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ME-OWA based DEMATEL reliability apportionment method

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ABSTRACT

The maximal entropy ordered weighted averaging (ME-OWA)-based decision making trial and evaluation laboratory (DEMATEL) method for reliability allocation has been examined. The assessment results show that most conventional reliability allocation methods have five fundamental problems. The first problem is the measurement scale; while the second problem is that the system allocation factors are not equally weighted to one another, the third problem is that most reliability allocations methods often neglect many important features, such as maintainability and risk issues. The fourth problem is that they do not consider indirect relations between subsystems or components, and the fifth problem is that they do not consider predicted failure rate in the apportionment process. This study evaluated reliability allocation using a fighter aircraft's digital flight control computer (DFLCC). The proposed method offers several benefits compared with current military and commercial approaches. The computational results clearly demonstrate the advantages of the proposed approach for solving the five fundamental problems.

1. Introduction

Reliability allocation is a top-down approach for apportioning accuracy goals in a system, which is essential when different design teams, subcontractors, or manufacturers are involved, as it effects the system safety and usability of the product. The purpose of the reliability allocation is to assign limited resources to the most important subsystems or components and ensure that the product can achieve its designed functions under specific operating conditions. The allocation technique significantly influences product life cycle cost and system operational effectiveness.

Apportioning system reliability is a multiple-criteria decision-making task, which is usually allocated on the basis of the component performance and/or cost as criteria. Fuqua (1987) introduced the "Reliability engineering for electronic design", and MIL-HDBK-338B (1988) defines four approaches to allocating reliability, which includes the equal apportionment technique, the ARINC apportionment technique, the feasibility-of-objectives (FOO) technique, and the minimization of effort algorithm. Kuo (1999) in his book "Reliability Assurance: Application for engineering and management", introduced four approaches to allocating reliability, which included the equal apportionment technique, the ARINC apportionment technique, the average weighting allocation method, and the pair comparison method. These conventional reliability methods have been widely and successfully applied in a great many domains (Anderson, 1976; Fuqua, 1987; Kuo, 1999; Smedley, 1992). In addition to these methods, Bracha (1964) introduced an allocated reliability method using four factors: state of the art, subsystem complexity as estimated by the number of parts, environmental conditions, and relative operating time, whereas Karmiol (1965) evaluated the complexity, state of the art, operational profile, and criticality of the system to mission objectives to apportion subsystem reliability. Boyd (1992) proposed the Boyd method to combine the equal method with the ARINC method, while Falcone, Silvestri, and Bona (2002) used the integrated factors method (IFM) using four factors, criticality (C), complexity (K), functionality (F), and effectiveness (O), to calculate system reliability for an aerospace prototype project.

However, the Karmiol method, the FOO technique, the average weighting allocation method, and the IFM method assume an equal interval between category labels; therefore, the operations of multiplication and division are not meaningful on ordinal numbers, and addition and subtraction, while sometimes meaningful, must be done carefully because they assume equal intervals between category labels. Thus these methods all share a common weakness in their measurement scale. Take the IFM as an example: the four system factors used in allocation reliability—criticality (C), complexity (K), functionality (F), effectiveness (O)—obtain the subsystem reliability $IG_i = K_i * F_i * O_i/C_i$ (IG_i: global index relative to the subsystem). In the use of the IFM method, the four system factors C, K, F, and O are rated from range 1 to 10. These four system factors are multiplied and divided to derive the IG_i; i.e., $IG_i = K_i * F_i * O_i/C_i$—the multiplied results inIG range from 1 to 1000. Higher

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IGs are assumed to be more of an overall rating than those having a lower *IG*. From a calculation perspective, the IFM method is simple, easy to understand, straightforward to use, and well documented for ease of reference. However, the method has problems with its measurement scale. The first problem is that the four system factors, *K*, *F*, *O*, and *C*, are evaluated according to discrete ordinal scales of measure, which represent serious flaws from a technical perspective; in particular, multiplication and/or division is not meaningful and in fact is misleading. The second problem is that the four system factors are not equally weighted, which makes the analysis and interpretation of the results problematic. For example, for two components with *IG* values of $(8 \times 2 \times 2)/2 = 16$ and $(7 \times 3 \times 2)/2 = 21$, respectively, the former should have had a higher reliability allocation overall rating than the latter, even though it has a lower *IG* value.

In resolving these two problems, Chang, Chang, and Liaw (2009) provided an innovative reliability allocation using the maximal entropy ordered weighted averaging (ME-OWA) method. This approach uses Yager's OWA (1988) and the ME-OWA (Chang, Cheng, & Chang, 2008; Fuller & Majlender, 2001) operators, which uses Lagrange multipliers on Yager's OWA equation to derive a polynomial equation, and determines the optimal weighting vector under the maximal entropy operator. This method is a both simple and effective approach that can efficiently resolve the two shortcomings of the conventional allocation methods. However, most current reliability allocation methods do not consider indirect relations between subsystems or components; also, most reliability allocation methods do not consider predicted failure rate in the evaluation process. The Battelle Memorial Institute, through its Geneva Research Centre, first developed the decision making trial and evaluation laboratory (DEMATEL) method (Gabus & Fontela, 1973). It is a potent method that gathers group knowledge for capturing the causal relationships between criteria, which is an important analytical tool to prioritize the alternatives based on the type of relationships and severity of influences. The DEMATEL method to analyze indirect relations has been successfully used in many industrial fields, such as marketing strategies, R&D projects, e-learning evaluations, managers' competencies, control systems, and airline safety problems (Chiu, Chen, Tzeng, & Shyu, 2006; Hori & Shimizu, 1999; Lin & Wu, 2008; Tzeng, Chiang, & Li, 2007).

Chang et al. (2009) introduced the ordered weighted geometric averaging (OWGA) operator and DEMATEL method to evaluate the ordering of risks for failure problems. This is a more general risk priority number (RPN) methodology and provides a more general and reasonable risk assessment ranking. The proposed approach, using ME-OWA, was used throughout the DEMATEL calculation to determine subsystem allocation weighting factors, and the DEMATEL technique also can consider indirect relations. After DEMATEL calculation processes, the proposed method also considers each subsystem's predicted failure rate. The higher ME-OWA-based DEMATEL values should have a higher reliability allocation overall rating and apportion a higher reliability ratio into subsystems or components. Meanwhile, when using the situation parameter (α), considering the indirect relationship and predicted subsystem failure rate at same time, the proposed method can efficiently resolve the five shortcomings of the conventional reliability allocation methods. This study evaluates reliability allocation using a fighter aircraft digital flight control computer (DFLCC). The results from comparison with conventional reliability methods show that the proposed method is an effective methodology, with proven effectiveness yet flexible and yielding accurate results.

The remainder of this paper is organized as follows: Section 2 introduces the ME-OWA operations and applications, Section 3 introduces the DEMATEL method, Section 4 introduces conventional reliability allocation methods, Section 5 proposes the ME-OWA-based DEMATEL method, and in Section 6, an example is drawn from an aircraft fighter's DFLCC using the proposed approach for reliability allocation assessment. Section 7 is the conclusion.

2. ME-OWA operators and its operations

2.1. ME-OWA operators

Yager (1988) first introduced the concept of OWA operators, which are important aggregation operators within the class of weighted aggregation methods. They have the ability to derive optimal weights of the attributes based on the rating of the weighting vectors after an aggregation process (see Definition 1).

Definition 1. An OWA operator of dimension n is mapped F: $R^n \to R$, which has an associated n weighting vector $W = [w_1, w_2, \ldots, w_n]^T$ of the properties $\sum_i w_i = 1, \forall w_i \in [0,1], i = 1, \ldots, \mathrm{n}$, such that

$$f(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i \tag{1}$$

where b_i is the ith largest element in the vector $(a_1, a_2, \ldots, a_n)$, and $b_1 \geqslant b_2 \geqslant \ldots \geqslant b_n$.

Yager (1988) also introduced two important characterizing measurements with respect to the weighting vector W of the OWA operator. One of these two measures is orness of the aggregation, which is defined in Definition 2.

Definition 2. Assume F is an OWA aggregation operator with a weighting function $W = [w_1, w_2, \ldots, w_n]$. The degree of orness associated with this operator is defined as:

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \tag{2}$$

where $orness(W) = \alpha$ is a situation parameter.

It is clear that $orness(W) \in [0,1]$ holds for any weighting vector.

The second characterizing measurement introduced by Yager (1988) is a measure of dispersion of the aggregation, which is defined in Definition 3.

Definition 3. Assume W is a weighting vector with elements $w_1, \ldots, w_n$; then the measure of dispersion of W is defined as:

$$dispersion(W) = -\sum_{i=1}^{n} w_i \ln w_i. \tag{3}$$

O'Hagan (1988) combined the principle of maximum entropy and OWA operators to propose a particular OWA weight that has maximum entropy with a given level of orness. This approach is based on the solution of the following mathematical programming problem:

$$\text{Maximize} \quad -\sum_{i=1}^{n} w_i \ln w_i \tag{4}$$

$$\text{Subject to}: \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i = \alpha, \quad 0 \leqslant \alpha \leqslant 1, \tag{5}$$

$$\sum_{i=1}^{n} w_i = 1, \quad 0 \leqslant w_i \leqslant 1, \quad i = 1, \ldots, n. \tag{6}$$

2.2. Determination of ME-OWA weights

Fuller and Majlender (2001) used the method of Lagrange multipliers on Yager's OWA equation to derive a polynomial equation, which can determine the optimal weighting vector under the maximal entropy. By their method, the associated weighting vector is easily obtained by Eqs. (7)–(9):

$$w_i = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \tag{7}$$

and

$$w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - nw_1} \tag{8}$$

then

$$w_1[(n-1)\alpha + 1 - nw_1]^n = ((n-1)\alpha)^{n-1}[((n-1)\alpha - n)w_1 + 1] \tag{9}$$

where w is the weight vector, n is the number of attributes, and α is the situation parameter.

3. DEMATEL methodology

The Battelle Memorial Institute, through its Geneva Research Centre (Gabus & Fontela, 1973), first developed the DEMATEL method. It is a potent method that gathers group knowledge for capturing the causal relationship between criteria and can precisely ascertain the cause–effect relationship of criteria when measuring a problem. In recent years, the DEMATEL method has been successfully applied in different industries and in many fields (Chiu et al., 2006; Hori & Shimizu, 1999; Lin & Wu, 2008; Tzeng et al., 2007). The original DEMATEL method was aimed at the fragmented and antagonistic phenomena of world societies and the search for integrated solutions. It is especially practical and useful for visualizing the structure of complicated causal relationships with matrices or digraphs. The matrices or digraphs portray a contextual relation between the elements of the system in which a numeral represents the strength of influence. Hence, the DEMATEL method can convert the relationship between the causes and effects of criteria into an intelligible structural model of the system, which are not only the direct influences taken into account but also the indirect influences among multiple factors. In this section, we briefly describe the DEMATEL method and procedure.

3.1. Outline of the DEMATEL method

The essential of the DEMATEL method is reviewed below (Seyed-Hosseini, Safaei, & Asgharpour, 2006).

Definition 4. The pair-wise comparison scale may be designated into four levels, where the scores of 0, 1, 2, and 3 represent "No influence", "Low influence", "High influence", and "Very high influence", respectively.

Definition 5. The initial direct-relation matrix Z is an $n \times n$ matrix that is obtained by pair-wise comparisons in terms of influences and directions between criteria, in which Z_{ij} is denoted as the degree to which the criterion D_i affects criterion D_j. Accordingly, all principal diagonal elements Z_{ii} of matrix Z are set to zero:

$$Z = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} 0 & z_{12} & \cdots & z_{1n} \\ z_{21} & 0 & 0 & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & 0 \end{bmatrix} \end{array}. \tag{10}$$

Definition 6. Let

$$s = \max_{1 \leqslant j \leqslant n} \left(\sum_{j=1}^{n} z_{ij} \right). \tag{11}$$

Then, the normalized direct-relation matrix X can be obtained through the following formula:

$$X = \frac{Z}{s}. \tag{12}$$

Definition 7. The total relation matrix T can be acquired by using formula (13), in which the I is denoted as the identity matrix:

$$T = \operatorname*{limit}_{k \to \infty}(X + X^2 + \cdots + X^k) = X(I - X)^{-1}. \tag{13}$$

Definition 8. Let t_{ij} $(i, j = 1, 2, \ldots, n)$ be the elements of the total-relation matrix T; then, the sum of the rows and the sum of the columns, denoted as R_i and C_j, respectively, can be obtained through the following two formulas:

$$D_i = \sum_{j=1}^{n} t_{ij} \quad (i = 1, 2, \ldots, n), \tag{14}$$

$$R_j = \sum_{i=1}^{n} t_{ij} \quad (j = 1, 2, \ldots, n). \tag{15}$$

Definition 9. A causal diagram can be acquired by mapping the ordered pairs of $(R + C,\ R - C)$, where the horizontal axis $(R + C)$ is named "Prominence" and the vertical axis $(R - C)$ is named "Relation."

In the causal diagram, the horizontal axis "Prominence" shows how important the criterion is, whereas the vertical axis "Relation" may divide the criteria into the cause and effect groups. When the value $(R - C)$ is positive, the criterion belongs to the cause group. If the value $(R - C)$ is negative, the criterion belongs to the effect group. Hence, causal diagrams can visualize the complicated causal relationships between criteria into a visible structural model and provide valuable insight for problem-solving. Furthermore, with the help of a causal diagram, this study will allow proper decisions to be made by recognizing the difference between cause and effect criteria.

3.2. The procedure of the DEMATEL method

The DEMATEL method can separate the relevant criteria of a system into the cause and effect groups to facilitate accurate decision-making. A DEMATEL procedure is explained as follows (Seyed-Hosseini et al., 2006):

(1) A system designer or decision-maker evaluates the relationship between sets of paired alternatives. As a result of this evaluation, a matrix M is obtained as the initial data of the DEMATEL analysis.
(2) The elements of the direct relative severity matrix (DRSM) are obtained by Eq. (11). It is the normalized version of matrix M.
(3) The elements of the direct and indirect relative severity matrix (DIRSM) are obtained by Eq. (12). The DIRSM consists of all of the relations, including direct and indirect relations between alternatives.
(4) Using the values of $R + C$ and $R - C$, where C is the sum of the columns and R is the sum of the rows of the DIRSM, a level of influence and a level of relationship are defined. The value

$R-C$ indicates the severity of influence for each alternative. Similarly, the value of $R+C$ indicates the degree of relation between each alternative with one another.

4. Conventional reliability allocation methods

This section presents a literature review of the applications of reliability allocation. Currently, many reliability allocation techniques are available, including the AGREE method (1957), ARINC apportionment technique (1976), Bracha method (1964), Karmiol method (1965), equalization allocation method, pair comparison allocation, the FOO technique, minimization of effort algorithm (1988), Boyd method (1992), average weighting allocation method (Kuo, 1999), Base method, and IFM method (Falcone et al., 2002), among others. The allocation technique significantly influences product life cycle cost and system operational effectiveness. Since the ARINC apportionment technique is an important method in military reliability allocation design, the basic definition and procedure of the ARINC apportionment technique and the ME-OWA apportionment method are reviewed in this section.

4.1. ARINC apportionment technique

This method assumes a series of subsystems with constant failure rates. Other fundamental assumptions are: (1) series subsystems, (2) constant failure rates, (3) same mission duration time T for each subsystem, and (4) a pre-defined, known allowable system failure rate: λ^*.

Suppose a system is composed of N subsystems. Let λ_i^* be the failure rate allocated to subsystem i. The objective is to choose λ_i^* such that (1988):

$$\sum_{i=1}^{n} \lambda_i^* \geqslant \lambda^*. \tag{16}$$

Determine the subsystem failure rates (λ_i) from past observation or estimation; assign a weighting factor (w_i) to each subsystem according to the failure rates determined by Eq. (17):

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n}\lambda_i}. \tag{17}$$

Allocate subsystem failure rate requirements as follows:

$$\lambda_i^* = w_i\lambda^*. \tag{18}$$

4.2. ME-OWA apportionment method

Chang et al. (2009) presented an innovative reliability allocation using the ME-OWA method for fighter aircraft RADAR programs. This method can determine the optimal weighting vector under maximal entropy, and the OWA operator has the ability to ascertain the optimal reliability allocation rating after an aggregation process. With the optimal weighting vector under maximal entropy with respect to different α values, sensitivity analysis enables the identification of different α values to evaluate their impact on the reliability allocation rating using Eqs. (7)–(9) with $n = 4$. Results from this analysis are presented in Table 1.

Table 1
The optimal weighting vector under maximal entropy ($n = 4$).

Weight	w_1	w_2	w_3	w_4
$\alpha = 0.5$	0.250000	0.250000	0.250000	0.250000
$\alpha = 0.6$	0.416657	0.233398	0.130859	0.073547
$\alpha = 0.7$	0.493805	0.237305	0.113770	0.054918
$\alpha = 0.8$	0.596466	0.251953	0.106445	0.045018
$\alpha = 0.9$	0.764099	0.182129	0.043457	0.010365
$\alpha = 1.0$	1.000000	0.000000	0.000000	0.000000

Suppose a system is composed of m subsystems; n is the number of system factors. Let R_s be the system's allocated rating, R_i be the allocated rating to the ith subsystem, and T be the mission duration; then the system failure rate λ_s is determined by Eq. (19) (1988):

$$\lambda_s = -\ln(R)/T. \tag{19}$$

This method uses four subsystem allocation factors, which are computed as a function of a numerical rating, such as system intricacy (I), state of the art (S), performance time (P), environment (E), mission time (T), complexity (K), functionality (F), effectiveness (E), and operational profile (O). Each rating is based on a scale from 1 to 10 and is estimated using design engineering and expert judgments. In order to compare the different method capabilities, the same influential system reliability factors were selected: I, S, P, and E. Also, the same estimated rating derived from design engineering and expert judgment. Based on Table 1 and Eq. (1), calculate the aggregated values by OWA weights with respect to different values of (α = 0.5, 0.6, 0.7, 0.8, 0.9, 1); $\alpha = 1$ is used to represent the situation when the decision-maker is maximally optimistic (a pure optimist), and $\alpha = 0.5$ is used when the decision-maker faces a moderate assessment. Use Eq. (20) to calculate complexity $C'_k, \forall k$.

$$C'_k = \frac{w'_k}{W'}, \quad \forall k. \tag{20}$$

Use Eq. (21) to calculate the allocated subsystem failure rate λ_k, $\forall k$.

$$\lambda_k = C'_k\lambda_s, \quad \forall k. \tag{21}$$

5. Proposed ME-OWA-based DEMATEL apportionment method

5.1. Advantages of the ME-OWA-based DEMATEL apportionment method

With the FOO technique, the average weighting allocation method and IFM method, such as the *ISPE* and *IG* values, are ordinal measurement scales; therefore, the operations of multiplication and division are not meaningful and misleading. As a result, some (I, S, P, E) scenarios produce an *ISPE* value that is lower than other combinations but potentially produce a higher reliability allocation overall rating. For example, the scenario with *ISPE* value $9 \times 5 \times 2 \times 2 = 180$ is lower than the scenario with *ISPE* value $7 \times 5 \times 3 \times 2 = 210$, even thought it should have a higher reliability allocation overall rating. Therefore, I, S, P, and E are not equally weighted with respect to one another in terms of overall rating. Meanwhile, most current reliability allocation methods do not consider indirect relations between subsystems or components; also, most reliability allocation methods do not consider predicted failure rate in the evaluation process.

In resolving the five conventional reliability allocation problems mentioned above, the proposed approach is to combine the ME-OWA, DEMATEL, and the ARINC methods. The ME-OWA is used to derive ISPE values and then uses the DEMATEL for capturing the causal relationship between criteria; this method also considers a subsystem's predicted failure rate at the same time. After calculating each subsystem's total indirect relationship, then apportion a reasonable reliability rating into subsystems or components.

To maintain the $R-C$ value in a positive state, this study proposes the $R-c$ value. c represents the average severity of influence for each alternative; the c value can be obtained through the following formula—then, the $R-c$ value is derived:

$$c_i = \frac{C_i}{\sum_{i=1}^{n} C_i}. \tag{22}$$

5.2. Procedures of the ME-OWA-based DEMATEL apportionment method

The detailed procedure of the proposed ME-OWA-based DEMATEL apportionment method is organized into 10 steps and is described as follows:

Step 1: List the structure of systems and subsystems.

Step 2: Define the system reliability and mission time.

Step 3: Compute the system failure rate from system specifications.
Based on Eq. (19), derive the system failure rate λ_s.

Step 4: Determine the scales for *I*, *S*, *P*, and *E*, respectively.
Subsystem allocation factors are computed as a function of numerical ratings of *I*, *S*, *P*, and *E*.

Step 5: Perform DEMATEL procedure.
The procedure of the DEMATEL process is as follows (Seyed-Hosseini et al., 2006):

(1) A system designer or decision-maker evaluates the relationship between subsystems of paired alternatives. The pair-wise comparison scale may be designated into 10 levels, where the scores of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 represent the influence levels “None”, “Very minor”, “Minor”, “Low”, “Moderate”, “Significant”, “Major”, “Extreme”, “Serious”, and “Hazardous”, respectively.
(2) Obtain the elements of DRSM by Eq. (11).
(3) Obtain the elements of DIRSM by Eq. (12).
(4) Calculate the values of $R + C$, $R - C$, and $R - c$.

Step 6: Compute reliability allocation values by assigning different values to the conditional parameter ($\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$).
Based on Table 1 and Eq. (1), calculate the aggregated values by OWA weights with respect to different values of ($\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$).

Step 7: Compute the allocation rating r_i for each subsystem and derive the overall rating w_i for the *k*th subsystem.
According to the results of DEMATEL’s calculation, using Eq. (22) to derive c_i, then calculate $R - c$ values and use Eq. (23) for allocation weighting factors; calculate the aggregated value by ME-OWA-based weights. The apportioning weighing ratio also consider the prediction failure rate in the apportionment process. Use Eq. (23) to calculate the allocate weighting factors w_i, $\forall i$:

$$w_i = \sum_{i=1}^{n} ISPE_i * D_i * \lambda_i \Big/ \sum_{i=1}^{n}\sum_{i=1}^{m} ISPE_i * D_i * \lambda_i \tag{23}$$

where *n*: number of *ISPE*, *m*: number of subsystems, $ISPE_i$: ME-OWA values for subsystem *i*, D_i: $R - c$ values for subsystem *i*, and λ_i: predicted failure rate for subsystem *i*.

Step 8: Compute the complexity C'_k for the *k*th subsystem, $\forall k$.
Use Eq. (20) to calculate the complexity $C'_k, \forall k$.

Step 9: Compute the allocated subsystem failure rate.
Use Eq. (21) to calculate the allocated subsystem failure rate λ_k, $\forall k$.

Step 10: Analyze the results and select the optimal reliability allocation decision.

6. A case study of the ME-OWA-based DEMATEL apportionment method

This paper presents a real-world illustrative example for implementation of the ME-OWA-based DEMATEL apportionment method. A case study of a DFLCC that is installed on a fighter aircraft, drawn from an aircraft company in Taiwan, was used to demonstrate the proposed approach. The DFLCC is a triplex redundant digital flight control computer. The DFLCC receives discrete, analog, and digital input signals from various sensors and computers throughout the fighter aircraft. It processes these input signals through a series of redundant management and control law algorithms, which were developed and supplied by the customer. The DFLCC produces various discrete, analog, and digital output signals that are sent to provide the pilot with the digital flight control system status and to the primary servo actuators and leading edge flap, used to control the aircraft’s motions. The DFLCC consists of eight major shop replaceable units (SRUs). The SRUs are the CPU circuit card assembly (CPU card), Input/Output Processor circuit card assembly (IOP card), ADIO circuit card assembly (ADIO card), 1553/DOUT circuit card assembly (1553/DOUT card), power supply assembly (P/S card), ACS circuit card assembly (ACS card), IBU circuit card assembly (IBU card), and motherboard. The DFLCC block diagram is show in Fig. 1.

To accurately assess reliability requirements of subsystems or components, system design engineering needs to clearly define the diagram for each system and subsystem. The diagram for the DFLCC is shown in Fig. 2.

6.1. ARINC apportionment technique analysis

To illustrate this method, consider a DFLCC that consists of eight major SRUs. The SRUs are the CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard. There are eight subsystems with predicted failure rates of $\lambda_1 = 22.4$ (CPU card), $\lambda_2 = 23.3$ (IOP card), $\lambda_3 = 15.3$ (ADIO card), $\lambda_4 = 13.1$ (1553/DOUT card), $\lambda_5 = 44.8$ (P/S card), $\lambda_6 = 35.5$ (ACS card), $\lambda_7 = 11.4$ (IBU card), and $\lambda_8 = 16.6$ (motherboard) failures per 10^6 h, respectively. The system has a mission time of 3 h, and a 0.999412 reliability is required. The apportioned system reliability goals are found by Eq. (19) as follows:

$$\lambda s = -\ln(R)/T = -\ln(0.999412)/3 = 196.058 \text{ failures per } 10^6 \text{ h}.$$

Based on the predicted subsystem failure rate for each subsystem, as follows:

$$\lambda_1 = 22.4, \quad \lambda_2 = 23.3, \quad \lambda_3 = 15.3, \quad \lambda_4 = 13.1, \quad \lambda_5 = 44.8$$
$$\lambda_6 = 35.5, \quad \lambda_7 = 11.4, \quad \lambda_8 = 16.6.$$

Using Eq. (17), the weighting factor (w_i) for each subsystem is as follows:

$$w_1 = \frac{22.4}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.122923$$

$$w_2 = \frac{23.3}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.127650$$

$$w_3 = \frac{15.3}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.083677$$

$$w_4 = \frac{13.1}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.071762$$

$$w_5 = \frac{44.8}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.245880$$

$$w_6 = \frac{35.5}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.194512$$

$$w_7 = \frac{11.4}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.062637$$

$$w_8 = \frac{16.6}{22.4 + 23.3 + 15.3 + 13.1 + 44.8 + 35.5 + 11.4 + 16.6} = 0.090958.$$

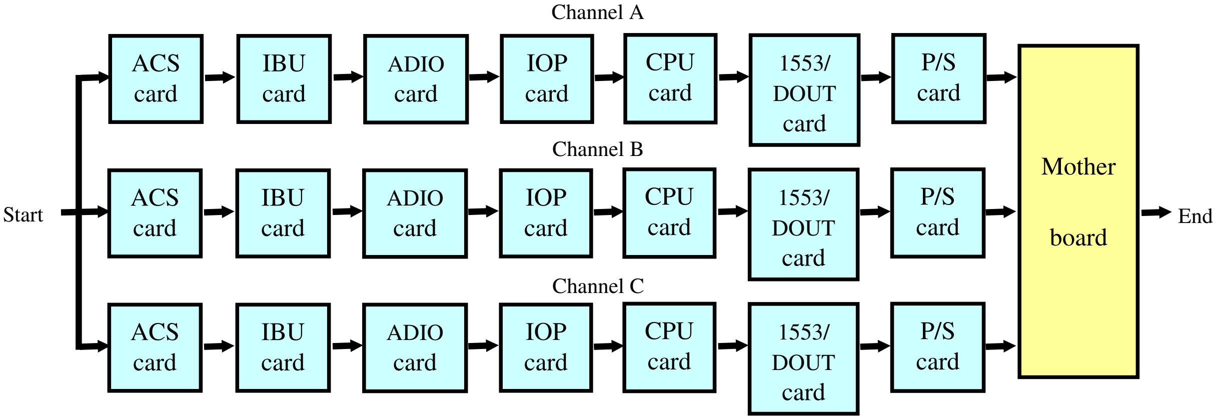

Fig. 1. The digital flight control computer (DFLCC) block diagram.

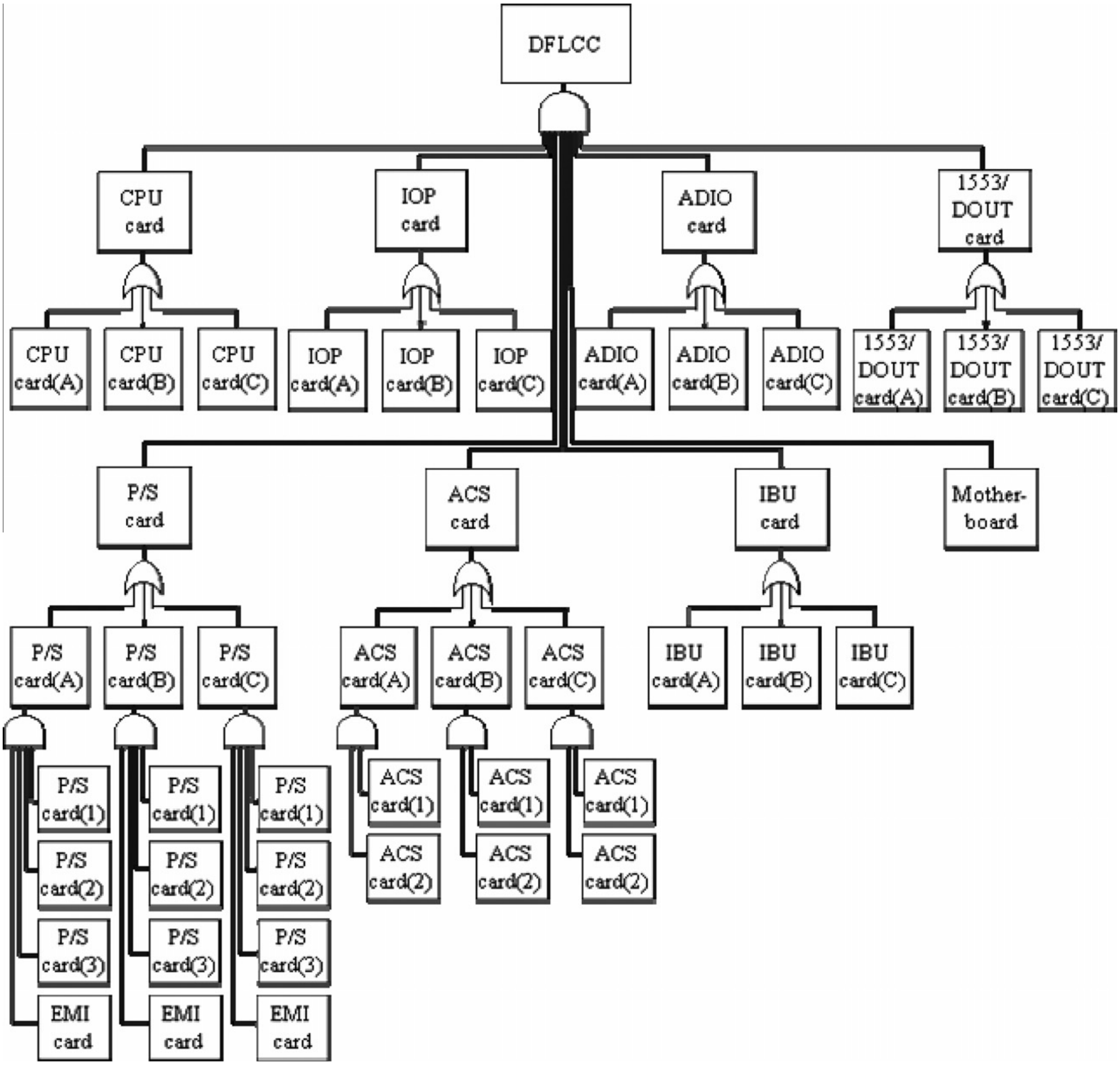

Fig. 2. Diagram for the digital flight control computer (DFLCC).

Using Eq. (18), the corresponding allocated subsystem reliability requirements are:

$$\lambda_1^* = w_1 \lambda s = 24.10014$$
$$\lambda_2^* = w_2 \lambda s = 25.02689$$
$$\lambda_3^* = w_3 \lambda s = 16.40557$$
$$\lambda_4^* = w_4 \lambda s = 14.06958$$
$$\lambda_5^* = w_5 \lambda s = 48.20675$$
$$\lambda_6^* = w_6 \lambda s = 38.13554$$
$$\lambda_7^* = w_7 \lambda s = 12.28047$$
$$\lambda_8^* = w_8 \lambda s = 17.83306.$$

6.2. ME-OWA method analysis

The proposed approach uses the ME-OWA operator for weight calculations. A sensitivity analysis using different values of α is presented to evaluate their impact on the reliability allocation rating. Based on Table 1, the optimal weighting under maximal entropy ($n = 4$), and Eq. (1), the failure rate of the CPU card is calculated as follows.

For $\alpha = 0.5$ (used when the decision-maker faces a moderate assessment), if I, S, P, and E for the subsystem CPU card are 10, 9, 8, and 7, respectively, and the weighting vector contains $w_1 = 0.25$, $w_2 = 0.25$, $w_3 = 0.25$, and $w_4 = 0.25$, then

$$w'_k = (10 \times 0.25) + (9 \times 0.25) + (8 \times 0.25) + (7 \times 0.25) = 8.5$$
$$C'_k = 8.5/45.5 = 0.187.$$

As a result, the failure rate of the CPU card (for $\alpha = 0.5$) = 196.058 × 0.187 = 36.62622 per 10^6 h.

$\alpha = 1$ is used to represent the situation when the decision-maker is maximally optimistic (a pure optimist). For $\alpha = 1$, the $OWA(a_1, a_2, a_3) = Max(a_1, a_2, a_3)$ if I, S, P, and E for the subsystem CPU card are 10, 9, 8, and 7, respectively. The weighting vector contains $w_1 = 1$, $w_2 = 0$, $w_3 = 0$, and $w_4 = 0$; consequently,

$$w'_k = (10 \times 1) + (9 \times 0) + (8 \times 0) + (7 \times 0) = 10$$
$$C'_k = 10/59 = 0.169.$$

As a result, the failure rate of the CPU card (for $\alpha = 1$) = 196.058 × 0.169 = 33.23017 per 10^6 h.

Following the calculation above, the aggregated values of OWA weights by different values of α ($\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$) are calculated for the subsystem CPU card. The resulting failure rates are 33.23017, 33.79919, 34.42269, 34.31116, 34.95256, and 36.62622 per 10^6 h, respectively. The failure rates for the CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard are also calculated, and the results are summarized in Table 2, columns (5) through (10). The reliability allocation results for the DFLCC system (ME-OWA method) are also shown in Table 2.

6.3. Proposed ME-OWA-based DEMATEL apportionment method analysis

The proposed approach is to combine the ME-OWA, DEMATEL, and ARINC methods. The ME-OWA uses the four system reliability factors I, S, P, and E after the estimated rating from design engineering and expert judgment and under situation parameters ($\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$) to derive the *ISPE* values. The results are summarized in Table 2, columns (5) through (10). Then, the DEMATEL is used for capturing the causal relationships between criteria. This method also considers a subsystem's predicted failure rate at the same time. Based on the results of system engineering and expert evaluation of the relationship between sets of paired subsystems or components, the initial direct-relation matrix Z is shown in Fig. 3.

According to Eq. (12), the elements of the direct relative severity matrix (DRSM) are obtained and shown in Fig. 4.

According to Eq. (13), the elements of the direct and indirect relative severity matrix (DIRSM) are obtained, which are shown in Fig. 5.

According to Eq. (10)–(15) and using Eq. (22), the outcome of DEMATEL implementation for the DFLCC with respect to direct and indirect relationships is shown in Table 3.

After calculating each subsystem's total indirect relationship, then apportion reasonable reliability ratings into subsystems or components. The calculation results for this model are as follows.

The subsystem failure rates (λ_k, $\forall k$) are estimated from past observations or estimations. The SRUs are the CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard. The predicted failure rates of the eight subsystems are $\lambda_1 = 22.4$ (CPU card), $\lambda_2 = 23.3$ (IOP card), $\lambda_3 = 15.3$ (ADIO card), $\lambda_4 = 13.1$ (1553/DOUT card), $\lambda_5 = 44.8$ (P/S card), $\lambda_6 = 35.5$ (ACS card), $\lambda_7 = 11.4$ (IBU card), and $\lambda_8 = 16.6$ (motherboard) failures per 10^6 h, respectively. The ISPE values for these eight subsystems are summarized in Table 2, columns (1) through (4). Then, using Eq. (1), we compute the overall rating and complexity factors and then ascertain the allocation failure rate for the CPU board as follows.

For $\alpha = 0.5$ (used when the decision-maker faces a moderate assessment), if I, S, P, and E for the subsystem CPU card are 10, 9, 8, and 7, respectively, the CPU card's predicted failure rate is $\lambda_1 = 22.4$. Based on Table 1 and Eq. (1) to derive *ISPE* = 8.5, according to Eq. (10)–(15), the outcome of the DEMATEL $R - c$ values is 5.25; then

Table 2
The reliability allocation results for DFLCC system (ME-OWA method).

Method	(1)	(2)	(3)	(4)	ME-OWA					
	Intricacy (I)	State-of-the-art (S)	Performance time (P)	Environment (E)	(5)	(6)	(7)	(8)	(9)	(10)
					$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1.0$
CPU	10	9	8	7	36.62622	34.95256	34.31116	34.42269	33.79919	33.23017
IOP	9	9	5	6	31.24001	31.26718	30.90323	31.12875	30.76176	29.90715
ADIO	5	6	3	4	19.39035	19.69331	19.55413	19.77479	19.86120	19.93810
1553/DOUT	7	8	7	6	30.16277	28.23552	27.62428	27.65346	27.01773	26.58414
P/S	4	5	10	9	30.16277	32.21480	32.23624	32.75853	33.23659	33.23017
ACS	3	4	4	3	15.08138	14.34666	14.06539	14.09318	13.75045	13.29207
IBU	2	3	3	3	11.84966	11.11608	12.70847	10.82105	10.41737	9.96905
Motherboard	3	4	9	4	21.54484	24.23189	24.65510	25.40554	27.21371	29.90715
Total	43	48	49	42	196.05800	196.05800	196.05800	196.05800	196.05800	196.05800

$$\begin{array}{cc} & \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ Z = \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} & \begin{bmatrix} 0 & 9 & 7 & 7 & 6 & 5 & 3 & 2 \\ 8 & 0 & 6 & 6 & 5 & 3 & 3 & 2 \\ 5 & 5 & 0 & 4 & 6 & 4 & 3 & 2 \\ 6 & 5 & 4 & 0 & 5 & 4 & 3 & 2 \\ 8 & 7 & 6 & 5 & 0 & 3 & 3 & 4 \\ 4 & 4 & 3 & 3 & 3 & 0 & 2 & 2 \\ 3 & 4 & 2 & 2 & 4 & 3 & 0 & 2 \\ 3 & 4 & 4 & 3 & 6 & 2 & 4 & 0 \end{bmatrix} \end{array}$$

Note: A: CPU card, B: IOP card, C: ADIO card, D: 1553/DOUT card, E: P/S card, F: ACS card, G: IBU card,and H: Motherboard.

Fig. 3. Corresponding initial direct-relation matrix *Z*.

$$\begin{array}{cc} & \begin{array}{cccccccc} \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{E} & \mathrm{F} & \mathrm{G} & \mathrm{H} \end{array} \\ Z = \begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \\ \mathrm{E} \\ \mathrm{F} \\ \mathrm{G} \\ \mathrm{H} \end{array} & \begin{bmatrix} 0 & 0.231 & 0.179 & 0.179 & 0.154 & 0.128 & 0.077 & 0.051 \\ 0.205 & 0 & 0.154 & 0.154 & 0.128 & 0.077 & 0.077 & 0.051 \\ 0.128 & 0.128 & 0 & 0.103 & 0.154 & 0.103 & 0.077 & 0.051 \\ 0.154 & 0.128 & 0.103 & 0 & 0.128 & 0.103 & 0.077 & 0.051 \\ 0.205 & 0.179 & 0.154 & 0.128 & 0 & 0.077 & 0.077 & 0.103 \\ 0.103 & 0.103 & 0.077 & 0.077 & 0.077 & 0 & 0.051 & 0.051 \\ 0.077 & 0.103 & 0.051 & 0.051 & 0.103 & 0.077 & 0 & 0.051 \\ 0.077 & 0.103 & 0.103 & 0.077 & 0.154 & 0.051 & 0.103 & 0 \end{bmatrix} \end{array}$$

Fig. 4. Corresponding DRSM of the DFLCC.

$$\begin{array}{cc} & \begin{array}{cccccccc} \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{E} & \mathrm{F} & \mathrm{G} & \mathrm{H} \end{array} \\ Z = \begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \\ \mathrm{E} \\ \mathrm{F} \\ \mathrm{G} \\ \mathrm{H} \end{array} & \begin{bmatrix} 1.568 & 0.759 & 0.648 & 0.626 & 0.640 & 0.484 & 0.381 & 0.295 \\ 0.668 & 1.501 & 0.569 & 0.549 & 0.560 & 0.399 & 0.343 & 0.264 \\ 0.548 & 0.551 & 1.380 & 0.455 & 0.523 & 0.377 & 0.310 & 0.239 \\ 0.568 & 0.552 & 0.474 & 1.363 & 0.503 & 0.379 & 0.310 & 0.239 \\ 0.706 & 0.692 & 0.603 & 0.561 & 1.482 & 0.422 & 0.366 & 0.324 \\ 0.412 & 0.415 & 0.352 & 0.339 & 0.359 & 1.210 & 0.226 & 0.188 \\ 0.374 & 0.396 & 0.314 & 0.301 & 0.364 & 0.269 & 1.167 & 0.182 \\ 0.457 & 0.480 & 0.429 & 0.391 & 0.483 & 0.300 & 0.308 & 1.171 \end{bmatrix} \end{array}$$

Fig. 5. Corresponding DIRSM of the DFLCC.

Table 3
The $R + C$, $R - C$ and $R - c$ values of the DFLCC by the ME-OWA based DEMATEL method.

No.	R	C	c	$R + C$	$R - C$	$R - c$
1	5.401	5.301	0.151	10.702	0.100	5.250
2	4.853	5.346	0.152	10.198	−0.493	4.700
3	4.382	4.768	0.136	9.150	−0.386	4.246
4	4.388	4.586	0.131	8.974	−0.199	4.257
5	5.156	4.914	0.140	10.070	0.242	5.016
6	3.502	3.839	0.109	7.341	−0.338	3.392
7	3.367	3.411	0.097	6.778	−0.044	3.270
8	4.018	2.902	0.083	6.920	1.117	3.936

$$w'_k = 8.5 \times 5.25 \times 22.4 = 999.6$$
$$C'_k = 999.6/4899.96 = 0.204.$$

As a result, the failure rate of the CPU card (for $\alpha = 0.5$) = 196.058 × 0.204 = 39.99635 per 10^6 h.

Following the calculation above, the aggregated values of OWA weights ($\alpha = 0.5$) for the subsystem CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard are 39.99635, 31.76884, 11.6974, 15.61947, 62.93992, 16.86418, 4.10171, and 13.07012 failures per 10^6 h, respectively. The results are summarized in Table 4, column (7).

$\alpha = 1$ is used to represent the situation when the decision-maker is maximally optimistic (a pure optimist). For $\alpha = 1$, the $OWA(a_1, a_2, a_3) = Max(a_1, a_2, a_3)$. If*I*, *S*, *P*, and *E* for the subsystem CPU card are 10, 9, 8, and 7, respectively, the CPU card's predicted failure rate is $\lambda_1 = 22.4$. Based on Table 1 and Eq. (1), *ISPE* = 10 is derived. According to Eq. (10)–(15) and using Eq. (22), the outcome of the DEMATEL $R - c$ values is 5.25; then

$$w'_k = 10 \times 5.25 \times 22.4 = 1176.01$$
$$C'_k = 1176.01/6426.23 = 0.183.$$

As a result, the failure rate of the CPU card (for $\alpha = 1$) = 196.058 × 0.183 = 35.87878 per 10^6 h.

Following the calculation above, the aggregated values of OWA weights ($\alpha = 1$) for the subsystem CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard are 35.87878, 30.0706, 11.89226, 13.61114, 68.55898, 14.69581, 3.41185, and 17.93859 failures per 10^6 h, respectively. The aggregated values of OWA weights by different values of ($\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 1$) for the subsystem CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard are also calculated, and the results are summarized in Table 4, columns (7) through (12). The reliability allocation results for the DFLCC system (ME-OWA-based DEMATEL method) are shown in Table 4.

6.4. Method comparison

As shown in Table 5, the failure rates using the ARINC apportionment technique for the CPU card, IOP card, ADIO card, 1553/DOUT card, P/S card, ACS card, IBU card, and motherboard are calculated, and the results are summarized in Table 5, row (13). The ME-OWA method results are shown in Table 5, rows (7) through (12). The ME-OWA-based DEMATEL method results are shown in Table 5, Rows (1)–(6). In this case study, using the ARINC apportionment technique, the appointing failure rates were calculated for the subsystem P/S card (48.20675) > ACS card (38.13554) > IOP card (25.02689) > CPU card (24.10014) > Motherboard (17.83306) > ADIO card (16.40557) > 1553/DOUT card (14.06958) > IBU card (12.28047). Compared with the ME-OWA method ($\alpha = 0.9$), the appointing failure rate of the subsystem CPU card (33.79919) > P/S card (33.23659) > IOP card (30.76176) > Motherboard (27.21371) > 1553/DOUT card (27.01773) > ADIO card (19.8612) > ACS card (13.75045) > IBU card (10.41737). Compared with the ME-OWA-based DEMATEL method ($\alpha = 0.9$), the appoint-

Table 4
The reliability allocation results for DFLCC system (ME-OWA based DEMATEL method).

Method	(1)	(2)	(3)	(4)	(5)	(6)	ME-OWA					
	I	S	P	E	$R-c$ (DEMATEL)	Predicted failure rate	(7) $\alpha = 0.5$	(8) $\alpha = 0.6$	(9) $\alpha = 0.7$	(10) $\alpha = 0.8$	(11) $\alpha = 0.9$	(12) $\alpha = 1.0$
CPU	10	9	8	7	5.250	22.400	39.99635	37.74110	37.22365	37.07745	36.36190	35.87878
IOP	9	9	5	6	4.700	23.300	31.76884	31.44028	31.22120	31.22401	30.81863	30.07060
ADIO	5	6	3	4	4.246	15.300	11.69740	11.74707	11.71919	11.76663	11.80378	11.89226
1553/DOUT	7	8	7	6	4.257	13.100	15.61947	14.45767	14.21155	14.12477	13.78338	13.61114
P/S	4	5	10	9	5.016	44.800	62.93992	66.46878	66.82744	67.42426	68.32559	68.55898
ACS	3	4	4	3	3.392	35.500	16.86418	15.86289	15.62541	15.54426	15.14792	14.69581
IBU	2	3	3	3	3.270	11.400	4.10171	3.80468	4.37027	3.69459	3.55246	3.41185
Motherboard	3	4	9	4	3.936	16.600	13.07012	14.53553	14.85930	15.20203	16.26432	17.93859
Total	43	48	49	42	34.067	182.400	196.05800	196.05800	196.05800	196.05800	196.05800	196.05800

Table 5
The allocation results for three methods.

	Method	Conditional parameter (α)	CPU	IOP	ADIO	1553/DOUT	P/S	ACS	IBU	Motherboard
1	ME-OWA based DEMATEL	0.5	39.99635	31.76884	11.69740	15.61947	62.93992	16.86418	4.10171	13.07012
2		0.6	37.74110	31.44028	11.74707	14.45767	66.48878	15.86289	3.80468	14.53553
3		0.7	37.22365	31.22120	11.71919	14.21155	66.82744	15.62541	4.37027	14.85930
4		0.8	37.07745	31.22401	11.76663	14.12477	67.42426	15.54426	3.69459	15.20203
5		0.9	36.36190	30.81863	11.80378	13.78338	68.32559	15.14792	3.55246	16.26432
6		1	35.87878	30.07060	11.89226	13.61114	68.55898	14.69581	3.41185	17.93859
7	ME-OWA	0.5	36.62622	31.24001	19.39035	30.16277	30.16277	15.08138	11.84966	21.54484
8		0.6	34.95256	31.26718	19.69331	28.23552	32.21481	14.34666	11.11608	24.23189
9		0.7	34.31116	30.90323	19.55413	27.62428	32.23624	14.06539	12.70847	24.65510
10		0.8	34.42269	31.12875	19.77479	27.65346	32.75853	14.09318	10.82105	25.40554
11		0.9	33.79919	30.76176	19.86120	27.01773	33.23659	13.75045	10.41737	27.21371
12		1	33.23017	29.90715	19.93810	26.58414	33.23017	13.29207	9.96905	29.90715
13	ARINC	–	24.10014	25.02689	16.40557	14.06958	48.20675	38.13554	12.28047	17.83306

ing failure rate of the subsystem P/S card (68.32559) > CPU card (36.3619) > IOP card (30.81863) > Motherboard (16.26432) > ACS card (15.14792) > 1553/DOUT card (13.78338) > ADIO card (11.80378) > IBU card (3.55246). The allocation results for the three methods are shown in Table 5.

From Table 5, because the ME-OWA-based DEMATEL method using *ISPE* values as a based, and followed by DEMATEL calculation process, which considers direct and indirect relationships between each of the subsystems at the same time. The results indicate that the P/S card and CPU card have higher failure rates, while the ADIO card and IBU card have lower failure rates. The P/S card $R-c$ value is 5.016 and has a higher predicted failure rate ($\lambda_5 = 44.8$) than the CPU card ($R-c$ values is 5.25 and predicted failure rate $\lambda_1 = 22.4$), whereas the IBU card $R-c$ value is 3.27 and has a lower predicted failure rate ($\lambda_7 = 11.4$) than the ADIO card $R-c$ values (4.246 and predicted failure rate $\lambda_3 = 15.3$). From a technical perspective, the subsystems have higher $R-c$ criteria (higher direct and indirect relationships) and higher predicted failure rates; the proposed ME-OWA-based DEMATEL apportionment method is more reasonable in appointing a more reliable ratio in subsystems.

As shown in Fig. 6, the results of the ME-OWA-based DEMATEL apportionment method, which incorporates ME-OWA, DEMATEL, and ARINC methods, this method obtains a correct and discriminating allocation ratio. The result is a more flexible and reasonable allocation rating than the conventional ARINC apportionment technique and the ME-OWA method. The results of the proposed ME-OWA-based DEMATEL method are compared with the ARINC apportionment technique and the ME-OWA method, shown in Fig. 6 below.

A comparison of the ARINC apportionment technique, the ME-OWA method, and the proposed ME-OWA-based DEMATEL apportionment method is summarized in Table 6. "O" indicates that the related factor is applicable, whereas "X" indicates that the related factor is not applicable.

The proposed method has concluded a number of advantages with its potentialities:

(1) Proposes a combined reliability allocation method using the ME-OWA-based DEMATEL method in apportioning system reliability, which combines the ME-OWA, DEMATEL, and the ARINC methods. It can overcome the three conventional shortcomings: measurement scale problem, the not equally weighted problem, and no consideration of indirect relationships between subsystems during the appointment processes. It also uses the ARINC's concept to consider the predicted failure rate for appointing reliability into subsystems or components.

(2) Considers situation parameter factors: the ME-OWA is based on estimated ratings from design engineering and expert judgment for appointing reliability. The four system reliability factors I, S, P, and E under the situation parameter (α = 0.5, 0.6, 0.7, 0.8, 0.9, 1) are used to derived the *ISPE* values. This assessment result shows that the ME-OWA-based DEMATEL method yielded results that not only were correct but also resulted in a discriminating allocation ratio that is flexible for real-world applications.

(3) Considers indirect relationships between subsystems and components: the combined DEMATEL calculates processes and can consider indirect relationships between subsystems and components. This calculation holds that the higher indirect relationship subsystems are appointed a higher allocation ratio, which can efficiently allocate limited resources in subsystems or components.

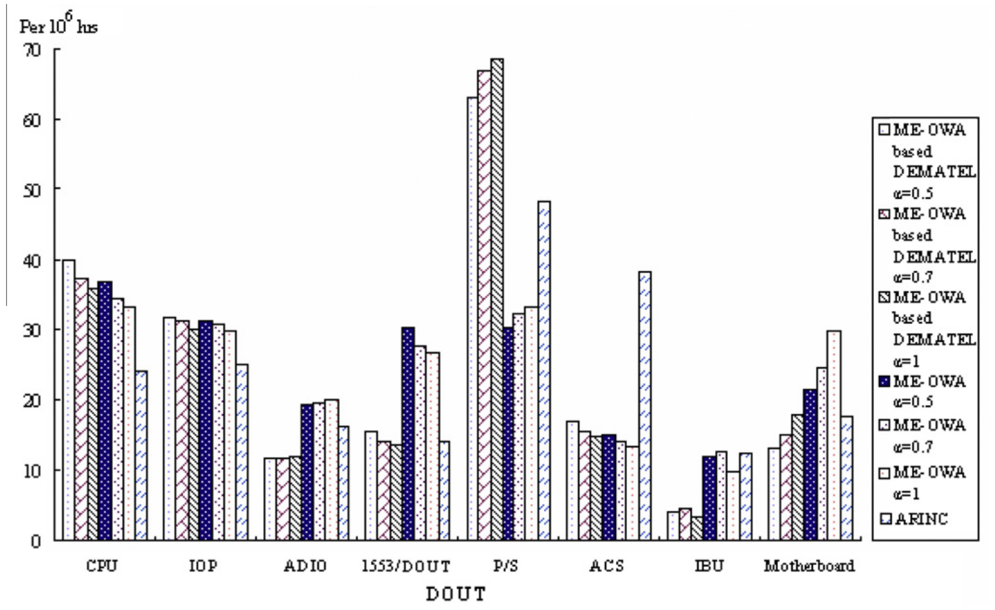

Fig. 6. Comparison of the five methods of reliability allocation.

Table 6
Comparison of the three methods.

Method	Consider factor			
	Measurement scale	Order weight	Indirect relationship	Predicted failure rate
Proposed method	O	O	O	O
ME-OWA	O	O	X	X
ARINC	X	X	X	O

Note: "O" represents that the factor is applicable, and "X" represents that the factor is not applicable.

(4) Provides an organized approach and a more flexible structure in reliability allocation: the ME-OWA-based DEMATEL method is applicable to the different design phases. The system reliability factors are not limited to only *I*, *S*, *P*, and *E*, and the number of factors is not limited to 4 items (3 or more items are acceptable). Depending upon the selection of applicable variables, such as system intricacy, complexity, state of the art (technology), cost, maintenance, risk, failure rate, design maturity, operating environment, and repair times, the allocating ratio is more suitable for different alternatives. There is no limitation for implementation of DEMATEL in very large and complex systems, and it can therefore provide an improved structured arrangement for reliability allocation.

7. Conclusion

This paper has successfully demonstrated the application of an ME-OWA-based DEMATEL apportionment method for reliability allocation using a fighter aircraft DFLCC. It is an easy, proven, and effective approach, which uses the ME-OWA to derive the ISPE values, followed by the DEMATEL calculation processes to ascertain indirect relations. The higher ISPE values, indirect relationship subsystems, and higher predicted failure rate translates into a higher reliability allocation overall rating.

The main advantages of the proposed approach are: (1) It provides an accurate yet flexible reliability allocation method, which combines the ME-OWA, DEMATEL, and the ARINC methods. The proposed approach can efficiently resolve the measurement scale problem, equally-weighted problem, and considers indirect relationships between subsystems during the appointment processes. (2) The ME-OWA-based DEMATEL method uses the conditional parameter (α) to derive the *ISPE* values. The assessment results show that the ME-OWA-based DEMATEL method yields results that not only are accurate but also yield discriminating allocation ratios that are flexible for real-world applications. The proposed ME-OWA-based DEMATEL method can better help managers or designers make correct decisions. (3) Using DEMATEL indirect relationships between subsystems and components can be considered. The calculation holds that the higher indirect relationship subsystems are appointed a higher allocation ratio, which can result in more efficient allocation of limited resources in subsystems or components. (4) It provides an organized approach and a more flexible structure in reliability allocation. The ME-OWA-based DEMATEL method is applicable to different design phases. The system reliability factors are not limited to only *I*, *S*, *P*, and *E* (3 or more items are acceptable). Depending upon the selection of applicable variables, such as system intricacy, complexity, state of the art (technology), cost, maintenance, risk, failure rate, design maturity, operating environment, and repair times, the allocating ratio is more suitable for different alternatives. There is no limitation for implementation of DEMATEL in very large and complex systems, and it can thereby provide an improved structured arrangement for reliability allocation. The ME-OWA-based DEMATEL apportionment method can also be used in a wide variety of different industries and fields. The results from the comparison with conventional reliability methods show that the proposed method is both accurate and flexible. Also, DEMATEL can be considered in Fuzzy environments that are suggested for further research.

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