that the rich literature on determining and eliciting value functions is not actually helping MCDM methods in this area. In this paper, first, a number of piecewise linear value functions with different shapes are proposed to be considered for the decision criteria. It is then shown, with some real-world examples, how such consideration might change the final results of a decision problem. A comparison between simple monotonic linear value functions, piecewise linear value functions, and exponential value functions is conducted, which shows the effectiveness of the proposed pricewise value functions. This is a significant contribution to this field and it is expected to be widely used by MCDM applications.

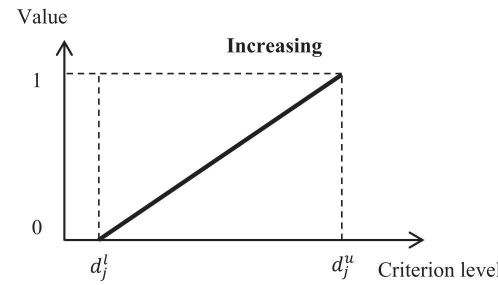


Fig. 1. Increasing value function.

In the next section, some piecewise linear value functions along with some real-world examples are presented, which is followed by some remarks in Section 3. In Section 4, some numerical analyses are used to show the applicability of considering the proposed value functions in a decision problem. In Section 5, the determination of the value functions is discussed. In Section 6, the paper is concluded, some limitations of the study are discussed, and some future research directions are proposed.

2. Piecewise linear value functions

In this section, a number of piecewise value functions are defined for decision criteria. We provide some example cases from the existing literature or practical decision-making problems to support[1] each value function. In all the following value functions we consider $[d_j^l,\ d_j^u]$ as the defined domain for the criterion by the decision-maker; x_{ij} shows the performance of alternative i with respect to criterion j; and u_{ij} shows the value of alternative i with respect to criterion j. For instance, if a decision-maker wants to buy a car considering price as one criterion, if all the alternatives the decision-maker considers are between €17,000 and €25,000, then the criterion might be defined for this range [17,000, 25,000].

2.1. Increasing

Increasing value function is perhaps the most commonly used function in MCDM applications. It basically shows that as the criterion level, x_{ij}, increases, its value, u_{ij}, increases as well. It is shown in Fig. 1 and formulated as follows:

$$u_{ij} = \begin{cases} \dfrac{x_{ij} - d_j^l}{d_j^u - d_j^l}, & d_j^l \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \quad (4)$$

For this function we can think of:

- Product quality in supplier selection (Xia & Wu, 2007). Considering a set of suppliers, a buyer may always prefer a supplier with a higher product quality compared to a supplier with lower product quality.
- Energy efficiency in alternative-fuel bus selection (Tzeng, Lin, & Opricovic, 2005). Considering a set of buses, a bus with more efficient fuel energy might always be preferred to a bus with less efficient fuel energy.

[1] It is worth-mentioning that the studies we discuss to support each value function have some theoretical or practical support for the proposed value functions. It does not, however, mean that those studies have used these value functions in their analysis.

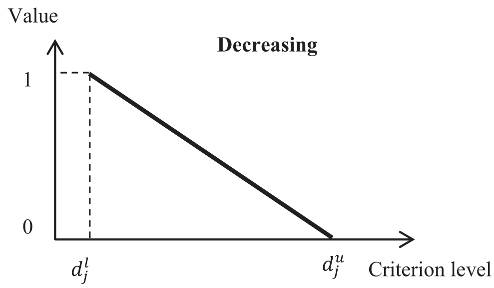


Fig. 2. Decreasing value function.

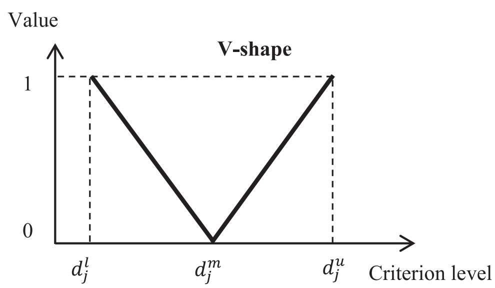


Fig. 3. V-shape value function.

2.2. Decreasing

Decreasing value function shows that as the criterion level, x_{ij}, increases, its value, u_{ij}, decreases. It is shown in Fig. 2 and formulated as follows:

$$u_{ij} = \begin{cases} \frac{d_j^u - x_{ij}}{d_j^u - d_j^l}, & d_j^l \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

For this function we can think of:

- Product price in supplier selection (Xia & Wu, 2007). Considering a set of suppliers, a supplier with a lower product price might always be preferred to a supplier with higher product price. So, a higher product price has a lower value.
- Maintenance cost in alternative-fuel bus selection (Tzeng et al., 2005). Considering a set of buses, a bus with less maintenance cost might be preferred to a bus with higher maintenance cost. So, a higher maintenance cost is associated with a lower value.

2.3. V-shape

V-shape value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, decreases gradually, and after that certain level, d_j^m, its value, u_{ij}, increases gradually. It is shown in Fig. 3 and formulated as follows:

$$u_{ij} = \begin{cases} \frac{d_j^m - x_{ij}}{d_j^m - d_j^l}, & d_j^l \le x_{ij} \le d_j^m, \\ \frac{x_{ij} - d_j^m}{d_j^u - d_j^m}, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

For this function, we could not find many examples, and it may represent a small number of very particular decision criteria. For this function we can think of:

- Relative market share in selecting a firm for investment (Wilson & Anell, 1999). Wilson and Anell (1999) found that for investment decision-making, firms with low and high market share are more desirable to the investors. This implies that the value of a firm decreases while its market share increases up to a certain level, d^m, and after that its value increases again.
- Firm size in R&D productivity (Tsai & Wang, 2005). Tsai and Wang (2005) found that both small and large firms have higher R&D productivity compared to medium-sized firms. This is true for both high-tech and traditional industries. This means that the relationship between size and value (measured by R&D productivity) is V-shape with a minimum level of value assigned to a certain size of d_j^m.

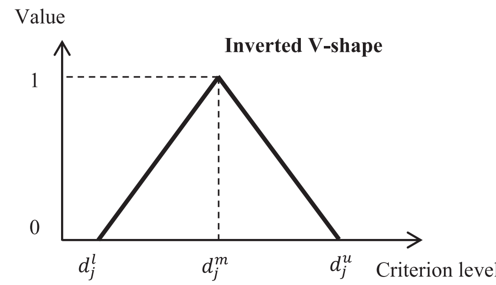


Fig. 4. Inverted V-shape value function.

2.4. Inverted V-shape

Inverted V-shape value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, increases, and after that certain level, d_j^m, its value, u_{ij}, decreases. It is shown in Fig. 4 and formulated as follows:

$$u_{ij} = \begin{cases} \frac{x_{ij} - d_j^l}{d_j^m - d_j^l}, & d_j^l \le x_{ij} \le d_j^m, \\ \frac{d_j^u - x_{ij}}{d_j^u - d_j^m}, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

For this function we can think of:

- Commute time in selecting a job (Redmond & Mokhtarian, 2001). For many people, the ideal commute, d_j^m, is larger than zero. This implies that commute times between zero and the optimal commute time, and between the optimal commute time and larger times, have lower value than the optimal commute time for such individuals. This suggests an inverted V-shape value function.
- Cognitive proximity in innovation partner selection (Nooteboom, 2000). For a company there is an optimal cognitive distance to the partner they are working on innovation (d_j^m). This implies that any distance less than d_j^m or larger than d_j^m has less value.

2.5. Increase-level

Increase-level value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, increases, and after that certain level, d_j^m, its value, u_{ij}, will remain at the maximum level. It is shown in Fig. 5 and formulated as follows:

$$u_{ij} = \begin{cases} \frac{x_{ij} - d_j^l}{d_j^m - d_j^l}, & d_j^l \le x_{ij} \le d_j^m, \\ 1, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

For this function we can think of:

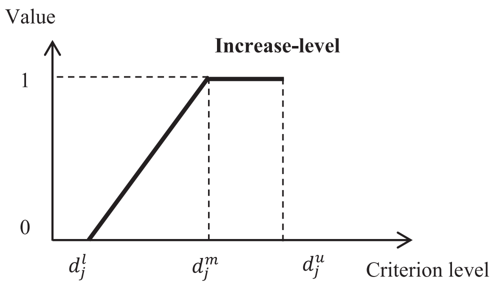


Fig. 5. Increase-level value function.

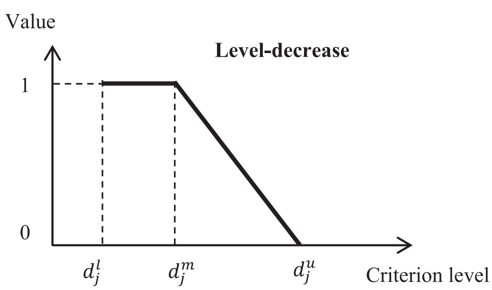


Fig. 6. Level-decrease value function.

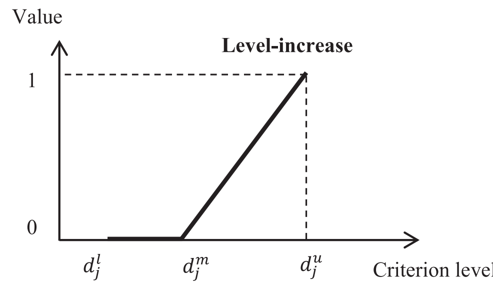


Fig. 7. Level-increase value function.

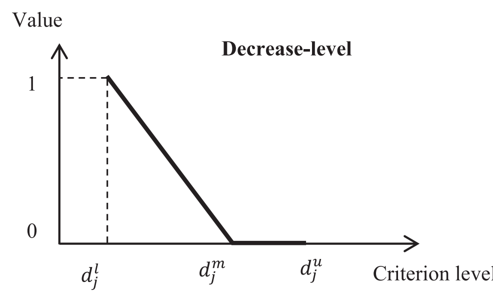


Fig. 8. Decrease-level value function.

- Fill rate in supplier selection (Chae, 2009). Although a buyer prefers suppliers with higher fill rate, which implies that as the fill rate increases its value increases, the buyer might be indifferent to any increase after a certain level, d_j^m, as usually buyers pre-identify a desirable service level which is satisfied by a certain minimum level of supplier's fill rate.
- Diversity of restaurants in hotel location selection (Chou, Hsu, & Chen, 2008). In order to find the best location for an international hotel, a decision-maker prefers locations with more divers restaurants. However, reaching a level, d_j^m, might fully satisfy a decision-maker implying that the decision-maker might not be sensitive to any increase after that certain level.

2.6. Level-decrease

Level-decrease value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, remains at maximum level, and after that certain level, d_j^m, its value, u_{ij}, decreases gradually. It is shown in Fig. 6 and formulated as follows:

$$u_{ij} = \begin{cases} 1, & d_j^l \le x_{ij} \le d_j^m, \\ \dfrac{d_j^u - x_{ij}}{d_j^u - d_j^m}, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \tag{9}$$

For this function we can think of:

- Distance in selecting a university (Carrico et al., 1997). While a student prefers a closer university to a farther university, this preference might start after a certain distance, d_j^m, implying that any distance between $[d_j^l, d_j^m]$ is optimal and indifferent for the student.
- Lead time in supplier selection (Çebi & Otay, 2016). Although a supplier with a shorter lead time is preferred, if the lead time is in a limit such that it does not negatively affect the company's production, the company might then be indifferent to that range.

2.7. Level-increase

Level-increase value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, remains at minimum level, and after that certain level, d_j^m, its value, u_{ij}, increases. It is shown in Fig. 7 and formulated as follows:

$$u_{ij} = \begin{cases} 0, & d_j^l \le x_{ij} \le d_j^m, \\ \dfrac{x_{ij} - d_j^m}{d_j^u - d_j^m}, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \tag{10}$$

For this function we can think of:

- Level of trust in making a buyer–supplier relationship (Ploetner & Ehret, 2006). Trust increases the level of partnership between a buyer and a supplier, however it is only effective after a certain threshold, d_j^m.
- Level of relational satisfaction in evaluating quality communication in marriage (Montgomery, 1981). Below a minimum level of relational satisfaction, d_j^m, quality communication cannot take place thus results in minimum value.

2.8. Decrease-level

Decrease-level value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, decreases, and after that certain level d_j^m, its value, u_{ij}, will remain at minimum. It is shown in Fig. 8 and formulated as follows:

$$u_{ij} = \begin{cases} \dfrac{d_j^m - x_{ij}}{d_j^m - d_j^l}, & d_j^l \le x_{ij} \le d_j^m, \\ 0, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \tag{11}$$

For this function we can think of:

- Carbon emission in transportation mode selection (Hoen, Tan, Fransoo, & van Houtum, 2014). In selecting a transportation mode, the more the carbon emission by the mode, the less the value of that mode. A decision-maker, however might assign

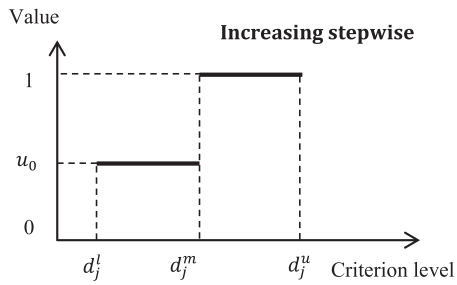


Fig. 9. Increasing stepwise value function.

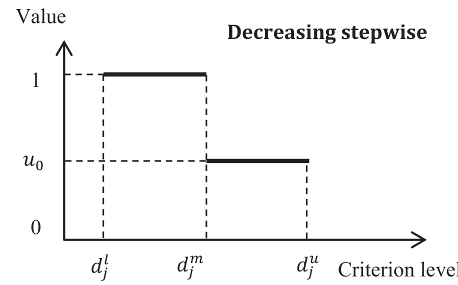


Fig. 10. Decreasing stepwise value function.

zero value to a mode with carbon emission higher than a certain level, d_j^m.

- Distance when selecting a school (Frenette, 2004). It has been shown that the longer the distance to the school the less the preference to attend that school. It is also clear that, for some people, there is no value after a certain distance, d_j^m.

2.9. Increasing stepwise

Increasing stepwise value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value remains at a certain level, u_0, and after that certain level, d_j^m, its value, u_{ij}, jumps to a higher level (maximum) and remains at the maximum. It is shown in Fig. 9 and formulated as follows:

$$u_{ij} = \begin{cases} u_0, & d_j^l \le x_{ij} \le d_j^m, \\ 1, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \quad (12)$$

where $0 < u_0 < 1$.

For this function we can think of:

- Suppliers capabilities in supplier segmentation (Rezaei & Ortt, 2012). Suppliers of a company are evaluated based on their capabilities, and then segmented based on two levels (low and high) with respect to their capabilities. As such a supplier scored between d_j^l and d_j^m is considered as a low-level capabilities supplier while a supplier scored between d_j^m and d_j^u is considered as a high-level capabilities supplier.
- Symmetry in selecting a close type of partnership (Lambert, Emmelhainz, & Gardner, 1996). In order to have a successful relationship between supply chain partners, there should be some demographical similarities (for instance, in terms of brand image, productivity) between them. So, more symmetry means closer relationship. However, if we consider two levels of closeness, it is clear that for some level of symmetry the value of closeness remains the same.

For the increasing stepwise value function, a criterion might have more than one jump. For instance, if a decision-maker wants to consider three levels low, medium, and high when segmenting the suppliers with respect to their capabilities, then an increasing stepwise function with two jumps should be defined for this criterion. The following value function is a general increasing stepwise value function with k jumps.

$$u_{ij} = \begin{cases} u_0, & d_j^l \le x_{ij} \le d_j^{m1}, \\ u_1, & d_j^{m1} \le x_{ij} \le d_j^{m2}, \\ \quad \vdots & \\ 1, & d_j^{mk} \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \quad (13)$$

where $0 < u_0 < u_1 < \ldots < 1$.

2.10. Decreasing stepwise

Decreasing stepwise value function shows that as the criterion level, x_{ij}, increases up to a certain level, d_j^m, its value, u_{ij}, remains at a the maximum level, and after that certain level, d_j^m, its value, u_{ij}, jumps down to a lower level, u_0, and remains at that level. It is shown in Fig. 10 and formulated as follows:

$$u_{ij} = \begin{cases} 1, & d_j^l \le x_{ij} \le d_j^m, \\ u_0, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \quad (14)$$

where $0 < u_0 < 1$.

For this function we can think of:

- Considering supply risk in portfolio modeling (Kraljic, 1983). For a company, an item with a higher level of risk results in less value, however, due to portfolio modeling, there is no difference between all levels of risk in the domain $[d_j^l,\ d_j^m]$. Similarly, all levels of risk in the domain $[d_j^m,\ d_j^u]$ result in the same value.
- Delay in logistics service provider selection (Qi, 2015). Some companies consider stepwise value function for delay in delivering the items by a logistics service provider, which means that the value of that provider decreases when delay increases, however it is constant within certain intervals.

For the decreasing stepwise function, a criterion might have more than one jump. The following value function is a general decreasing stepwise function with k jumps.

$$u_{ij} = \begin{cases} 1, & d_j^l \le x_{ij} \le d_j^{m1}, \\ u_1, & d_j^{m1} \le x_{ij} \le d_j^{m2}, \\ \quad \vdots & \\ u_k, & d_j^{mk} \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \quad (15)$$

where $0 < u_k < \ldots < u_1 < 1$.

3. Some remarks on the value functions

Here, a number of remarks are discussed, shedding light on some aspects of the proposed value functions, which might be used in real-world applications.

Remark 1. Shape and parameters of a value function is decision-maker-dependent, implying that (i) while a decision-maker considers, for instance, a level-increasing function for the size of garden when buying a house, another decision-maker considers an increasing stepwise function, and (ii) while two decision-makers consider increasing stepwise function for the size of garden when buying a house, the parameters they consider for their functions (d_j^l, d_j^m, d_j^u) might be different.

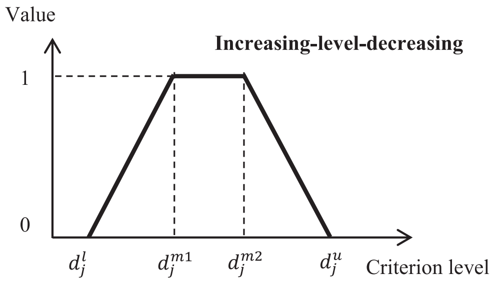


Fig. 11. Increasing-level-decreasing value function.

Table 1
Suppliers performance with respect to different decision criteria (x_{ij}).

	Criteria				
Supplier	Quality	Price (€/item)	Trust	CO_2 (g/item)	Delivery (day)
1	85	27	4	1000	3
2	90	28	2	1500	4
3	80	26	5	2000	3
4	75	25	5	1000	2
5	95	29	7	1700	3
6	99	30	6	2000	1

Remark 2. A decision-maker might consider a hybrid value function for a criterion. For instance, a criterion might be characterized with an increasing-level-decreasing, which is a combination of increasing-level and level-decreasing. This function can also be considered as a special form of inverted V-shape function. It is shown in Fig. 11 and formulated as follows:

$$u_{ij} = \begin{cases} \frac{x_{ij} - d_j^l}{d_j^m - d_j^l}, & d_j^l \le x_{ij} \le d_j^{m1}, \\ 1, & d^{m1} \le x_{ij} \le d^{m2}, \\ \frac{d_j^u - x_{ij}}{d_j^u - d_j^m}, & d^{m2} \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \tag{16}$$

For instance, a decision-maker has to select the best R&D partner among 10 partners. One of the criteria is distance and the company gives less preference to very close or very distant partners, which are distributed in the range [10 km, 2000 km]. The company considers an optimal distance of [200 km, 500 km]. This implies that distance follows an 'increasing-level-decreasing' function for this decision-maker: $[d_j^l, \ d_j^{m1}, d_j^{m2}, d_j^u] = [10, \ 200, \ 500, \ 2000]$.

4. Numerical and comparison analyses

In this section, we show how to incorporate the proposed piecewise value functions into account when applying an MCDM method, and we show that the results might be different when we consider the proposed piecewise value functions.

We consider an MCDM problem, where a buyer should select a supplier from among six qualified suppliers considering five criteria: quality which is measured by $1-\alpha$, where α shows the lot-size average imperfect rate; price (euro) per item; trust, which is measured by a Likert scale (1: very low to 7: very high); CO_2 (gram) per item; delivery (day), the amount of time which takes to deliver items from the supplier to the buyer (all the criteria are continuous except trust). Table 1 shows the performance of the six suppliers with respect to the five criteria.

The buyer has used an elicitation method[2] to find the weights which are as follows:

$$w^*_{quality} = 0.20, \ w^*_{price} = 0.30, \ w^*_{trust} = 0.27, \ w^*_{CO_2} = 0.08, \ w^*_{delivery} = 0.15.$$

And we assume that the decision-maker considers piecewise value functions for these criteria (see, Table 2).

So, as can be seen from Table 2, the decision-maker considers a level-increase linear function for quality with the lowest and highest values of 0 and 100, respectively. For the decision-maker any number below 85 has no value at all. For criterion price, the decision-maker gives the highest value to any price below 15 (although in the existing set of suppliers there is no supplier with a price within this range), after which the value decreases till it reaches to the maximum price of 30. For criterion trust which is measured using a Likert scale (1: very low to 7: very high), any number less than 3 has no value for the decision-maker, while the value gradually increases between 3 and 7. For CO_2 emission, there is a decreasing value function from 0 to 1500 g per item, after which till 2000 g, all the numbers have zero value. Finally, for delivery there is a simple decreasing function with minimum and maximum values of 0 and 5 days.

By using the following equation, we can find the overall value of each supplier and then rank them to find the best supplier.

$$U_i = \sum_{j=1}^{n} w_j u_{ij} \tag{17}$$

where, u_{ij} is the value of the performance of supplier i with respect to criterion j (using the equations in Table 2 for the data in Table 1), and w_j is the weight of criterion j as follows:

$$w^*_{quality} = 0.20, \ w^*_{price} = 0.30, \ w^*_{trust} = 0.27, \ w^*_{CO_2} = 0.08, \ w^*_{delivery} = 0.15.$$

The value scores and the aggregated values are presented in Table 3 (see also Fig. 13 for the final results).

As can be seen from Table 3, supplier 6 with the greatest overall value of 0.51 is ranked as the first supplier. Suppliers 5, 4, 3, 1, and 2 are ranked in the next places.

4.1. Comparing with the simple linear value functions

In existing literature, considering the nature of the criteria, the values are calculated, for instance, using the following simple linear value function:

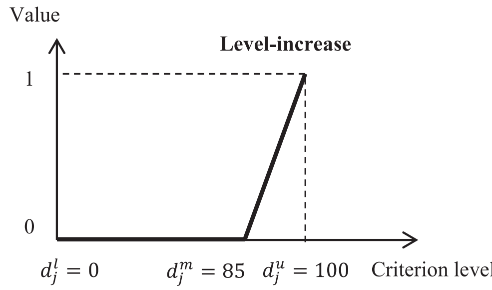
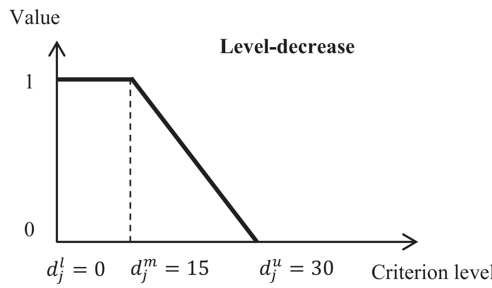
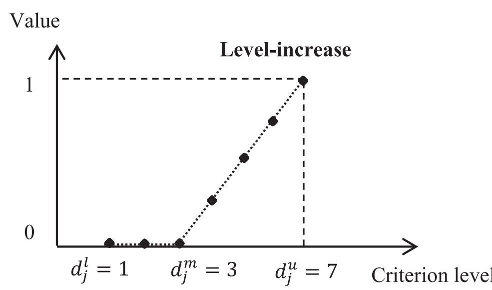
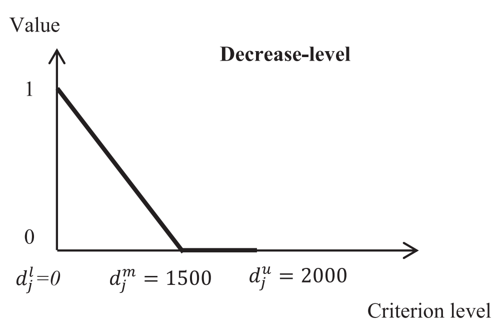
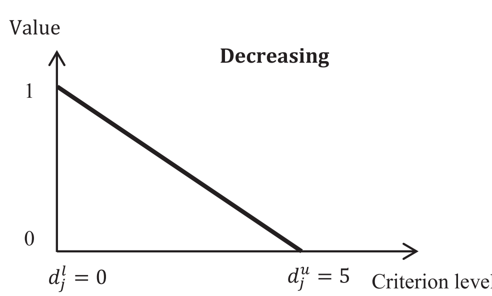
$$u_{ij} = \begin{cases} \frac{x_{ij} - d_j^l}{d_j^u - d_j^l}, & \text{if more } x_{ij} \text{ is more desirable (such as quality)}, \\ \frac{d_j^u - x_{ij}}{d_j^u - d_j^l}, & \text{if more } x_{ij} \text{ is less desirable (such as price)}. \end{cases} \tag{18}$$

Eq. (18) is used to find the values of the criteria for each supplier using the data in Table 1. Considering the criteria weights, and u_{ij} (Eq. (18) for the data in Table 1) using Eq. (17) the value scores and also the aggregated overall score of each alternative can be calculated which are shown in Table 4 (see also Fig. 13 for the final results).

In Table 4 it is assumed that quality and trust are criteria for which the higher the better, while for the other criteria (price, CO_2, and delivery), the lower the better. In fact, we consider simple linear functions (increasing and decreasing respectively) for the two groups of criteria.

[2] Please note that we report some weights for the criteria as the aim of the study is not the weighing part.

Table 2
Piecewise value functions.

Shape	Value function
	$u_{ij} = \begin{cases} 0, & d_j^l \le x_{ij} \le d_j^m, \\ \frac{x_{ij} - d_j^m}{d_j^u - d_j^m}, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} 0, & 0 \le x_{ij} \le 85, \\ \frac{x_{ij} - 85}{100 - 85}, & 85 \le x_{ij} \le 100, \\ 0, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} 1, & d_j^l \le x \le d_j^m, \\ \frac{d_j^u - x_{ij}}{d_j^u - d_j^m}, & d_j^m \le x \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} 1, & 13 \le x_{ij} \le 15, \\ \frac{30 - x_{ij}}{30 - 15}, & 15 \le x_{ij} \le 30, \\ 0, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} 0, & d_j^l \le x_{ij} \le d_j^m, \\ \frac{x_{ij} - d_j^m}{d_j^u - d_j^m}, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} 0, & 1 \le x_{ij} \le 3, \\ \frac{x_{ij} - 3}{7 - 3}, & 3 \le x_{ij} \le 7, \\ 0, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} \frac{d_j^m - x_{ij}}{d_j^m - d_j^l}, & d_j^l \le x_{ij} \le d_j^m, \\ 0, & d_j^m \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1500 - x_{ij}}{1500 - 0}, & 0 \le x_{ij} \le 1500, \\ 0, & 1500 \le x_{ij} \le 2000, \\ 0, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} \frac{d_j^u - x_{ij}}{d_j^u - d_j^l}, & d_j^l \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} \frac{5 - x_{ij}}{5 - 0}, & 0 \le x_{ij} \le 5, \\ 0, & \text{otherwise.} \end{cases}$

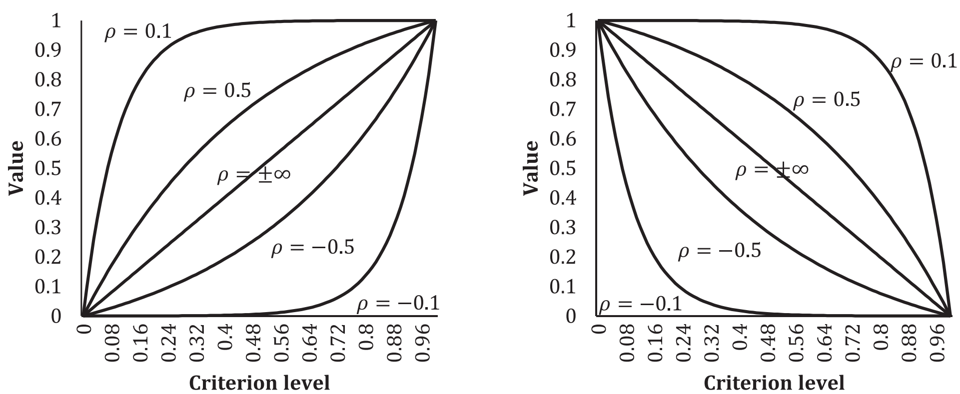
According to Table 4, the best supplier is supplier 4, which is ranked as the 3rd one considering the piecewise value functions (Table 3). The ranking of the other suppliers is also different. So, as can be seen, such differences are associated to the way we calculate the value of the criteria. If we look at the criterion trust, for instance (Table 3), we can see that only the numbers greater than 3 can be used for compensating the other criteria. In other words, the values 1, 2 and 3 for this criterion have no selection power. No supplier can compensate its weakness in other criteria by having a value between 1 and 3 for trust. However, such important issue is entirely ignored in the simple way of determining the value functions which is very popular in existing studies. This consideration

Table 3
Value scores, u_{ij}, and the aggregated overall score considering the proposed piecewise value functions.

Supplier	Quality	Price	Trust	CO_2	Delivery	Aggregated value	Rank
1	0.00	0.20	0.25	0.50	0.40	0.23	5
2	0.33	0.13	0.00	0.00	0.20	0.14	6
3	0.00	0.27	0.50	0.00	0.40	0.28	4
4	0.00	0.33	0.50	0.50	0.60	0.37	3
5	0.67	0.07	1.00	0.00	0.40	0.48	2
6	0.93	0.00	0.75	0.00	0.80	0.51	1

Table 4
Value scores, u_{ij}, and the aggregated overall scores considering the simple linear value functions.

Supplier	Quality	Price	Trust	CO_2	Delivery	Aggregated value	Rank
1	0.42	0.60	0.40	1.00	0.33	0.50	4
2	0.63	0.40	0.00	0.50	0.00	0.29	6
3	0.21	0.80	0.60	0.00	0.33	0.49	5
4	0.00	1.00	0.60	1.00	0.67	0.64	1
5	0.83	0.20	1.00	0.30	0.33	0.57	2
6	1.00	0.00	0.80	0.00	1.00	0.57	3



a. A monotonically increasing exponential value function   b. A monotonically decreasing exponential value function

Fig. 12. Exponential value functions.

is even of a higher importance for compensatory methods such as AHP and BWM.

4.2. Comparing with the exponential value functions

Another important way to approximate the value functions in practice is the use of *exponential value functions* (Kirkwood, 1997; Pratt, 1964). The exponential value functions can specifically be used when the preferences are monotonically increasing or decreasing. Although this approach is not popular in MCDM domain, and we were not able to find any application of these value functions particularly in MCDM field, we would like to compare our results to the results of applying these functions, which are, to some extent, close to some of our proposed piecewise value functions (such as level-increase, level-decrease, increase-level, and decrease-level). Using the same notations as before and considering a shape parameter ρ which is called 'risk tolerance', a monotonically increasing value function can be shown as follows:

$$u_{ij} = \begin{cases} \dfrac{1-\exp\left[-\left(x_{ij}-d_j^l\right)/\rho\right]}{1-exp\left[-\left(d_j^u-d_j^l\right)/\rho\right]}, & \rho \neq \text{Infinity} \\ \dfrac{x_{ij}-d_j^l}{d_j^u-d_j^l}, & \text{otherwise.} \end{cases} \tag{19}$$

A monotonically decreasing exponential value function can be shown as follows:

$$u_{ij} = \begin{cases} \dfrac{1-\exp\left[-\left(d_j^u-x_{ij}\right)/\rho\right]}{1-exp\left[-\left(d_j^u-d_j^l\right)/\rho\right]}, & \rho \neq \text{Infinity} \\ \dfrac{d_j^u-x_{ij}}{d_j^u-d_j^l}, & \text{otherwise.} \end{cases} \tag{20}$$

Fig. 12 shows the monotonically increasing exponential value functions (for different values of ρ) (a), and the monotonically decreasing exponential value functions (for different values of ρ) (b).

Risk-averse decision-makers have $\rho > 0$ (hill-like functions in Fig. 12), while risk-seeking decision-makers have $\rho < 0$ (bowl-like functions in Fig. 12). $\rho =$ Infinity (straight-line in Fig. 12) shows the value for the risk neutral decision-makers. In fact, $\rho =$ Infinity produces the simple linear value functions which are very popular in MCDM field.

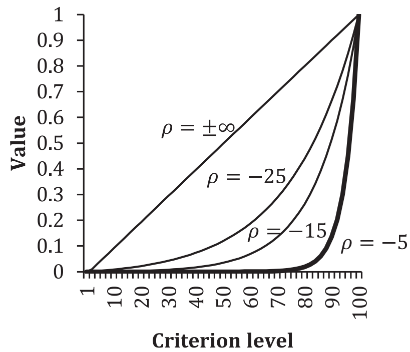
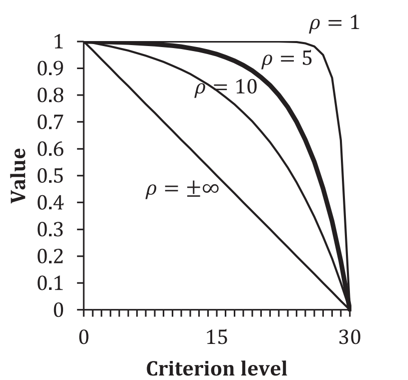
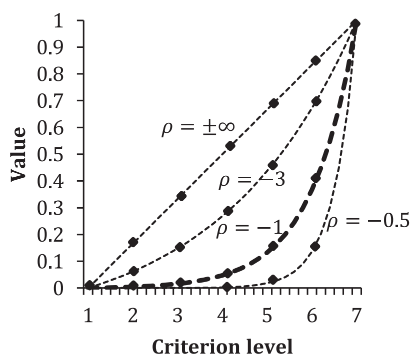
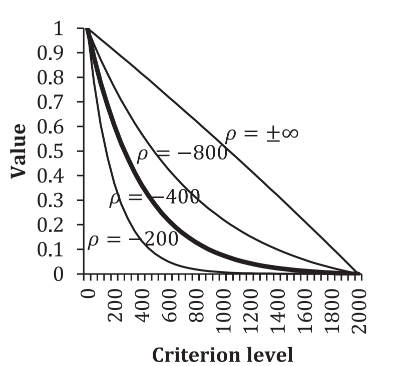
In order to do the comparison analysis, we use exponential value functions for the criteria of the aforementioned example (Table 1) to check the similarities and differences. To make a fair comparison, we try to generate[3] the corresponding exponential value functions of the piecewise value functions (Table 2) as

[3] To see how these value functions are elicited considering the decision-maker risk tolerance, refer to Kirkwood (1997).

close as possible. For quality, a monotonically increasing exponential value function with negative ρ would be appropriate. For price a monotonically decreasing exponential value function with a positive ρ, for trust, a monotonically increasing exponential value function with a negative ρ, for CO_2, a monotonically decreasing exponential value function with a negative ρ, and, finally, for delivery a monotonically decreasing exponential value function with $\rho =$ Infinity would be suitable. Table 5 shows the functions, where functions with different $\rho's$ are shown and a more suitable one is shown in bold.

Using the exponential value functions of Table 5, for the data of Table 1, we get the value scores and the aggregated values as presented in Table 6 (see also Fig. 13 for the final results).

Table 5
Exponential value functions.

Shape	Function
	$u_{ij} = \begin{cases} \frac{1-\exp[-(x_{ij}-d_j^l)/\rho]}{1-exp[-(d_j^u-d_j^l)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{x_{ij}-d_j^l}{d_j^u-d_j^l}, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1-\exp[-(x_{ij}-0)/\rho]}{1-exp[-(100-0)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{x_{ij}-0}{100-0}, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} \frac{1-\exp[-(d_j^u-x_{ij})/\rho]}{1-exp[-(d_j^u-d_j^l)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{d_j^u-x_{ij}}{d_j^u-d_j^l}, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1-\exp[-(30-x_{ij})/\rho]}{1-exp[-(30-0)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{30-x_{ij}}{30-0}, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} \frac{1-\exp[-(x_{ij}-d_j^l)/\rho]}{1-exp[-(d_j^u-d_j^l)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{x_{ij}-d_j^l}{d_j^u-d_j^l}, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1-\exp[-(x_{ij}-1)/\rho]}{1-exp[-(7-1)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{x_{ij}-1}{7-1}, & \text{otherwise.} \end{cases}$
	$u_{ij} = \begin{cases} \frac{1-\exp[-(d_j^u-x_{ij})/\rho]}{1-exp[-(d_j^u-d_j^l)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{d_j^u-x_{ij}}{d_j^u-d_j^l}, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1-\exp[-(2000-x_{ij})/\rho]}{1-exp[-(2000-1500)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{2000-x_{ij}}{2000-0}, & \text{otherwise.} \end{cases}$

(continued on next page)

Table 5 (*continued*)

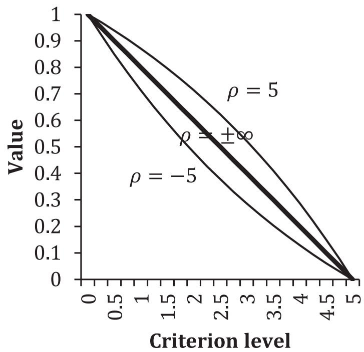
Shape	Function
	$u_{ij} = \begin{cases} \frac{1-\exp[-(d_j^u - x_{ij})/\rho]}{1-exp[-(d_j^u - d_j^l)/\rho]}, & \text{Infinity} \\ \frac{d_j^u - x_{ij}}{d_j^u - d_j^l}, & \text{otherwise.} \end{cases} = \begin{cases} \frac{1-\exp[-(5 - x_{ij})/\rho]}{1-exp[-(5-0)/\rho]}, & \rho \neq \text{Infinity} \\ \frac{5 - x_{ij}}{5-0}, & \text{otherwise.} \end{cases}$

Table 6
Value scores, u_{ij}, and the aggregated overall scores considering the exponential value functions.

Supplier	Quality	Price	Trust	CO_2	Delivery	Aggregated value	Rank
1	0.85	0.45	0.05	0.08	0.40	0.22	5
2	0.90	0.33	0.00	0.02	0.20	0.16	6
3	0.80	0.55	0.13	0.00	0.40	0.26	4
4	0.75	0.63	0.13	0.08	0.60	0.32	3
5	0.95	0.18	1.00	0.00	0.40	0.46	1
6	0.99	0.00	0.37	0.00	0.80	0.38	2

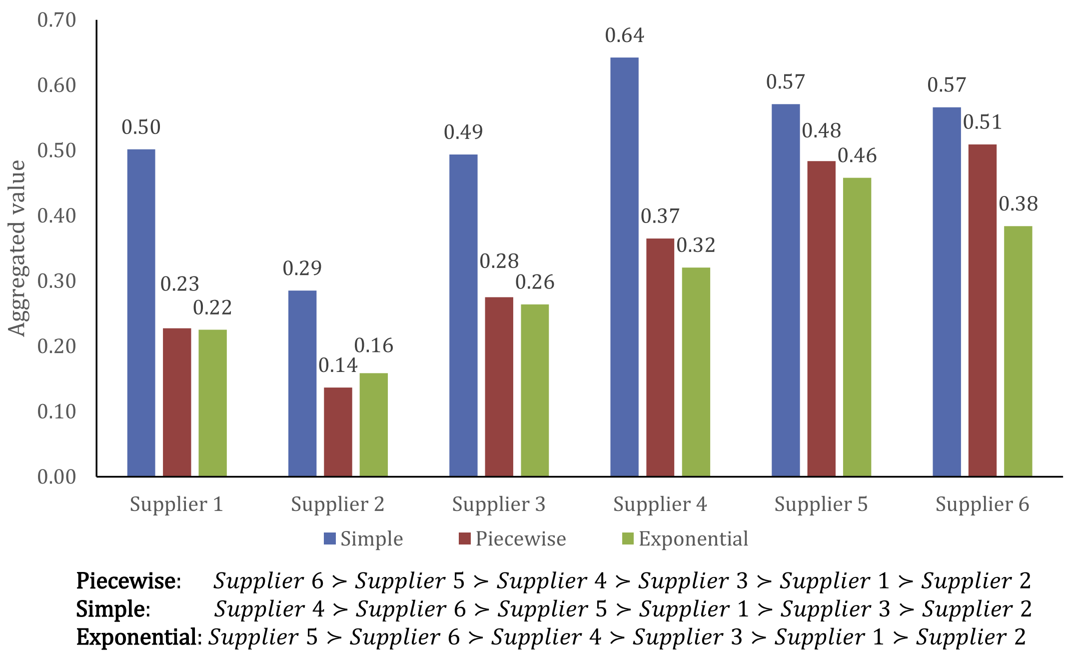
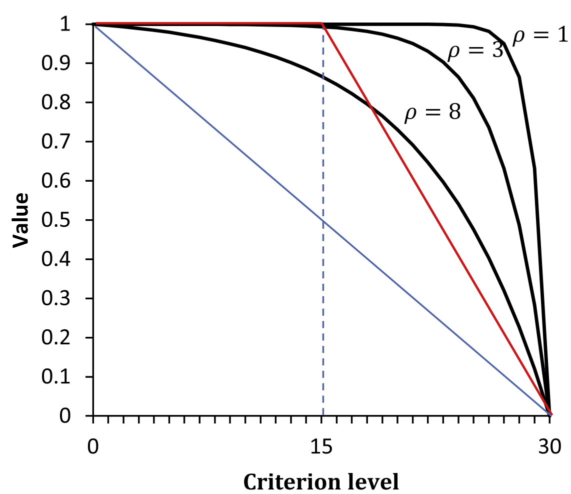


Fig. 13. Final results using three types of value functions.

As can be seen from Table 6, supplier 5 with the greatest overall value of 0.46 is ranked as the first supplier, which is different from what we get from the proposed piecewise value functions (Table 3). While supplier 5 was ranked the 2nd based on our proposed value functions, using the exponential value functions, this supplier becomes number 2. Other suppliers (1, 2, 3, 4) have the same ranking based on the two approaches. The differences are obviously associated to the way we get the values of the criteria. We also checked some other close ρ values for the exponential value functions. There are some changes in the aggregated values, yet, the ranking is the same.

As it can be seen, the results of the two approaches (piecewise value functions and exponential functions are much closer to each other than to the results of the regular simple linear value functions).

Our observation is that the exponential value functions can play a role close to a number of proposed value functions in this study such as increase-level, decrease-level, level-increase, and level-decrease. In order to make an exponential value functions close to one of the mentioned proposed value functions we should choose ρ values close to zero. If we try to make the ρ as close as we perfectly make the “level” part of the criterion, then the



Level-decrease value function (red):

$$u_{ij}=\begin{cases}1, & 0\le x_{ij}\le 15,\\ \dfrac{30-x_{ij}}{30-15}, & 15\le x_{ij}\le 30,\\ 0, & \text{otherwise.}\end{cases}$$

Exponential decreasing value function (black):

$$u_{ij}=\begin{cases}\dfrac{1-\exp[-(30-x_{ij})/\rho]}{1-exp[-(30-0)/\rho]}, & \rho\neq \text{Infinity}\\ \dfrac{30-x_{ij}}{30-0}, & \text{otherwise.}\end{cases}$$

Simple linear value function (blue):

$$u_{ij}=\begin{cases}\dfrac{30-x_{ij}}{30-0}, & 0\le x_{ij}\le 30,\\ 0, & \text{otherwise.}\end{cases}$$

Fig. 14. Fitting an exponential value function to a level-decrease value function.

other part becomes very steep and not representative. On the other hand, if we want to choose a ρ value which better represents the slope of the function for the "increase" or the "decrease" part, then it is impossible to cover the "level" part of the value function properly.

For example, let us consider the criterion price again. In Fig. 14, it can be seen that, if we consider $\rho = 1$, the level part is fully covered, but the decrease part of the exponential value function is very much different from the decrease part of the linear function. Even if we choose $\rho = 3$, which does not fully cover the level part of the proposed function, the decrease part is really different. On the other hand, we can find $\rho = 8$ as a close one to the decrease part of the linear function, but this time it is not possible to cover the level part properly. So, although a very good approximation, the exponential value functions might not be suitable for cases in which a decision-maker has a clear value-indifference interval (a level part) for a criterion. However, these functions are indeed suitable when the decision-maker has different preferences on the lower and on the upper parts of the criterion measure.

From the figure we can also see that while we could make the two piecewise and exponential value functions, to some degree, close to each other, they are too different from the simple linear value function. It is also clear that none of the simple linear value functions or the exponential value functions can represent the V- or inverted V-shape value functions.

As a general conclusion, we think that the proposed piecewise linear functions have two salient features: (i) simplicity; and (ii) representativeness. That is, it is easy to work with linear functions, and it is easy for a practitioner to find a more representative function from the proposed library of the pricewise value functions for a particular criterion. The cut-off points can also be estimated by the decision-maker. The simple monotonic-linear value functions, which are dominant in existing literature, are very simple. However, they might not be representative in some cases. Finally, the exponential value functions might have a better representativeness (compared to the simple monotonic-linear value functions). However, they are not simple. Working with non-liner functions is not easy for practitioner, and, more importantly, it is very difficult for a practitioner to estimate a value for ρ (the shape parameter of the exponential value functions), as it cannot be easily interpreted by a practitioner (please note that we consider a value function as an input for an MCDM problem in this study).

5. Determining the value functions

One of the big challenges in real-world decision-making is to find a proper value function for a decision criterion. This, perhaps, has been one of the main reasons why the use of simple linear value functions in multi-criteria decision-making is dominant. The linear value functions are easy for modeling purposes and can, to some extent, represent the reality. More complicated value functions, although might be closer to reality of the decision-maker's preferences, are more difficult to be elicited and are difficult for modeling purposes. We refer the interested readers to some existing procedures for identifying value functions (Fishburn, 1967; Keeney & Nair, 1976; Kirkwood, 1997; Pratt, 1964; Stewart & Janssen, 2013). We think that the proposed value functions in this paper do not have the disadvantage of nonlinearity and at the same time have the advantages of being closer to the real preferences of the decision-maker as they provide some diversity and flexibility in modeling the functions. As we do not consider the nonlinearity of the value functions we do not use the concept of risk tolerance in determining the value functions as it has been used by others. We rather propose a simple procedure, which is more practical.

A decision analyst, could first show the value functions in Table 2 to the decision-maker to see which one most suits the preference structure of the decision-maker. Once the decision-maker selects a particular value function, the other details of the function, such as the lower bound, the upper bound and the thresholds can be determined. We should highlight again that in most MCDM methods, the value function is not elicited. It is rather simply assumed to have a particular shape, and this is why we think having a pre-specified set of standard value functions which can be used as subjective approximation of the real preferences of the decision-maker can make a significant impact on the results.

6. Conclusion, limitations and future research

This study proposes a set of piecewise value functions for multi-criteria decision-making (MCDM) problems. While the existing applications of MCDM methods usually use two general simple increasing and decreasing linear value functions, this study provides several real-world examples to support the applicability of some other forms of value functions for the criteria used in MCDM. It is also explicated how, in some decision problems, a combination of two or more value functions can be used for a particular decision criterion. The proposed functions can be used for different MCDM methods in different decision problems. A numerical example of supplier selection problem (including a comparison between simple monotonic linear value functions, piecewise linear value functions, and exponential value functions) showed how the use of the proposed value functions could affect the final results. Considering these value functions could better represent the real preferences of the decision-maker. It can also help reduce the inappropriate compensations of the decision criteria, for instance, through using a level-increasing function which assigns zero value to any value of the criterion below a certain threshold. The proposed value functions are presented in a general form such that they can be tailor-made for a specific decision-maker. That is to say, not only it is possible for two different decision-makers to use two different value functions for a single criterion. It is also possible to use different domain (e.g. min and max) values for that particular value function.

Despite the advantages of the proposed value functions, they have some limitations. Although the proposed value functions consider some real-world features of the decision criteria, they are linear which might be, to some degree, a simplification. We think that the bigger problem in existing literature of MCDM is monotonicity assumption and not linearity assumption. Nevertheless, more research needs to be conducted to empirically find the share of each. Furthermore, to formulate the decision criteria one should pay enough attention to check the real contribution of the decision criterion into the ultimate goal of the decision-making problem. For instance, if a criterion contributes to another criterion which has a real role in making the decision, one should exclude the initial one. For a detailed discussion on this matter, interested readers are referred to Brugha (1998). One interesting future direction would be to apply the proposed value functions in some real-world MCDM problems and compare their fitness to the other value functions. In this regard, finding a more systematic approach to determine the value functions in practice would be also very interesting. It would be also interesting to study the cases in which there are more than one decision-maker. As different decision-makers may choose different value functions, different domains, and different thresholds for a single criterion, proposing a way to find the final output of the MCDM problem for the group would be an interesting future research. Finally, finding a sensitivity analysis for the proposed value functions is recommended. Considering the studies of Bertsch and Fichtner (2016), Bertsch, Treitz, Geldermann, and Rentz (2007), Insua and French (1991), Wulf and Bertsch (2017) could give interesting ideas to make such sensitivity analysis framework.

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