Research Article

# Three Hybrid Metaheuristic Algorithms for Stochastic Flexible Flow Shop Scheduling Problem with Preventive Maintenance and Budget Constraint 

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#### Abstract

Stochastic flexible flow shop scheduling problem (SFFSSP) is one the main focus of researchers due to the complexity arises from inherent uncertainties and also the difficulty of solving such NP-hard problems. Conventionally, in such problems each machine's job process time may encounter uncertainty due to their relevant random behaviour. In order to examine such problems more realistically, fixed interval preventive maintenance (PM) and budget constraint are considered.PM activity is a crucial task to reduce the production efficiency. In the current research we focused on a scheduling problem which a job is processed at the upstream stage and all the downstream machines get busy or alternatively PM cost is significant, consequently the job waits inside the buffers and increases the associated holding cost. This paper proposes a new more realistic mathematical model which considers both the PM and holding cost of jobs inside the buffers in the stochastic flexible flow shop scheduling problem. The holding cost is controlled in the model via the budget constraint. In order to solve the proposedmodel, three hybrid metaheuristic algorithms are introduced. They include a couple of well-known metaheuristic algorithms which have efficient quality solutions in the literature. The two algorithms of them constructed byincorporationof the particle swarm optimization algorithm (PSO) and parallel simulated annealing (PSA) methods under different random generation policies. The third one enriched based on genetic algorithm (GA) with PSA. To evaluate the performance of the proposed algorithms, different numerical examples are presented. Computational experiments revealed that the proposed algorithms embedboth desirable accuracy and CPU time. Among them, the PSO-PSAП outperforms than other algorithms in terms of makespan and CPU time especially for large size problems.


Keywords: Stochastic flexible flow shop; Budget constraint, Preventive maintenance; Genetic algorithm; Simulated annealing; Particle swarm optimization.

## 1. Introduction

The flexible flow shop scheduling problem (FFSSP) consists of a flow manufacturing line with one or more parallel machine on some processing stages (or workstation) in series. Multiple products (or jobs) are produced in each stage. The objective function of FFSSP is on minimizing the total completions of all jobs (makespan or $C_{m a x}$ )(Hoogeveen et al., 1996; Gupta, 1998). This kind of issue often takes into account an NP-hard problem. This means that at a reasonable computational time, an optimal answer cannot be obtained. (Brucker and Kramer, 1995)have proved that a two-stage FFSSP remains NP-hard even there is only one machine on the first stage and two machines on the second stage.Most researchers have focused on the system which their relevant process time deploy a given deterministic value. However, in real circumstances, processing time of jobs at each stage is the main part of makespan. Most research focused on the system with deterministicprocessing time. However, in a real system, theprocessing time is arandom variable due to random behaviour of tool wearing,
operator skill, material variability and so on (Koulamas and Kyparisis, 2000). According to Choi and Wang (2012) makespan estimationmight become invalid under differentcircumstances. Another important factor that effects on makespan is interruption caused by machine breakdowns (Fahmy and Sharif, 2009). One of the most important ways to deal with these types of random interruptions is on following preventive and scheduled tasks. Hence, unavailability of machines, due to preventive maintenance, could be incorporated as a set of constraints in the mathematical models. Taking into account such disruptions due to preventive maintenance and stochastic processing time makes the problem more real, but more complicated and has been kept on the research gap at yet.The significant contributions of this paper are to propose a more realistic scheduling model for such production and delivering an efficient solution methods.The proposed model integrates FFSSP with preventive maintenance (PM) and budget constraint under stochastic processing time. Hence,consideringqueues between consecutive stages is a respectable alternative. When a job is processed at upstream stage and all the downstream machines get to busy state or comes under

[^0]any PM activity, therefore the job should wait in the bufferswhichcause holding cost.Because the total budget in handhas a given level this leads to interrupt the operations and cause increasing of the makespan. Consequently, in the proposed model the jobs are scheduled in such a way that there is no interruption in their operations. By considering all of these affecting factors, the optimized makespan will be acquired by the model. Obviously, such FFSSP also ruins an NP-hard model. Therefore, three hybrid populations based metaheuristic algorithms are introduced as solution methods. The flexible flow shop scheduling problem has been studied extensively in the literature. Koulamas and Kyparisis(2000) developed a heuristic method for a two and a three-stage FFSSP with random processing time based on the makespan objective function. They proved the effect of the proposed heuristic by finding the solutions which were better than the available lower bounds. A Tabu search (TS) algorithm together with a procedure for a constructing a complete schedule to solve the FFSSP with limited buffers has been given by Wardono and Fathi(2004) that minimizes job completion time. Their algorithm acts based on the stage-oriented decomposition approach. Akrami et al. (2006) presented a mixed-integer linear mathematical programming for FFSSP. They assumed that there are limited buffers among stages. Two meta-heuristics, based on genetic algorithm (GA) and TS algorithm, were presented to optimality solve the model. A flow shop scheduling was investigated with limited intermediate buffers was studied by Wang and Tang (2009). They focused on the makespan minimizing as a performance criterion and they applied a TS algorithm to optimize the problem. In order to improve the diversity of the TS, a scatter search mechanism was applied. The computational results showed the high efficiency of their proposed hybrid metaheuristic. Al-Hinai and ElMekkawy(2011) proposed a flexible flow shop problem with random machine failures and a two stage hybrid GA was applied. The first stage was on minimizing the primary objective; the makespan, and the second stage was optimized the biobjective function and integrates machines assignments and operations sequencing with the expected machine breakdown. Tran and Ng (2011)presented a water-flow algorithm to solve the FFSSP with intermediate buffers. They pooled the amount of precipitation and falling force to form flexible erosion capability. This work helped the erosion process of the algorithm to focus on exploiting promising regions strongly. They also utilized an improved procedure for constructing a complete schedule from a permutation that represents the sequence of jobs at the first stage of the scheduling problem. Kianfar et al. (2012) investigated a FFSSP with non-deterministic arrival of jobs and sequence dependent setup times. They used average tardiness of jobs as the objective function. To optimize the problem, they presented a novel dispatching rule and hybrid GA. A computer simulation model was also developed to evaluate the presented dispatching rule. The results showed that their proposed
dispatching rule can lead to much better results in comparison with the traditional dispatching rules. Singh and Mahapatra (2012) proposed a novel PSO to solve FFSSP. An efficient mutation operator was embedded in PSO to prevent solutions from falling into the local optimums. The performance of the PSO was evaluated against GA by a set of test problems taken from the literature. According to the obtained results, the percentage deviation of the proposed PSO of the lower bound is equal to 2.961 and the same measure for GA is equal to3.559.In order to deal with uncertain job processing times in FFSSP's, Choi and Wang (2012) presented a decomposition-based approach. The method combines two reactive approaches with a reactiveproactive approach. Lin and Ying (2013) proposed a hybrid algorithm based on the features of artificial immune systems and the annealing process of simulated annealing algorithms (SA) to optimize the FFSSP with limited buffer storage between stages. Almeder and Hartl(2013) studied the FFSSP with limited buffer storage in the metal-working industry. The partitioned the problem as a two stage problem. The first stage contains a single machine and its buffer. The semi-finished parts are stored in this buffer until a machine of the second stage is available. The second stage contains two parallel nonidentical machines. They applied a hybrid approach based on discrete-event simulation and variable neighbourhood search to optimize the problem. The hybrid approach was led to an improvement between $3 \%$ and $10 \%$ compared with the current production plan of the company. By considering unrelated parallel machines, sequencedependent setup times, probable reworks and different ready times, Rabiee et al. (2014) investigated the no-wait two-stage FFSSP. They proposed a novel hybrid algorithm based on imperialist competitive algorithm (ICA), SA, variable neighbourhood search (VNS) and GA. The performance of the proposed hybrid algorithm was evaluated against ICA, SA, VNS, GA and ant colony optimization (ACO) and high efficiency of their proposed hybrid algorithm was revealed. Arnaout (2014) tackled a rescheduling problem for the flow shop problem associated with stochastic processing and setup time. They proposed a new repair rule which and compared it with the existing algorithms. The results obtained from the computational experiments showed that the proposed repair rule can perform better those available algorithms in literature. Rahmani and Heydari (2014) studied FFSSP under uncertain processing times and unexpected arrivals of new jobs. They proposed a new approach to find robust schedules in this situation. Their approach was a proactive-reactive method which uses a two-step procedure. In order to consider stochastic processing time, Wang and Choi (2014) introduced a novel decompositionbased the Holonic approach (DBHA) to solve a FFSSP with stochastic processing time. Their proposed method was based on genetic algorithm control (GAC) and the shorting processing time contract net procontrol (SPTCNP). Also, K-means clustering is utilized to divide machines into different clusters according to their
stochastic environments. The gained results showed that DBHA had better quality solutions against GAC and SPTCNPA simulation based optimization approach for stochastic hybrid FFSSP in a real-world semiconductor back-end assembly facility was presented by Lin and Chen (2015). In their approach, the optimization strategy, based on genetic algorithm, was used to find the optimal assignment of the production line and machine type at each stage. The simulation model was used to evaluate the performance of solutions. They applied a real case study to prove the necessity of using simulation optimization approaches for practical applications. A novel algorithm was developed by Li and Pan (2015) to solve the hybrid flow shop with limited buffers. They combine two metaheuristic algorithms of artificial bee colony and TS and used makespan as the performance criterion. Their proposed algorithm was evaluated against several algorithms reported in the literature and the experimental results showed the high effectively and efficient performance of the proposed algorithm. Sangsawang et al. (2015) proposed two metaheuristic algorithms to optimize a two-stage re-entrant FFSSP. The first algorithm was a hybridization of GA and adaptive auto-tuning based on a fuzzy logic controller. The second one was a hybridization of particle swarm optimization (PSO) and Cauchy distribution. Experimental results revealed that both hybrid algorithms could present better solutions and are more powerful than the classical metaheuristics. The statistical analyses indicated that the hybrid PSO and hybrid GA can improve the best solutions in the literature by averages of $15.60 \%$ and $15.51 \%$, respectively. Zabihzadeh and Rezaeian(2015) developed a mixed integer linear programming model for FFSSP. They assumed that there are some robots between stages for unloading, transferring and loading parts. Their proposed model has ability of determining the number of robots and jobs sequence. They applied two meta-heuristic algorithms; GA and ACO, to solve their model. Computational results showed that the GA is more efficient than ACO to optimize the model. Tang et al. (2016) presented a new model for dynamic FFSSP's. By taking emergency maintenance, the proposed model minimized both objectives of energy consumption and makespan. Since they developed model was NP-hard, a multi-objective PSO algorithm to optimize the model. For finite capacity material requirement planning system in a flexible flow shop, Sukkerd and Wuttipornpun(2016) presented a hybrid GA and a Tabu search algorithm (HGATS). The results showed that the HGATS can outperform on comparing with the existing algorithms. Correspondingly, computational time of HGATS is acceptable when applied to real industrial systems. Rahmani and Ramezanian(2016) addressed a stochastic FFSSP in which new jobs arrive into the process as disruptions. They developed a mathematical model to minimize total weighted tardiness. A variable neighbourhood search algorithm was used to solve the model. The efficiency of the proposed algorithm evaluated through a set of test problems. González-Neira
et al. (2016) presented a multi-criteria FFSSP in which criterion is quantitative and the other is qualitative. They assumed that job processing time deploys by a stochastic manner.The integral analysis method (IAM) was implemented to solve the relevant problem. In the IAM method, the problem first was introduced, then, cardinal analysis, ordinal analysis and integration analysis are done. Results showed that IAM method is able to select the alternatives with high efficiency in terms of both types of criteria.
The most important criticism of scheduling problems is the gap between academic and practical problems. Even though the developed models have tried to be more realistic, but they fail to find the exact makespan in real life problems. This comes from the fact that the model cannot consider all the factors affected on makespan. As a result, there is a gap between obtained makespan by mathematical models and the makespan occurs in reality. In order to bridge this gap, this research presents an integrated mathematical model which is capable to consider all of the influential factors on makespan. The proposed model can simultaneously consider preventive maintenance, stochastic processing time and budget constraint. By integrating all of these subjects the model will reflect the real performance of a FFSSP.
The rest of the paper is organized as follows: In section 2 the problem definition, mathematical model formulation and assumption of the model is presented. In section 3, the proposed hybrid metaheuristic algorithms in which three hybrid metaheuristic algorithms, including combining parallel SA with two types of PSO algorithms (PSO-PSAI and PSO-PSAП), and a GA with parallel SA (GA-PSA) are specially explained. In section 4, computational results are presentedwhich actually compare the results of proposed metaheuristic algorithms. First, the small scale problem size is solved with CPLEX software. Then, the metaheuristic algorithms are used to find the near optimal solutions for the large scale of the problems. Analysis of the results and comparisons demonstrate the performance of the proposed solving methodologies on different problem sizes. Finally, conclusions and future researches are presented in the last section.

## 2. Mathematical Model Formulation

In this section the problem under consideration is described. Consider a problem of $J$ jobs and $S$ working stages as shown in Figure 1. Each stage $s$ has a number of $N_{s}$ identical parallel machines that operate in parallel. All jobs visit all stages from the first to the last stage. They are processed by one machine at each stage. Each job processed all the stages and every machine process maximum one job at a time. Thereis a buffer between two consecutive stages. The machines are under PM tasks. . Consequently, each machine could be available or unavailable at each time. All jobs are independent of each other and they are available at time zero. The processing time of jobs is considered to be stochastic. In order to
model such randomly distributed processing time, the stochastic programing technique is applied in which each possible processing time is called as a scenario. The probability of occurrences of each scenario is shown by $\operatorname{Pr}(\pi)$. Therefore, the averaged processing time of each job at each stage is calculated according to its processing times in different scenarios and the probability of occurrences of each scenario. The objective is to set a sequence of jobs, allocating jobs to machines at each stage and determining the time between two consecutive maintenance activities in such a way that total completion time being minimized. Figure 2 shows an example of the Gantt chart of proposed stochastic flexible flow shop scheduling problem (SFFSSP) with 3 stages and 5 jobs. There is 2,3 and 2 machines in stages 1,2 and 3 , respectively. In machine 1 in stage 1 , maintenance occurs after processing job 3. The other maintenance occurs in stages 2 and 3 . The job 2 is processed in stage 1 from time 2 to 5 and is started in stage 2 from time 7. Therefore, job 2 is sent to the buffer of stage 1 and cause holding cost for each period of time, until machine 1 in stage 2 gets empty. Sometimes more than one job is placed in the buffers. For example, in the buffer of stage 2, from time 11 and 12 , jobs 1 and 3 are placed in and increasing the holding cost for staying of each period of time in the buffer.
Other constraints as well assumptions are listed as follows:

- There is a set of jobs denoted $J(j=1,2, \ldots, J)$ jobs which are available at time zero and no job may be cancelled before completion.
- The set of $L$ consecutive stages denoted by ( $s=1$, $2, \ldots, L$ ),
- Each stage is equipped with non-identical parallel machines denoted by ( $m=1,2, \ldots, N_{s}$ ).
- The set of $R$, denoted by $(r=1,2, \ldots, R)$ indicated the number of intermediate buffers.
- All jobs have to process serially through all machines at each stage for only once time.
- The interruption processing of each job on all machines at each stage is not acceptable. On the other hand, once a job is started, it must be processed to completion without any interruption either on or between machines.
- All machines are continuously available at time zero, in another word; non-machines are failing at the starting time.
- Each machine could process only one job at the same time, and each job has to visit each machine exactly once.
- The preventive maintenance activities are performed on each machine at the fixed intervals $\left(P_{m s}\right)$.
- Once the preventive maintenance activity is carried out, there is no probability of a subsequent machine breakdown.
- All jobsat each intermediate buffer havea same holding cost, but different at each stage.


Fig. 1.The framework of the flexible flow shop problem of this study

### 2.1. Notations

To present the model using mathematical terms, consider the following notations.

## Indices

$s$ Indexofstages $\{s=1,2, \ldots, L\}$
$m$ Index of machines at $s\left\{m=1,2, \ldots, N_{s}\right\}$
$j$ Index of jobs $\{d, j=1,2, \ldots, J\}$
$r$ Index of intermediate buffers $\{r=1,2, \ldots, R\}$
$h$ Index of job sequence $\left\{h, u=1,2, \ldots, K_{m}\right\}$
$n$ Index of maintenance activity which is done on machine $j\left\{n=1,2, \ldots, V_{m s}\right\}$
$\pi$ Index of probabilistic scenarios $\{\pi=1,2, \ldots, \Pi\}$

## Parameters

$p_{j}^{S}(\pi)$ The processing time of job $j$ at stage $s$ in scenario $\pi$
$\operatorname{Pr}(\pi)$ The probability of occurrences scenario $\pi$
$h_{r s}^{j}$ Holding cost of job jinintermediate buffer of $r$ at stage $s$
$\mu_{m s}$ The repair rate of machine $m$ at stage $s$
$\lambda_{m s}$ The failure rate of machine $m$ at stage $s$
$D_{m s}$ The duration time of maintenance activity of machinematstages
$\bar{n}_{m s}$ The mean number of jobs that are performed on machine $m$ at stage $s$
$\beta$ The minimum of availability of the system
$M \quad$ An arbitrary large position number
B Total budget
Dependent decision variables
$S_{j}^{S}$ Starting time for the processing of job $j$ at stage $s$
$C_{j}^{S}$ Completion time of job $j$ at stage $s$
$P_{m s}$ The time between two consecutive maintenance activities on machine $m$ at stage $s$
$m_{m s}$ The number of maintenance activities on machine $m$ at stage $s$
$d_{r s}^{j}$ Waiting time of job $j$ atintermediate bufferr at stage $s$
$Z_{j}^{r}(t)\left\{\begin{array}{lc}1 & \text { if job } j \text { is in intermediate buffer } r \text { at time } t \\ 0 & \text { otherwise }\end{array}\right.$
$T_{m s}$ The completion time of the last PM action on machine $m$ at stage $s$
$A_{m s}(t)$ The availability of machine $m$ at stage $s$ at time $t$
$A_{s}(t)$ The unavailability of stage $s$ at time $t$
$A_{s y s}(t)$ The unavailability of system at time $t$
W 0 or 1

Independent decision variables
$X_{j h}^{m s} \begin{cases}1 & \text { if job } j \text { is processed in sequence } h \text { by machine } m \text { at stage } s \\ 0\end{cases}$
$Y_{n}^{m s}\left\{\begin{array}{l}1 \\ 0\end{array}\right.$
otherwise
machine $m$ at stage $s$ is executed
otherwise

### 2.2 Mathematical model

In our proposed model, availability of machine $m$ in stage $s$ at time $t$ under PM activities could be calculated according to Villemeur(1991) by Eq. 1.
$A_{m s}(t)=\frac{\mu_{m s}}{\mu_{m s}+\lambda_{m s}}+\frac{\lambda_{m s}}{\mu_{m s}+\lambda_{m s}} \exp \left[-\left(\mu_{m s}+\lambda_{m s}\right)\left(t-T_{m s}\right)\right]$
(1)re
we considered system configuration as a parallel-series system in which machines ateach stage are parallel and stages are a series. A stage is said to be unavailable if all of its machines are unavailable. Therefore, the unavailability of stage $s$ is calculated by Eq. 2 and unavailability of the total system by Eq. 3 .
$A_{s}(t)=\prod_{m=1}^{N_{s}}\left(1-A_{m s}(t)\right)$
$A_{s y s}(t)=1-\prod_{s=1}^{L}\left(1-A_{s}(t)\right)$
Therefore, the proposed model as an extension to basic model was taken fromGonzález-Neira et al. (2016) will be as follows.
$\operatorname{Min}\left\{\max \left(C_{j}^{s}\right)\right\}$
St:
$\sum_{\substack{m=1 \\ N_{s}}}^{N_{s}} \sum_{h=1}^{K_{m}} X_{j h}^{m s}=1$
$\forall j \in J ; s \in L$
$\sum_{m=1}^{N_{s}} \sum_{j=1}^{J} X_{j h}^{m s}=1$
$\forall s \in L ; h \in K_{m}$
$C_{j}^{s} \geq S_{j}^{s}+\sum_{\pi}^{\Pi} \operatorname{Pr}(\pi) p_{j}^{s}(\pi)$
$C_{j}^{s-1}-M\left(1-Z_{j}^{r}(t)\right) \leq t \leq S_{j}^{s}+M\left(1-Z_{j}^{r}(t)\right)$
$\forall j \in J ; s \in L ; \forall r \in R \mid s=r$
$S_{j}^{s}+\left(1-X_{j h}^{m s}\right) M \geq C_{d}^{s}-\left(1-X_{d u}^{m s}\right) M$
$\forall j, d \in J\left|j \neq d ; h, u \in K_{m}\right| u<h, m \in N_{s} ; s \in L$
$\sum_{j=1}^{J} \sum_{r=1}^{R} d_{r s}^{j} h_{r s}^{j} \leq B$
$d_{r s}^{j} \geq S_{j}^{s+1}-C_{j}^{s}$
$\left(n P_{m s} Y_{n}^{m s}-C_{j}^{s}\right) X_{j h}^{m s} \geq-M(1-W)$
$\forall j \in J ; \forall m \in N_{s} ; \forall s \in L ; h \in K_{m} ; 0 \leq n \leq V_{m s}$

$$
\begin{align*}
& \left(C_{j}^{s}-p_{j}^{S}(\pi)-n P_{m s} Y_{n}^{m s}-D_{m s}\right) X_{j h}^{m s} \geq-M(W)  \tag{12}\\
& \quad 0 \leq n \leq V_{m s} \tag{13}
\end{align*}
$$

$P_{m s}=\frac{\sum_{j=1}^{J} \sum_{h=1}^{K_{m}} X_{j h}^{m s} \sum_{\pi}^{\Pi} \operatorname{Pr}(\pi) p_{j}^{s}(\pi)}{m_{m s}}$
$\forall m \in N_{s} ; \forall s \in L$
$T_{m s} \leq n P_{m s}$
$1-A_{s y s}(t) \geq \beta$
$C_{j}^{s}, d_{r s}^{j} \geq 0 \quad \forall j \in J ; \forall s \in L$
$X_{j h}^{m s} \in\{0,1\} \forall j \in J ; \forall m \in N_{s} ; \forall s \in L ; h \in K_{m}$
$Y_{n}^{m s} \in\{0,1\} \forall m \in N_{s} ; \forall s \in L ; n \in V_{m s}$
Eq. (4) indicates the objective function. The constraint (5) ensures that the assignment of each job to one and only one machine at each stage. Constraints set (6) determines that the position of a machine sequence is takes by only one job at each stage. Constraints set (7) states that it is not allowed starting processing jobs at the next stage unless they have completed processing at the previous stage. Constraint sets (8) and (9) indicate that no interference should be taken among jobs on a common machine at any stage if the machine is available. On the other words, the difference between the processing times of any two jobs assigned to the same machine should not have any overlap.The constraint set (10) guarantees that the holding costs should be less than the available budget. Constraints set (11) determines the waiting time of jobs in each buffer.Constraints set (12) and (13) ensure that there are no overlap among operations and maintenance task. Constraint sets (14)-(16) are related to control the system availability.Finally, constraint set (17)-(19) controls the decision variable types.


Fig. 2. An example of proposed SFFSSP Gantt chart of this study

## 3. Solution Methodologies

As mentioned earlier, the proposed modelof SFFSSP is an NP-hard problem and solving this problem with exact methods in reasonable CPU time is impossible, so we should use heuristic or metaheuristic algorithms to solve $\mathrm{it}($ Pinedo\& Chao, 1999). Pinedo (2002) proved that some exact methods have been developed for solvingSFFSSP's, but they are not suitable for more than 20 jobs and 10 machines. Also, heuristic algorithms are often may be trapped onsome local solutions, but metaheuristic algorithms can be described as a master strategy that guides and modifies subordinate heuristics to explore the solutions beyond the local optimally (Osam et al., 1996).We describe threehybridmetaheuristic algorithms, including combining parallel SA with two types of PSO algorithms (named PSO-PSAI and PSO-PSAП), and a GA with parallel SA (named GA-PSA) which have good solution quality in the literature(Singh and Mahapatra, 2012; Kianfar et al., 2012; Rabiee et al., 2014; Tang et al., 2016; Sukkerd and Wuttipornpun, 2016). Theproposed metaheuristic algorithms are explained in the nextsubsections on details.

### 3.1. Hybrid PSO-PSA algorithms

PSO algorithm is an evolutional solution method performed on a population of candidate solutions called particles. These particles move around in the search-space according to simple routine. The particles movements are guided by the best found positions in the search-space, which are continually updated as better positions are found by the particles. At each iteration, the in position of a particle ( X vector) is updated by calculating the velocity (Vel vector) using the differences between the current position of the particle and the two following vectors (Kennedy and Eberhart, 1995):

- The best position experienced by the particle in all previous iterations. This is called the particle best (Pbest).
- The best position experienced by all particles in population in all previous iterations. This is called the global best (gbest).

Generally, a PSO is a continuous algorithm inherently, while SA is a discrete one. Experiments show that combining PSO with a discrete algorithm such as SA creates better performances. Poli, Kennedy and Blackwell (2007) presented a review on the variation and the hybridization of the PSO. We proposed two types of hybrid PSO-PSA algorithms named PSO-PSAI and PSOPSAП to have both advantages of these methods. The basic idea of the hybrid PSO-PSA algorithm is running the PSO algorithm and improving the best results by applying parallel SA (PSA). In order to have variety in the proposed algorithm, we consider every bad, normal and good solutions could be selected with same chances.Combining PSO and PSA decreases the probability to be trapped in the local optimal solutions. Also, by introducing a suitable neighbourhood formation structure, the search process is enhanced and finds the near global optimum solution. The essential components of our proposed PSO-PSAI and PSO-PSAП are similar, except in generating initial solutions as described below. The pseudo-code for our PSO-PSA algorithm is shown Figure 6.

### 3.1.1. Initial solution of PSO-PSAI

The random generation policy is used to generate the initial population of PSO-PSAI.

### 3.1.2. Initial solution of PSO-РSAП

In PSO-PSAП a new procedure is applied to generate initial solutions. First, we consider SFFSSP with relaxed conditions in which the binary constraints of model are
relaxed. In other word the $X_{j h}^{m s}$ is considered to be a real number between 0 and 1 . Next, the relaxed model was solved by CPLEX. The optimum values of decision variables are numbers between 0 and 1 which are looked as a probability distribution. These values are used as primal inputs of initial solutions for PSO-PSAП. For example, if is $\mathrm{X}_{23}^{12}=0.8$ then we set $\mathrm{X}_{23}^{12}=1$ with the 0.8 and $X_{23}^{12}=0$ with the 0.2 . The initial solutions obtained by this methodare distributed in the high quality of solution space. In order to perform a comprehensive search the low quality of solution space must be investigated. Therefore, some initial solutions are generated, unlike the obtained probability distributions. For example, if $X_{23}^{12}=0.8$, then we set $X_{23}^{12}=1$ with the 0.2 and $\mathrm{X}_{23}^{12}=0$ with the 0.8 , the two groups if initial solutions are merged and formed initial solutions of PSOРSАП.
The input parameters of both proposed PSO-PSAI and PSO-PSAП are: the population size $\left(N_{p o p}\right)$, the number of successive iterations in which the best solution does not change $(M)$, the cognition learning factor $\left(C_{1}\right)$, The social factor $\left(C_{2}\right)$ the inertial weight $(w)$, the population size $\left(N_{p o p}\right)$, the number of internal loop (In loop), and the temperature decreasing rate $(\alpha)$. The other components of proposed PSO-PSAI and PSO-PSAП are the same and described as follows:

### 3.1.3. The solution structure of PSO-PSA

The solution is represented by two sub-matrixes, each of them are associatedto a special area of decision variable. The first sub-matrix presents the sequence of jobs contains $S^{*} J$ matrix where $S$ is the number of stations and $J$ is the number of jobs. An enhanced version of random key representation, proposed by Norman and Bean (1999) is used to show the sequence of jobs which is capable to preserve the solution feasibility. In this way, each job at each station is assigned a real number between $\left(1, N_{s}\right)$. The integer part is the number determines the machine number to which the job is assigned and the fractional part is used to sort the jobs assigned to that machine. For example, consider a problem with 4 jobs, 5 stations and 4 machines at each station. An example of a solution for this problem is presented in Figure 3. According to Figure 4, in station 2, the jobs 1 and 4 are both processed on machine 3 , and the job 2 is processed on machine 1 , and the job 3 is processed on machine 2 . Also the order of jobs to be scheduled on machine 3 is job 1 followed by job 4.The second sub-matrix is related to the number of maintenance activity on machines. It is a $S^{*} M$ matrix where $S$ is the number of stations and $M$ is the number of machines at each station. Each cell of this matrix is filled with a random number between 1 and the maximum number of allowable maintenance activity. Figure 4 is an example of a problem with 5 stations and 4 machines at each station. As shown in Figure 4, three, one, three and four maintenance activities are performed on machines 1 ,

2, 3 and 4 in station 1 , respectively. Therefore, the decision variables $Y_{1}^{11}, Y_{2}^{11}$ and $Y_{3}^{11}$ will be equal to one.

|  | Job 1 | Job 2 | Job 3 | Job 4 |
| :--- | :---: | :---: | :---: | :---: |
| Station 1 | 4.41 | 2.29 | 3.12 | 1.65 |
| Station 2 | 3.37 | 1.84 | 2.62 | 3.73 |
| Station 3 | 1.97 | 3.42 | 3.85 | 4.24 |
| Station 4 | 2.54 | 1.16 | 3.96 | 2.52 |
| Station 5 | 1.04 | 3.73 | 2.14 | 4.91 |

Fig.3. An example of representation of solution
(Sequence of jobs)

|  | Machine 1 | Machine 2 | Machine 3 | Machine 4 |
| :---: | :---: | :---: | :---: | :---: |
| Station 1 | 3 | 1 | 3 | 4 |
| Station 2 | 2 | 2 | 4 | 3 |
| Station 3 | 1 | 4 | 2 | 3 |
| Station 4 | 3 | 3 | 4 | 2 |
| Station 5 | 2 | 1 | 3 | 3 |

Fig. 4. An example of representation of solution (Maintenance activity)

### 3.1.4. Particle movements

For particle movements, the following formulas are used to update the velocity and position vectors of a particle: $\operatorname{Vel}_{i}(k+1)=w * \operatorname{Vel}_{i}(k)+C 1 * r 1 *\left(\right.$ Pbest $\left._{i}-X_{i}(k)\right)+$ $C 2 * r 2 *\left(\right.$ gbest $\left.-X_{i}(k)\right) \quad$ (19)
$X_{i}(k+1)=X_{i}(k)+\operatorname{Vel}_{i}(k+1)$
In equation (19), $V e l_{i}(k)$ is the velocity of particle $i$ in the $k^{\text {th }}$ iteration and $X_{i}(k)$ states the position of particle $i$ in iteration $k$. Also, Pbest $_{i}$ is the vector for the best known position of particle $i$ and gbest is the best position vector of all particles in the whole population. $w$ is called the inertia weight that determines the impact of the current velocity of a particle on its velocity at the next iteration. The parameters $C_{1}$ and $C_{2}$ are acceleration coefficients which have constant values to determine the impact of Pbest and gbestvalues in defining the velocity, respectively. $r_{1}$ and $r_{2}$ are two random numbers uniformly distributed in $[0,1]$.

### 3.1.5. Local search improvements

One of the challenges of the algorithms is trapped in local optimal solutions. In the other words, it is possible that the near optimum solution which has found yet, is selected and the algorithm accepts this solution as a final optimum solution and stops. The hybrid algorithms are used to combine the base algorithm with different strategies to improve the algorithm performances. We utilized medium radius local search for the proposed hybrid PSO-PSA algorithms. In this method, the local searches are not used on each swarm, and we used the double change technique. If the improvements on the convergence of fitness function are achieved, we replace it to the previous swarm, but, if no improvements have been seen, we accept this by Bultzen probability.

### 3.1.6. Initial temperature

A suitable initial temperature is the one that results in an average increase of acceptance probability near to one. The value of initial temperature will clearly depend on the scaling of fitness and, hence, it should be problemspecific. Therefore, we first generate a large set of random solutions, then a standard division of them is calculated and is used to determine the initial temperature in the way that the acceptance probability of primary generations reach to 0.999 . Consequently, the initial $T_{k}$ is set to 1000based on some preliminary examinations.

### 3.1.7. Cooling Schedule

The performance of this algorithm also depends on the cooling schedule, which is essentially the temperature updating function. In the proportional decrement scheme, temperatures at the k and $k+1$ steps of the outer loop, $T_{k}$ and $T_{k+1}$, are related by:
$T_{k+1}=\alpha T_{k}$
Where $\alpha$ is cooling rate and is obtained by some experiment.

### 3.1.8. Stopping criteria

To limit the number of iterations of PSO-PSA algorithms, some convergence experiment was performed and the best criterion was applied as follows:
PSO-PSA will be stopped when the best solution does not change after a pre-determined number of successive iterations $(M)$. Also, the PSA is allowed to search in a temperature level, for In-loop iterations. The optimum value of $M$ and In-loop is determined by Taguchi experiments. The pseudo-code of the proposed PSO-PSA is described in Figure 5.

## Procedure of hybrid PSO-PSA algorithms

$P$ : initial particle with random positions and velocities
Untiltermination condition is met Do For each particle ido Update the velocity of particle $i$ Update the position of particle $i$ Evaluate particle $i$ Update Pbest and gbest Endfor
Start PSA with some of the best and the worst chromosomes

## Set repetition counter $k=0$

Untiltermination condition is met Do
Set repetition counter $M=0$
Until $M=$ in_loop $\mathbf{D o}$
Generate a neighbour solution: $\omega_{2}$
Calculate $\Delta_{\omega l, \omega 2}=f(\omega)-f\left(\omega_{0}\right)$
If $\Delta_{\omega l, \omega 2} \leq 0$, then $\omega_{1}=\omega_{2}$
If $\Delta_{\omega l, \omega_{2}}>0$, then $\omega_{I}=\omega_{2}$ with probability $\exp \left(-\Delta_{\omega I, \omega_{2}} / t_{k}\right)$
$m=m+1$
End
$T_{k+1}=\alpha T_{k}$
$k=k+1$
End
Transfer the improved solution to the particles
End

### 3.2. Hybrid GA-PSA algorithm (GA-PSA)

Genetic algorithm (GA) has no ability to search effectively to find the best global optimum solution. Also, this algorithm isn't a capable to complete local searches on solutions. Therefore, we can combine the power of GA in global search with simulated annealing (SA) local searches to address the global optimum solution. This hybrid algorithm which combines GA and SA has both advantages of these algorithms and help to improve solution performances.In this hybrid algorithm, first the GA generates initial solutions with crossover and mutation operators. Next, some of these solutions have been selected for the parallel SA (PSA) as initial solutions. Then, the parallel local search process on the selected solution starts. To have variety in the proposed algorithm, we consider every bad, normal and good solutions could be selected with same chances. The PSA procedure in the same applied in PSO-PSA. The other components of PSO-SA are as follows:

### 3.2.1. Initialization

The input of GA-PSA is the population size $\left(N_{p o p}\right)$, the number of successive iterations in which the best solution does not change $(M)$, the crossover probability ( $P c$ ), and the similarity coefficient $(S C)$, and the mutation probability ( Pm ) the number of internal loops (In loop), and the temperature decreasing rate $(\alpha)$ are first initialized. Then, to generate an initial population, a random generation policy is utilized in this step. Since the solutions obtained by a metaheuristic algorithm are sensitive to their parameter values, a statistical procedure based on the Taguchi parameter tuning method is used to tune the parameters.

### 3.2.2. Selection operator

One of the most key elements of a GA is the selection operator which is used to select chromosomes (parents) which lead to generate new chromosomes (offspring). The proposed selection operator is roulette wheel selection method in which parent chromosomes are probabilistically selected based on their fitness function value. The better chromosomes are selected with the highest probability. Using the roulette wheel selection each chromosome in the population occupies a slot with a slot size proportional to the chromosome fitness. When the wheel is randomly spun, the chromosome corresponding to the slot where the wheel stopped is selected as the first parent. This process is repeated to find the second parent. Clearly, since better chromosomes have larger slots, they have better chances to be chosen in the selection process.

### 3.2.3. The chromosome structure

The chromosome structure of the GA-PSA is just like that used in "solution structure" in PSO-PSA.

### 3.2.4. Chromosome evaluation

In order to evaluate chromosomes, each chromosome is simulated for 30 times. Then, the obtained results are averaged and considered as the chromosome fitness function. In the other word, by changing the stochastic input parameters, the fitness function of the chromosome will change. Therefore, the fitness function of each chromosome is not under deterministic value and show the stochastic nature of the model.

### 3.2.5. $\quad$ The crossover operator

As defined in the previous section, the chromosomes have two parts, so a crossover operator is applied in these two parts. For each part of the chromosome a single-point crossover is applied. For the first part of the chromosome, a cross point between $(1, J)$ is generated ( $J$ is the number of jobs). Then, the crossover operator is applied according to Figure 8. The same procedure is applied to generate the second of the offspring chromosomes.

### 3.2.6. The mutation operator

The mutation operator is used in only some iterations. The similarity coefficient (SC) determines if mutation is applied or not. We can calculate the SC as follows:
$S C_{a b}=\frac{\sum_{j=1}^{J} \sum_{h=1}^{K_{m}} \sum_{m=1}^{N_{s}} \sum_{s=1}^{L} \partial\left(X_{j h}^{m s}(a), X_{j h}^{m s}(b)\right)}{S \times J}$
Where $X_{j h}^{m s}(\mathrm{a})$ and $X_{j h}^{m s}(b)$ are decision variables in chromosomes $a$ and $b$. For comparing the similarity between two chromosomes, we consider the similarity of each gene that can be expressed as follows:
$\partial\left(X_{j h}^{m s}(a), X_{j h}^{m s}(b)\right)=\left\{\begin{array}{rr}1 & \text { if } X_{j h}^{m s}(a)=X_{j h}^{m s}(b) \\ 0 & \text { otherwise }\end{array}\right.$
The average similarity coefficient of the population is calculated as follows:
$\overline{S C}=\frac{\sum_{a=1}^{N-1} \sum_{b=a+1}^{N} S C_{a b}}{\binom{N}{2}}$
Where $N$ is the number of chromosomes in the population. Considering a pre-defined threshold similarity coefficient, the specified mutation operator will be automatically applied. Two swapping types are used in proposed GAPSA. Swapping type 1 is used to define the neighbourhood $N(s)$ in local search (PSA) and swapping type 2 is used as mutation operator of GA.

- Swapping type 1

The mutation operator is applied on both two parts of chromosomes. To this aim, a column of each chromosome sub matrix is randomly selected and is inversely arranged. Figure 6 shows an example of the mutation operator.

$\left\lvert\,$| Parent |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| 4.41 | 2.29 | 3.12 | 1.65 |  |
| 3.37 | 1.84 | 2.62 | 3.73 |  |
| 1.97 | 3.42 | 3.85 | 4.24 |  |
| 2.54 | 1.16 | 3.96 | 2.52 |  |
| 1.04 | 3.73 | 2.14 | 4.91 |  |
|  |  |  |  |  |
| Offspring    <br> 1.04 2.29 3.12 1.65 <br> 2.54 1.84 2.62 3.73 <br> 1.97 3.42 3.85 4.24 <br> 3.37 1.16 3.96 2.52 <br> 4.41 3.73 2.14 4.91 |  |  |  |  |$>.$|  |
| :--- |\right.

Fig.6. An example of mutation operator of swapping type 1

> (Sequence of jobs)

- Swapping type 2

The swapping type 2 works which select two rowsof each chromosome sub matrix is randomly selected and swapped (see Figure7).

| Parent |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4.41 | 2.29 | 3.12 | 1.65 |  |
| 3.37 | 1.84 | 2.62 | 3.73 |  |
| 1.97 | 3.42 | 3.85 | 4.24 |  |
| 2.54 | 1.16 | 3.96 | 2.52 |  |
| 1.04 | 3.73 | 2.14 | 4.91 |  |
|  |  |  |  |  |


| Offspring |  |  |  |
| :---: | :---: | :---: | :---: |
| 1.65 | 3.12 | 2.29 | 4.41 |
| 2.54 | 1.84 | 2.62 | 3.73 |
| 1.97 | 3.42 | 3.85 | 4.24 |
| 3.37 | 1.16 | 3.96 | 2.52 |
| 4.41 | 3.73 | 2.14 | 4.91 |

Fig. 7. An example of mutation operator of swapping type 2 (Sequence of jobs)

| Parent 1    <br> 4.41 2.29 3.12 1.65 <br> 3.37 1.84 2.62 3.73 <br> 1.97 3.42 3.85 4.24 <br> 2.54 1.16 3.96 2.52 <br> 1.04 3.73 2.14 4.91 <br>     <br> Parent 2    <br> 3.93 4.42 1.11 2.25 <br> 3.72 2.85 3.07 4.66 <br> 2.96 1.84 4.52 3.38 <br> 2.77 3.77 2.86 2.93 <br> 4.13 2.45 1.55 3.72 |
| :--- |


| Offspring 1 |  |  |  |
| :---: | :--- | :--- | :--- |
| 4.41 | 2.29 | 1.11 | 2.25 |
| 3.37 | 1.84 | 3.07 | 4.66 |
| 1.97 | 3.42 | 4.52 | 3.38 |
| 2.54 | 1.16 | 2.86 | 2.93 |
| 1.04 | 3.73 | 1.55 | 3.72 |
| Offspring 2    <br> 3.93 4.42 3.12 1.65 <br> 3.72 2.85 2.62 3.73 <br> 2.96 1.84 3.85 4.24 <br> 2.77 3.77 3.96 2.52 <br> 4.13 2.45 2.14 4.91 |  |  |  |$\ggg>$

Fig. 8. An example of crossover operator (Sequence of jobs)

### 3.2.7. Stopping criteria

The stopping criteria in the GA-PSA is similar to the ones described for PSO-PSA. Figure 9 shows the pseudo-code of the proposed GA-PSA algorithm.


Fig. 9. Pseudo-code of GA-PSA

## 4. Computational Evaluation

All experimental results have been carried out on an ASUS laptop with a 2.4 GHz , core i 5 processor using a 4GB of RAM. All metaheuristic algorithms have been implemented in MATLAB Software (Version 7.10.0.499, R2010a) and the linear programming models have been solved using CPLEX 12. Also, the Minitab 17 software has been used to apply theTaghuchi design of experiment method for parameters tuning.Two sets of test problems are applied to solve the model. These test problems are
generated based on the data given in Table 1. Here, we considered 5 scenarios, each of which has equal probability to be occurring.Three machines workat each station, considering the number of jobs and the numbers of stations, the two test sets aregeneratedin small and large problem sizes. First, 18small sized test problems are generated and the solutions, obtained by the algorithms, are evaluated against global optimum amounts.Furthermore, 60 large sizedtest problems are applied to compare the performance of the algorithms. We considered two kinds of total budget called hereinafter by type 1 and 2 . Both budget types are associated to the number of jobs. The budget types 1 and 2 are respectively calculating by multiplying of number of jobs in 1200 and 1000 . Therefore, the budget type 2 is more strict than type 1 .

Table 1
Factor and their levels

| Factors | Levels |
| :--- | :--- |
| Number of jobs $(j)$ | $[4,5,6,20,50,60,100]$ |
| Number of stations $(s)$ | $[2,3,4,6,9]$ |
| Holding cost $(\$ / \mathrm{sec})$ | Uniform $[50,100]$ |
| Process times $(\mathrm{sec})$ | Uniform $[1,99]$ |
| The minimum of availability | 0.95 |
| The repair rate of machine $(\mathrm{sec})$ | Uniform $[10,50]$ |
| The failure rate of machine $(\mathrm{sec})$ | Uniform $[150,450]$ |
| Number of scenarios | 5 |

### 4.1. Parameter tuning

As mentioned before, the initial parameters of hybrid PSO-PSA algorithms (including both PSO-PSAI and PSO-PSAП) are the population size ( $N_{\text {pop }}$ ), the number of successive iterations in which the best solution does not change $(M)$, the cognition learning factor $\left(C_{1}\right)$, The social factor $\left(C_{2}\right)$ the inertial weight ( $w$ ), the population size ( $N_{p o p}$ ), the number of internal loop (In loop), and the temperature decreasing rate $(\alpha)$. Also, the population size ( $N_{p o p}$ ), the number of successive iterations in which the best solution does not change $(M)$, the crossover probability $(P c)$, and the similarity coefficient $(S C)$, and the mutation probability $(P m)$ the number of internal loops (In loop), and the temperature decreasing rate ( $\alpha$ ) were used in GA-PSA.To investigate the influence of those parameters on the performance of the algorithms, we implement the Taguchi's method in the design of experiments (DOE) (Montgomery, 2005).In the Taguchi's method, the results are converted into an estimator called single to noise ratio $(\mathrm{S} / \mathrm{N})$. The $\mathrm{S} / \mathrm{N}$ ratio shows both the mean and the variation in the response variables. To minimize the objective function the $\mathrm{S} / \mathrm{N}$ ratio is calculated as the following formula:
$S /{ }_{N}=-10 \log \left(1 / n \sum_{i=1}^{n} \frac{1}{y_{i}^{2}}\right)$
Which, $n$ and $y_{i}$ indicate number of replications and process response value at $i$ 'th replication. We chose the
orthogonal array $L_{27}$ for both PSO-PSA and GA-PSA. In Taguchi's design of experiments, the relative percentage deviations (RPD's) transform into S/N ratio, we figure out the response value of each parameter in the proposed algorithms.After testing the different parameter levels and analysing the obtained results, the better initial parameter levels were gained. The initial and optimumparameter levels of each proposed hybrid algorithm are shown in Tables 2.According to the parameter values in Table 2, we illustrate the trend of each factor level of PSO-PSA and GA-PSA are in Figures 10 and 11, respectively. Also, the results of each test problem solved by the proposed algorithms are shown in Figure 12. The statistical results of the objective function (makespan)and CPU time for the small and large sized problems are presented in Tables 5 and 6.Additionally, we utilized the relative percentage deviation (RPD) to test the performances of applied metaheuristics as below:
$R P D=\frac{A l g_{\text {sol }}-L_{\text {sol }}}{L_{\text {sol }}} \quad i=1, \ldots, n$


Fig. 10. Factor level trend of PSO-PSAalgorithm


Fig. 11. Factor level trend of GA-PSAalgorithm

Table 5
Solutions found by the algorithms of the small-sized test problems

| Problem \# | Budget type | $\begin{gathered} \text { NOJ } \\ (j) \\ \hline \end{gathered}$ | $\begin{gathered} \text { NOS } \\ (s) \\ \hline \end{gathered}$ | CPLEX |  |  | PSO-PSAП |  | PSO-PSAI |  | GA-PSA |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Makespan | CPU time | Gap (\%) | Makespan | CPU time | Makespan | CPU <br> time | Makespan | CPU <br> time |
| 1 | 1 | 4 | 2 | 205 | 22 | 0 | 205 | 22 | 205 | 26 | 205 | 29 |
|  | 2 |  |  | 210 | 22 | 0 | 210 | 23 | 229 | 37 | 233 | 39 |
| 2 | 1 |  |  | 209 | 35 | 0 | 209 | 37 | 216 | 52 | 219 | 55 |
|  | 2 |  |  | 217 | 36 | 0 | 218 | 35 | 225 | 47 | 228 | 49 |
| 3 | 1 |  | 3 | 214 | 95 | 0 | 214 | 97 | 215 | 109 | 227 | 115 |
|  | 2 |  |  | 240 | 91 | 0 | 241 | 94 | 248 | 117 | 253 | 122 |
| 4 | 1 |  |  | 218 | 155 | 0 | 218 | 156 | 227 | 160 | 249 | 163 |
|  | 2 |  |  | 223 | 150 | 0 | 224 | 152 | 237 | 182 | 250 | 186 |
| 5 | 1 |  | 4 | 232 | 185 | 0 | 235 | 191 | 243 | 305 | 247 | 321 |
|  | 2 |  |  | 236 | 184 | 0 | 238 | 187 | 246 | 287 | 249 | 328 |
| 6 | 1 |  |  | 249 | 235 | 0 | 249 | 238 | 257 | 398 | 260 | 415 |
|  | 2 |  |  | 250 | 234 | 0 | 252 | 236 | 264 | 382 | 262 | 402 |
| 7 | 1 | 5 | 2 | 237 | 645 | 0 | 238 | 651 | 248 | 1039 | 251 | 1100 |
|  | 2 |  |  | 241 | 647 | 0 | 245 | 650 | 252 | 1077 | 255 | 1138 |
| 8 | 1 |  |  | 253 | 822 | 0 | 256 | 831 | 265 | 1005 | 270 | 1176 |
|  | 2 |  |  | 262 | 830 | 0 | 268 | 833 | 273 | 1015 | 276 | 1165 |
| 9 | 1 |  | 3 | 270 | 882 | 0 | 274 | 890 | 280 | 996 | 284 | 1034 |
|  | 2 |  |  | 280 | 890 | 0 | 282 | 893 | 292 | 1016 | 298 | 1086 |
| 10 | 1 |  |  | 289 | 918 | 0 | 292 | 923 | 302 | 987 | 305 | 1150 |
|  | 2 |  |  | 304 | 922 | 0 | 308 | 931 | 317 | 1083 | 321 | 1154 |
| 11 | 1 |  | 4 | 335 | 1012 | 0 | 337 | 1024 | 346 | 1056 | 350 | 1149 |
|  | 2 |  |  | 348 | 1048 | 0 | 350 | 1055 | 355 | 1088 | 369 | 1047 |
| 12 | 1 |  |  | 326 | 943 | 0 | 329 | 951 | 336 | 1008 | 339 | 1062 |
|  | 2 |  |  | 351 | 950 | 0 | 353 | 958 | 363 | 1032 | 368 | 1094 |
| 13 | 1 | 6 | 2 | 349 | 1083 | 0 | 352 | 1099 | 361 | 1125 | 367 | 1139 |
|  | 2 |  |  | 362 | 1116 | 0 | 366 | 1128 | 374 | 1176 | 379 | 1182 |
| 14 | 1 |  |  | 351 | 1043 | 0 | 355 | 1074 | 363 | 1133 | 369 | 1273 |
|  | 2 |  |  | 366 | 1055 | 0 | 371 | 1094 | 381 | 1156 | 385 | 1258 |
| 15 | 1 |  | 3 | 350 | 1103 | 24.54 | 354 | 1127 | 367 | 1176 | 369 | 1201 |
|  | 2 |  |  | 372 | 1144 | 28.55 | 376 | 1185 | 392 | 1192 | 395 | 1282 |
| 16 | 1 |  |  | 347 | 1078 | 29.17 | 351 | 1112 | 358 | 1185 | 363 | 1246 |
|  | 2 |  |  | 360 | 1123 | 31.47 | 363 | 1175 | 371 | 1211 | 374 | 1267 |
| 17 | 1 |  | 4 | 361 | 1197 | 68.35 | 364 | 1225 | 373 | 1312 | 377 | 1424 |
|  | 2 |  |  | 367 | 1255 | 66.92 | 371 | 1292 | 379 | 1332 | 383 | 1487 |
| 18 | 1 |  |  | 358 | 1205 | 74.18 | 363 | 1243 | 370 | 1294 | 376 | 1431 |
|  | 2 |  |  | 394 | 1270 | 78.53 | 398 | 1293 | 384 | 1345 | 399 | 1481 |
| Average | 1 |  |  | 268.28 | 703.22 | 10.90 | 288.61 | 716.17 | 296.22 | 798.11 | 301.50 | 860.17 |
|  | 2 |  |  | 299.06 | 720.39 | 51.37 | 301.89 | 734.11 | 310.11 | 820.33 | 315.39 | 875.94 |

Gap (\%) = CPLEX optimality gap


Fig. 12. The performance of proposed hybrid algorithmsof the large-sized test problems in terms of average makespan

Table 6
Solutions found by the algorithms of the large-sized test problems

| Problem \# | Budget type | $\begin{gathered} \hline \text { NOJ } \\ (j) \end{gathered}$ | NOS <br> (s) | PSO-PSAП |  | PSO-PSAI |  | GA-PSA |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Makespan | CPU time | Makespan | CPU time | Makespan | CPU time |
| 1 | 1 | 20 | 3 | 958 | 2231 | 937 | 2265 | 1009 | 2319 |
|  | 2 |  |  | 1005 | 2277 | 996 | 2309 | 1055 | 2393 |
| 2 | 1 |  |  | 941 | 2226 | 959 | 2261 | 992 | 2318 |
|  | 2 |  |  | 994 | 2202 | 1002 | 2278 | 1038 | 2373 |
| 3 | 1 |  |  | 964 | 2230 | 988 | 2278 | 1031 | 2302 |
|  | 2 |  |  | 1032 | 2282 | 1042 | 2266 | 1086 | 2285 |
| 4 | 1 |  |  | 984 | 2236 | 981 | 2279 | 1035 | 2333 |
|  | 2 |  |  | 1034 | 2224 | 1034 | 2273 | 1088 | 2383 |
| 5 | 1 |  |  | 952 | 2227 | 985 | 2291 | 993 | 2329 |
|  | 2 |  |  | 993 | 2265 | 1071 | 2281 | 1039 | 2382 |
| 6 | 1 |  | 6 | 1127 | 2385 | 1138 | 2450 | 1161 | 2483 |
|  | 2 |  |  | 1179 | 2463 | 1197 | 2408 | 1231 | 2479 |
| 7 | 1 |  |  | 1152 | 2397 | 1131 | 2438 | 1201 | 2508 |
|  | 2 |  |  | 1223 | 2467 | 1174 | 2464 | 1276 | 2579 |
| 8 | 1 |  |  | 1080 | 2395 | 1097 | 2428 | 1119 | 2469 |
|  | 2 |  |  | 1147 | 241 | 1142 | 2426 | 1165 | 2486 |
| 9 | 1 |  |  | 1188 | 2373 | 1149 | 2443 | 1245 | 2471 |
|  | 2 |  |  | 1252 | 2434 | 1223 | 2406 | 1297 | 2532 |
| 10 | 1 |  |  | 1164 | 2380 | 1170 | 2431 | 1104 | 2468 |
|  | 2 |  |  | 1218 | 2379 | 1228 | 2426 | 1159 | 2464 |
| 11 | 1 |  | 9 | 1174 | 2417 | 1178 | 2458 | 1216 | 2529 |
|  | 2 |  |  | 1210 | 2490 | 1234 | 2524 | 1278 | 2531 |
| 12 | 1 |  |  | 1120 | 2416 | 1155 | 2471 | 1178 | 2521 |
|  | 2 |  |  | 1190 | 2490 | 1225 | 2450 | 1249 | 2480 |
| 13 | 1 |  |  | 1197 | 2433 | 1233 | 2443 | 1257 | 2471 |
|  | 2 |  |  | 1260 | 2476 | 1304 | 2441 | 1307 | 2457 |
| 14 | 1 |  |  | 1143 | 2421 | 1145 | 2465 | 1169 | 2500 |
|  | 2 |  |  | 1215 | 2437 | 1213 | 2495 | 1233 | 2549 |
| 15 | 1 |  |  | 1187 | 2417 | 1233 | 2476 | 1236 | 2511 |
|  | 2 |  |  | 1250 | 2412 | 1293 | 2485 | 1309 | 2521 |
| 16 | 1 | 50 | 3 | 2617 | 2903 | 2684 | 3021 | 2767 | 3197 |
|  | 2 |  |  | 2706 | 2990 | 2793 | 3030 | 2912 | 3137 |
| 17 | 1 |  |  | 2652 | 2911 | 2730 | 3028 | 2813 | 3134 |
|  | 2 |  |  | 2758 | 2945 | 2881 | 3058 | 2936 | 3216 |
| 18 | 1 |  |  | 2562 | 2910 | 2639 | 3074 | 2697 | 3251 |
|  | 2 |  |  | 2693 | 2956 | 2753 | 3124 | 2799 | 3339 |
| 19 | 1 |  |  | 2635 | 2908 | 2697 | 3010 | 2768 | 3159 |
|  | 2 |  |  | 2740 | 2952 | 2813 | 3013 | 2916 | 3130 |
| 20 | 1 |  |  | 2589 | 2901 | 2680 | 3050 | 2757 | 3180 |
|  | 2 |  |  | 2676 | 2936 | 2789 | 3042 | 2858 | 3270 |
| 21 | 1 |  | 6 | 2839 | 3010 | 2920 | 3141 | 3055 | 3300 |
|  | 2 |  |  | 2992 | 3097 | 3035 | 3085 | 3218 | 3378 |
| 22 | 1 |  |  | 2815 | 3009 | 2918 | 3115 | 2994 | 3240 |
|  | 2 |  |  | 2922 | 3085 | 3036 | 3075 | 3106 | 3233 |
| 23 | 1 |  |  | 2620 | 3010 | 2698 | 3116 | 2770 | 3255 |
|  | 2 |  |  | 2725 | 3103 | 2801 | 3146 | 2873 | 3319 |
| 24 | 1 |  |  | 2843 | 3031 | 2933 | 3199 | 3050 | 3352 |
|  | 2 |  |  | 2996 | 3019 | 3035 | 3286 | 3169 | 3444 |
| 25 | 1 |  |  | 2568 | 3022 | 2637 | 3184 | 2750 | 3300 |
|  | 2 |  |  | 2687 | 3073 | 2727 | 3260 | 2860 | 3269 |
| 26 | 1 |  | 9 | 2827 | 3181 | 2911 | 3324 | 3004 | 3491 |
|  | 2 |  |  | 2943 | 3178 | 3024 | 3427 | 3153 | 3571 |
| 27 | 1 |  |  | 2825 | 3186 | 2898 | 3318 | 3009 | 3516 |
|  | 2 |  |  | 2989 | 3225 | 3064 | 3399 | 3187 | 3460 |
| 28 | 1 |  |  | 3109 | 3164 | 3214 | 3288 | 3325 | 3399 |
|  | 2 |  |  | 3296 | 3137 | 3373 | 3273 | 3478 | 3465 |
| 29 | 1 |  |  | 2827 | 3181 | 2931 | 3280 | 3051 | 3390 |
|  | 2 |  |  | 2948 | 3203 | 3043 | 3369 | 3210 | 3338 |
| 30 | 1 |  |  | 3087 | 3193 | 3178 | 3344 | 3299 | 3517 |
|  | 2 |  |  | 3245 | 3292 | 3322 | 3380 | 3489 | 3476 |
| 31 | 1 | 60 | 3 | 4117 | 3578 | 4525 | 3834 | 4900 | 4070 |
|  | 2 |  |  | 4388 | 3690 | 4793 | 3926 | 5726 | 4084 |
| 32 | 1 |  |  | 3861 | 3596 | 4194 | 3837 | 4468 | 4035 |
|  | 2 |  |  | 4059 | 3616 | 4466 | 3763 | 4823 | 4125 |
| 33 | 1 |  |  | 4026 | 3594 | 4304 | 3847 | 4752 | 4097 |
|  | 2 |  |  | 4327 | 3470 | 4684 | 3786 | 5026 | 4179 |
| 34 | 1 |  |  | 3828 | 3586 | 4192 | 3876 | 4507 | 4134 |
|  | 2 |  |  | 4043 | 3670 | 4452 | 3968 | 4910 | 4243 |
| 35 | 1 |  |  | 4048 | 3589 | 4424 | 3839 | 4787 | 4112 |



NOJ $(j)=$ number of jobs
NOS ( $s$ ) =number of stations

After solving all test problems, the results showed that in the all test problems, PSO-PSAПhas better performances in terms of makespan and CPU time. The comparisons of makespan show that the PSO-PSAП provides better solution quality with difference average RPD (ARPD) of 5.24 , and 12.02 in comparison to PSO-PSAI, and GAPSA, respectively. Also, we can see that in the comparison of CPU times, PSO-PSAП give better results in terms of difference ARPD of 6.04, and 13.42 against PSO-PSAI, and GA-PSA. Figures 13 and 14 show the mean plot of the CPU time and makespan of the proposed metaheuristic algorithms, respectively.According to computational results, it can be inferred that in smallsized test problems, all the algorithms approximately have
the same computational effort. Therefore, it can be proved that all the metaheuristic algorithms are able to reach optimal/near optimal solutions.But, as the sizes of the problems are increased, the difference between algorithms is more revealed so that PSO-PSAП overcomes all other algorithms. However, the PSO-PSAI, and GA-PSA are highly affected by the problem size so that by increasing it , the CPU time is exponentially increased. Figure 15 depicts the $95 \%$ confidence intervals of makespan and Figure 16 showsthe $95 \%$ confidence intervals for RPD among the proposed algorithms.


Fig. 13. Means plot of CPU time of hybrid algorithms


Fig. 14. Means plot of makespan of hybrid algorithms

## 5. Conclusion

In this study, we developed the stochastic flexible flow shop scheduling problem (SFFSSP) considering fixed interval preventive maintenance (PM) and budget constraint. This new type of problem which is the main contribution of the research is presented an integrated mathematical model which is capable to consider all of the influential factors on makespan. In theproposed SFFSSP, there is a buffer between each stage, if all machines are busy or under PM action, the job can wait in the buffer. The budget constraint controls the holding costs of total jobs in all buffers should not be greater than available budget. The proposed model considers not only the preventive maintenance, but also stochastic processing time and budget constraint. By integrating all of these subjects the model will reflect the real performance of a SFFSSP. Since the problem was strongly NP-hard, three hybrid algorithms, including PSA with two types of PSO algorithms (PSO-PSAI and PSO-SAП), and a GA (GAPSA) were proposed to solve the model, which have suitable quality solutions in the literature.To compare the computational results, we tested two types of test problems containing 18 small sized test problems and 60 large sized test problems. As the results showed, the PSOPSAП algorithm provided better quality solutions in both
makespan and CPU time among the test problems. Also, the higher performance of proposed PSO-PSAП algorithm with respect to other algorithms is more revealed in the large sized test problems.
The presented model is still open to considering other options, such as sequence dependent setup time, machine random breakdown, and the problem of job availability at time zero. Also, it might be exciting in working on biobjective SFFSSP's, which the other objective function could be minimizes the maximum tardiness.Another research direction could be incorporating different transportation types to transport jobs between each stage. Additionaleffort can try to solve model by developing a new solution methodology such as a new hybrid algorithm or a new population-based algorithm can be investigated.We assumed that each job has a same holding cost at each intermediate buffer, but it is different at each stage. Another aspect deserving future efforts is to consider that the holding costs of each job are different at any intermediate buffer.


Fig 15. The $95 \%$ confidence intervals of makespanof the smallsized test problems


Fig 16. The 95\% confidence intervals of makespanof the largesized test problems

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