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Expert systems and finite element structural analysis – a review

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Abstract. Finite element analysis of many engineering systems is practised more as an art than as a science. It involves high level expertise (analytical as well as heuristic) regarding problem modelling (e.g. problem specification, choosing the appropriate type of elements etc.), optical mesh design for achieving the specified accuracy (e.g. initial mesh selection, adaptive mesh refinement), selection of the appropriate type of analysis and solution routines and, finally, diagnosis of the finite element solutions. Very often such expertise is highly dispersed and is not available at a single place with a single expert. The design of an expert system, such that the necessary expertise is available to a novice to perform the same job even in the absence of trained experts, becomes an attractive proposition.

In this paper, the areas of finite element structural analysis which require experience and decision-making capabilities are explored. A simple expert system, with a feasible knowledge base for problem modelling, optimal mesh design, type of analysis and solution routines, and diagnosis, is outlined. Several efforts in these directions, reported in the open literature, are also reviewed in this paper.

Keywords. Finite element method; optimal mesh design; structural analysis; expert system; knowledge base.

1. Introduction

The recent trend in the field of software development for scientific and engineering applications is to relieve the lay user of the need for expert knowledge necessary for the optimal use of the software as well as the theoretical and mathematical background for using the code. For this purpose, a combination of knowledge-based expert system (KBES) control and the automation of data generation and problem optimisation (e.g. adaptive mesh generation), with the computational method of solving the problem (e.g. finite element method), can make the entire system a powerful, efficient and cost-effective tool for design, analysis and diagnosis of complex problems.

Today, the finite element method has become an essential (and rather inevitable) tool for modelling and evaluating the physical performance of structural, mechanical and thermal systems in several engineering disciplines – aerospace, mechanical, civil, nuclear, marine etc. It has considerable impact on other engineering fields like electromagnetism and fluid mechanics as well. It has become an integral part of the present day CAD/CAM cycles, and has even been used to simulate manufacturing processes like casting, forging etc. The scope of the present paper, however, is confined to the role expert systems can play to simplify finite element structural analysis.

Structural analysis is the process of determining the response of a fully specified structure to its environment. Full specification of a structure needs a priori description of the topology of the structural domain, the material properties and the boundary conditions. The loads acting on the structure – including the equivalent loads produced by thermal effects, material changes like shrinkage and creep, initial strain/stress etc. – form the environment. The structural deformation and stresses (or stress resultants) are the standard response quantities of interest.

The finite element analysis of a structural system has attained a high level of sophistication and involves a considerable amount of experience and judgement in taking a priori and/or a posteriori decisions regarding modelling of the system with proper elements, data management for optimum mesh size for achieving the desired accuracy, choice of the type of analysis required and choice of the correct combination of the solution tools and finally the diagnosis of the results obtained. A great deal of expertise and a considerable amount of heuristic knowledge are essential for such an exercise.

Today, with the availability of numerous general purpose finite element programs (e.g. NASTRAN, ASKA, SAP, NISA, ANSYS, COSMOS, MARC etc.), coupled with their broad range of capabilities, the need for expertise in finite element analysis has been multipled. Often an experienced analyst might have attained expertise in one field of application using only one such general purpose software packages. Thus, apart from the novice user, even the experienced user often confronts difficulties while modelling a new class of problems with a new package. Thus, one of the major requirements is to accumulate, organise and thereby transfer the expertise of such experienced users to the less experienced ones, for the same class of problems and the same finite element programs, and, eventually aggregate such expertise over many different classes of problems and many different finite element packages.

Knowledge-based expert systems hold the promise of providing a methodology for such finite element modelling aids. Often these are called *intelligent pre- and post-processors* (Adeli 1988), in the sense that they act as knowledge-based interfaces to the algorithmic programs. Architecturally, such expert systems are closely related to the so-called classification or diagnostic systems (Harmon & King 1985) since they are data-driven (either input or generated data).

Several expert systems have been developed in practice with reference to modelling structural problems using some specific finite element programs (Bennett *et al* 1978; Bennett & Engelmore 1979, pp. 47–49; Rivlin *et al* 1980; Corlett *et al* 1984; Fenves 1985, 1986; Holt & Narayana 1986; Rehak 1986; Taig 1986; Wilson & Itoh 1986; Logan & Genberg 1987; Chen & Hajela 1988). SACON (Bennett *et al* 1978; Bennett & Engelmore 1979, pp. 47–49) is one of the earliest. It used the popular diagnostic expert system shell EMYCIN (Zumsteg & Flaggs 1985) and interacted with the user for proper application of the MARC finite element program for structural analysis (Bennett *et al* 1978; Bennett & Engelmore 1979, pp. 47–49; Rivlin *et al* 1980; Fjellheim & Syversen 1983). Most of these are knowledge-based consultation systems and are developed for experimental purposes. Their applications are often confined to initial modelling of the problem for a particular finite element program.

Another area of the finite element method which demands experience, judgement and heuristic knowledge is that of identifying the singularities present, the locations of stress concentrations and their relative strengths and the designing of an optimal mesh for achieving the specified accuracy. Several strategies of adaptive refinement for this purpose (e.g. increasing the number of elements of the same order, *h-type*, or increasing the order of the element interpolations, *p-type*, or the combination of the two, *hp-type*, refinement) are developed in the literature (Babuska & Rheinboldt 1978; Atluri *et al* 1986; Babuska *et al* 1986; Zienkiewicz & Zu 1987). Recently, a few attempts have also been made towards developing expert-like systems to identify the singularities in the structure and arrive at optimal refined mesh configurations (Babuska & Rheinboldt 1978; Babuska & Rank 1987; Rank & Babuska 1987; Szabo, PROBE). These systems mainly deal with the rules for adaptive refinement based on analytical error norms (e.g. strain energy norms, stress gradient norms etc.) and heuristic knowledge about the strength of the singularities and/or stress-concentrations. However, expert systems for such tasks are not yet well-established in practice.

In this paper, the role that expert systems can play in general finite element structural analysis is discussed. An overview of several aspects of finite element analysis which need both theoretical/analytical and heuristic expertise is given. Finally a proposal for a simple expert-system architecture for such a task is presented. This architecture was the basis for building an experimental expert-system module for problem modelling and optimal mesh design for a special purpose 3-D finite element package at the National Aeronautical Laboratory in Bangalore (Prathap & Naganarayana 1991). This is discussed briefly as a typical case study.

2. Finite element analysis – scope for expert systems

The main areas of finite element analysis which demand a high level of experience and judgement capabilities (figure 1) are problem modelling (e.g. problem specification, choosing the appropriate type of elements etc.), optimal mesh design for achieving

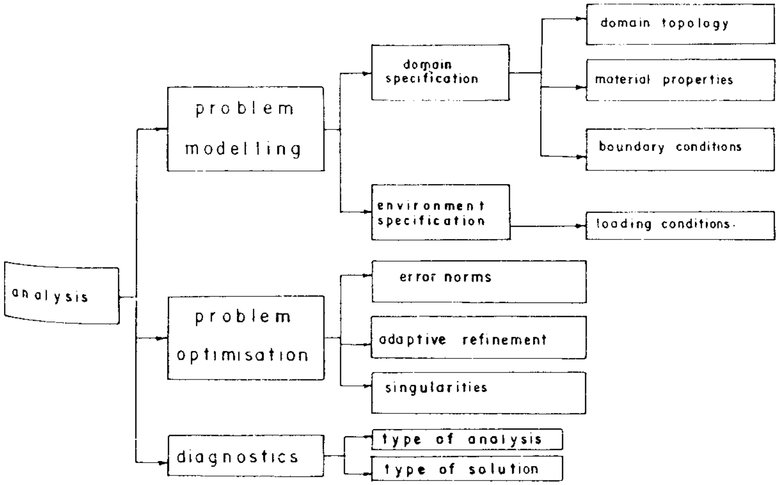


Figure 1. Scope of expert systems in finite element analysis.

specified accuracy, selection of appropriate type of analysis and solution routines and, finally, diagnosis of the finite element solutions.

2.1 *Problem modelling*

Modelling any problem requires expertise in selecting and/or development of the optimal theoretical basis (e.g. plate theories, shell theories etc.) and hence the type of elements to be used. Considerable knowledge regarding topological configuration and material constitution of the structure, boundary conditions and loading environment is required for this purpose.

A decision on the theoretical basis is crucial for making the analysis meaningful, optimally accurate as well as cost-effective. It should be noted that the cost of finite element analysis is directly related to the continuity requirements and the total degrees of freedom used for modelling the problem (table 1). An intelligent use of the available and/or generated knowledge is necessary to decide whether 3-dimensional, 2-dimensional or 1-dimensional modelling is appropriate for the problem, depending on the geometrical configuration and the material constitution of the structure, the boundary conditions and the loading environment. If 2-D or 1-D analysis seems sufficient with some loss of accuracy and/or compatibility in the displacement and the stress recovery, one has to decide whether a refined higher order shell, plate or beam theory is necessary for achieving the required accuracy. In addition, the degree of importance and risk associated with the type of application can also dictate the type of theoretical support required. It should be kept in mind that the time and cost of analysis increases sharply with an increase in the total degrees of freedom and continuity requirements for modelling the structure (table 1). Thus, 3-D analysis may be unwarranted, for example, when one of the dimensions of the domain is 'small' as compared with the others. If this is not done, the cost of computation increases dramatically requiring much more computer time and larger storage requirements.

Other deciding factors for choosing 1-D, 2-D or 3-D elements are the surface boundary conditions and loading. For example if 3 adjacent surfaces of a prismatic slab are completely or partially supported or loaded, 3-D analysis may become mandatory. Often, depending on the geometrical and material properties of the structure, the boundary conditions and the loading environment, one can reduce a 3-D problem to a 2-D problem in a plane-stress or plane-strain sense (Timoshenko & Goodier 1970). Accordingly, a 2-dimensional plane-stress or a plane-strain element may suffice for the modelling requirements. Depending on the loading and the type of element choosen, only the *active* components of strain energy need to be computed e.g. for purely in-plane loading on a plane-stress model of a plate, one requires to compute only the membrane component of the strain energy and the in-active degrees of freedom (transverse deflection and all rotations) should be suppressed. Such decisions reduce the computer time and cost effectively, in particular, when one deals with large problems.

Table 1. Comparison of different theories regarding the total d.o.f. and continuity requirement.

Dimension	Theoretical basis	Continuity requirement	Total d.o.f. (20 nodes per edge of hexahedral domain)	d.o.f. per node
1-D	Elementary	C^1	20	1
	First order	C^0	40	2
	Third order	C^0	60	3
2-D	Elementary	C^1	400	1
	First order	C^0	1,200	3
	Third order-I	C^0	2,000	5
	Third order-II	C^0	2,400	6
3-D		C^0	24,000	3

Very often, prescribing the boundary conditions in idealised terms, e.g. simple support or clamped, can be unrealistic and unreliable. Boundary conditions become much more complex when they become nonlinear. On the other hand, for some problems, one may go for a combination of simple boundary conditions without losing accuracy of solution. Similarly, modelling of loads acting on a structure can also be non-unique. Normally, the general loads acting upon a structure (e.g. airplane wing, landing gears etc.) do not fit into any standard category. In such cases, an expert may arrive at a decision regarding modelling of the boundary conditions and the loads based on the structure, its environment, his experience and intuition, and his knowledge of the information available in the literature. Hence, in such cases, it may be preferable to go according to the rule of thumb followed by experts in the relevant fields of application. Recently, Daniel (1991) has proposed an expert system for choosing proper boundary conditions for a given structure.

Sometimes, the global geometry of the structure and the possible local singularities may lead to conflicting opinions about using any particular theory for modelling and/or the method of solution. For example, a 2-D structural domain with a small, but critical, crack embedded deep in the domain may just require a global 2-D analysis and a local 3-D analysis around the crack. Sometimes, for a small crack in a relatively large 2-D or 3-D structural domain for which only the global behaviour of the structure is of interest, it may be desirable to have a combination of the boundary element and the finite element methods in modelling the problem. This achieves a cheap but sufficiently accurate solution. On the other hand, if the *strength* of the singularity is low enough, a refined mesh may be sufficient for achieving the specified accuracy.

Problem modelling cannot be complete without defining proper correlation (geometrical as well as material) between any two points in the domain. Experience is required in choosing the interpolation functions for generating the geometrical configuration of the structure (e.g. automatic volume, surface and line generation) from the minimum data input. This is turn can play a decisive role in choosing the type and order of the elements (as is apparent above) and the type of the basic mesh required.

Material modelling requires special attention, in particular when the structure is laminated. Material correlation in the thickness direction of the laminate should be evolved from the lay-up data (e.g. thickness, orientation and material properties of each layer for each of the orthotropic layers and the global constitutive relationships for each of the anisotropic layers) from which one has to deduce knowledge on the type of the laminate structure e.g. symmetric, antisymmetric, cross-ply etc. Accordingly computations can be optimised later by generating only the required data and following only the required steps of analysis.

2.2 *Mesh optimisation*

Mesh optimisation gains importance, in particular, when *singularities* are embedded in the problem domain. In general, the singularities create error-prone zones in their proximity. This demands data refinement, localised to such zones, for achieving the specified accuracy of solution in an optimal sense. The domain singularities, with reference to solid and structural mechanics, fall in one of the four general categories:

Material singularities – Hard grains, material voids etc.
Geometric singularities – Cracks, notches, voids etc.
Boundary conditions – Point supports, discontinuous supports etc.
Loading environment – Concentrated forces, discontinuously distributed loads etc.

Often, an expert uses his experience, and the analytical as well as heuristic knowledge available in the literature with reference to both the method of analysis and the problem under consideration so as to optimise the data refinement in the error-prone zones to achieve accuracy of solution in an efficient and cost-effective manner. Sometimes it may not be convenient to determine the strength of the singularities analytically. In such cases, the rule of thumb followed by the acknowledged experts in the corresponding field may have to be sought for mesh optimisation.

In general, it is impossible to construct an *a priori* error norm for finite element modelling of a structure of complex geometry. In such cases one has to use *a posteriori* information from cheap coarse mesh computations of the model, construct the error norms, identify the error-prone zones and optimise the mesh appropriately to achieve the specified accuracy in the most efficient and cost-effective manner.

A considerable amount of research and development has taken place in literature with reference to the error norms for finite element analysis of structure with singularities and several adaptive mesh refinement techniques (e.g. *h*-type, *p*-type and *hp*-type) have been proposed to achieve the specified accuracy (Babuska & Rheinboldt 1978; Atluri *et al* 1986; Babuska *et al* 1986; Zienkiewicz & Zu 1987). Recently some efforts have been made toward developing some expert-like systems for co-ordinating such activities in finite element analysis (Babuska & Rank 1987; Rank & Babuska 1987; Szabo (year not given). However, the resulting expert systems are primitive in the sense that they only consider the analytical knowledge with reference only to the method of analysis (i.e. adaptive finite element methods).

2.3 *Type of analysis and solution*

Another area in which a great deal of expert knowledge is essential is the understanding of how a structure with certain geometric and material properties responds to a given load under given boundary conditions. The major parameters involved in taking such decisions are the material properties, the domain geometry, the boundary conditions and the loads. One has to have a clear understanding of the independent as well as the collective effects of these parameters on structural behaviour before deciding on the type of analysis required.

Knowledge of the microstructural as well as the macrostructural properties of the material is required in deciding whether the material behaviour is linear or nonlinear, elastic or plastic, compressible or incompressible etc. One has to have an *a priori* knowledge of whether there are certain zones in the domain which require nonlinear elastic or plastic analysis though the global structural behaviour is linearly elastic. For example, the zone in the proximity of the crack tips in an elastic domain may require plastic analysis in a local sense, in particular, under fatigue load. Such knowledge becomes important, for the design/analysis of failure of a structural component. Sometimes the loads can also lead to such requirements, though the material is linearly elastic, depending on their magnitude. Normally a material database is required for such purpose.

The next important aspect is about the geometric nonlinear behaviour of a structure. Based on the structural geometry, boundary supports and the type of loads on the structure, one has to decide whether the structure is undergoing relatively small or large deflections and accordingly choose the theory for the analysis. The characteristic properties that influence geometric nonlinearity of a structure are mainly the structural geometry and magnitude and direction of the load. A good example of geometric nonlinearity is seen in the buckling of structures. Often such problems are approximately solved as an eigenvalue problem for certain simple cases (e.g. buckling of beams) with reasonably accurate solutions. On the other hand, the true geometric nonlinear considerations often become mandatory for analysing buckling of complex structures like shells.

Normally one needs *a posteriori* information about the deflections and stresses for taking a decision on nonlinear analysis for a structure, in particular if deformation or strain under load is large. If the deflection at any point of the structure is of the same order as the minimum characteristic structural dimension (e.g. thickness of a plate) one may need geometric nonlinear analysis. On the other hand, if any of the stress components is close to the corresponding elastic design limits for the material (e.g. corresponding yield stress) material nonlinear analysis may be required. Again this material nonlinearity can fall into any category like elasto-plastic, visco-elastic, plastic etc. depending on the structure and its environment. Often, an expert uses his past experience and depends on several heuristic rules for taking such decisions.

Apart from the magnitude, location and direction of the loads which normally influence the geometric (sometimes material) nonlinearity, the type of the loads with respect to time also influences the type of analysis required. Expert decisions have to be taken about whether a static analysis is enough or a dynamic analysis is required depending on the type of loads. If dynamic analysis is required, the type of analysis is again influenced by the type of application as well as the duration of application of the load (e.g. eigenvalue analysis, frequency response analysis, impact load analysis, flutter analysis, fatigue damage analysis etc.).

The presence of certain singularities like a crack may require special knowledge such as fracture mechanics – which may again be linear or nonlinear – to be incorporated into the analysis system. The analysis may differ for different types of cracks. The analysis of crack initiation and propagation becomes very important, in particular, for enduring dynamic loads causing fatigue in the material structure.

Depending on the type of numerical method and the type of analysis one has to select an optimum combination of solution routines. Exact or direct solution techniques may be desirable in conventional finite element modelling of linear problems, probably enhanced with an 'out-of-core' algorithm for very large problems. On the other hand, iterative solution routines may be preferable for conventional finite element modelling of nonlinear problems (e.g. Newton–Raphson method) and for adaptive finite element modelling (e.g. element-by-element conjugate gradient method with hierarchical preconditioning of the stiffness matrix). For dynamic analysis one may need additional solution routines like time-step integration, eigenvalue solution etc. depending on the type of analysis required.

Finally, expertise is needed in determining the tools to be used for tackling the problem in numerical and logical manner, for example, what combination of computer languages, what sort of data management, how the already available software can be optimally used etc. Today, most of the expert systems are written in C-language owing to its flexibility and efficiency in handling both numerical as well as logical operations. Blackboard architecture (Nilssion 1980; Nii 1986) is popular for the management of data and computational as well as logical software.

2.4 *Diagnostics*

The usual practice of analysis, especially for a complex problem, is to start with a simplified approximate model and then choosing optimally efficient combinations of more sophisticated tools of analysis after an intelligent diagnosis of the initial results. Interpretation of the final results also demands high levels of intelligence and expertise. Thus, it is often the diagnosis and interpretation of the finite element results for a complex problem that calls for high levels of intelligence, expertise and judgement capabilities. On the other hand, this is the most difficult part to code, since such knowledge is normally heuristic and changes from person to person, context to context and finally organisation to organisation (depending on the policies). In addition, most of the expertise on diagnosis is highly problem-dependent and cannot be generalised for all finite element applications as such. Therefore, developing a general purpose expert system for diagnosis of finite element results may not be a practical proposition and in fact hardly any effort has been documented in this area in the open literature. Thus one may have to confine the system to tackling specific problems like analysing rocket tanks (Desaleux & Fouet 1966) etc. using an appropriate finite element package.

3. **Expert systems**

As discussed earlier, if the expertise has to be transparent to a novice user of an analysis package, it becomes mandatory to coordinate the knowledge regarding the package and the field of interest and make it readily available. Again, as explained earlier, an expert system is a very useful tool for such an exercise.

In addition, a knowledge-based expert system combined with automated problem modelling and problem optimisation can make the analysis process very efficient and cost-effective when the process is of repetitive nature (figure 2). Such a system can have many advantages over conventional numerical analysis. Once the expert system is calibrated by the expert, he can release the software for use to less qualified assistants and coordinate several different tasks simultaneously. Since the algorithmic computations are also optimised, the total time and cost of the whole analysis process is drastically reduced. Finally, such systems can be effectively used to train novice users in specific fields.

Various interpretations and definitions of expert systems can be found in the literature. For example:

> "An interactive computer program incorporating judgement, experience, rules of thumb, intuition and other expertise to provide knowledgeable advice about a "variety of tasks" (Feigenbaum 1981).

> "An intelligent computer program that uses knowledge and inference procedure to solve problems that are difficult enough to require significant human expertise for their solution" (Gasching *et al* 1981).

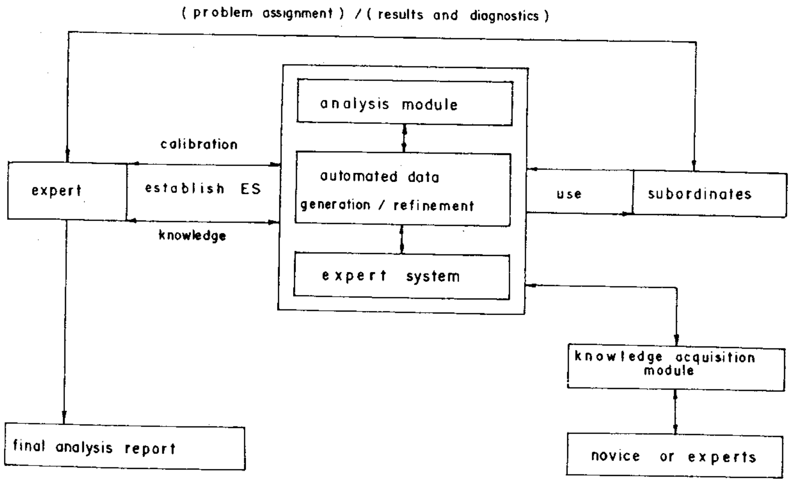


Figure 2. Role of expert systems in finite element analysis.

> "An expert system solves real-world, complex problems using a complex model of expert human reasoning, reaching the same conclusions that the human expert would reach if faced with a comparable problem" (Weiss & Kulikowski 1984).

Keeping the dominantly computational nature of the finite element analysis in mind, the last interpretation for the expert system appears to be most appropriate in the present context. In this section, the aspects involved in building an expert system are discussed with particular reference to finite element analysis of structural problems.

It is convenient to handle an expert system if its frame is divided into a domain-dependent part, the knowledge base and a domain-independent part, the *Inference Engine* (Adeli 1988; Levine *et al* 1988). The inference engine forms the *context*, *working memory* and *decision paths* for the system while the knowledge-base provides the technical informations and rules for reaching the specified goals in a particular domain of application, as an *expert* would do. Essentially, the inference engine coordinates different tasks (both computational and logical) and the knowledge base, normally constituted by database and rulebase, forms the body of knowledge for executing the tasks identified by the inference engine.

The general configuration for an expert system for finite element analysis is shown in figure 3. It consists of an inference engine having two parts. The *logical module* coordinates the information provided by the knowledge-base, database, rule-base and user interaction enhanced with an expert advice facility to select the appropriate finite elements, methods/types of analysis, a coarse initial mesh etc. Essentially, this module produces an optimum initial model for the given problem. On the other hand, the *computational module* coordinates the computational, algorithmic and graphical software through a recurring cycle of finite element analysis, error estimations and mesh refinement resulting in an optimally accurate solution for the problem. This module essentially involves problem optimisation and solution. Finally the system can be enhanced with the assistance of post-processing and diagnosis facilities. Other major modules in such an expert system are the Advise/Help, Explanation and

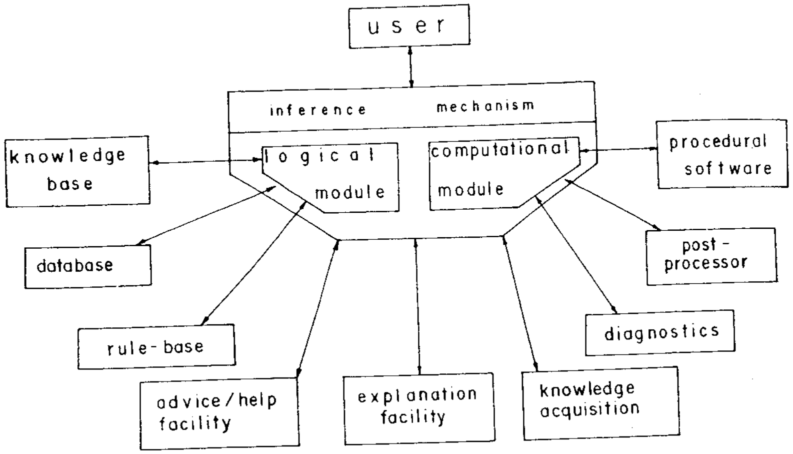


Figure 3. A typical expert system for finite element analysis.

Learning facilities. Figure 4 shows the details of the design of such an expert adaptive finite element structural analysis system.

3.1 *Knowledge base*

Knowledge can be categorised into two types. *Surface Knowledge* consists of rules regarding the necessary tasks, data input and the sequence of the tasks for the given particular problem (e.g. determining relative dimensions of the structure). This can be normally handled by a rule-based expert system through a user-interactive logical questionnaire. On the other hand, *Deep Knowledge* involves knowledge about the underlying problem structure (e.g. error estimation and recognition of critical zones). Each can be *Declarative* (facts about objects, events, e.g. whether strain energy density at a point is high or low) or *Procedural* (information about courses of action e.g. the mesh refinement strategy depending on the error norms). Surface knowledge is often heuristic while deep knowledge is analytical. The process of building a knowledge-base involves acquisition of relevant and reliable knowledge from the experts and other sources like the literature etc. and its representation in a form that the inference engine can process in an efficient manner.

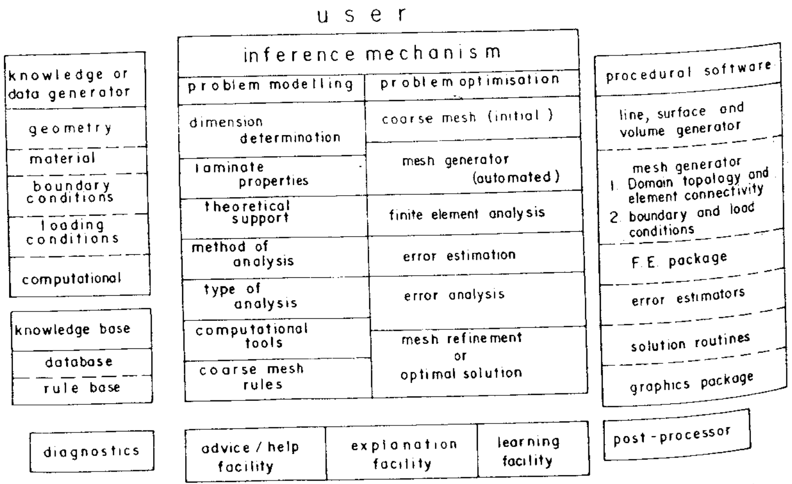


Figure 4. Blackboard architecture for expert system in adaptive finite element analysis.

3.1a *Knowledge acquisition*: The most difficult challenge in building a knowledge-base is to acquire reliable knowledge relevant to the domain under consideration. This requires careful survey of different reliable sources of knowledge such as (Adeli 1987):

(1) Technical literature (e.g. books, manuals, journal articles etc.);
(2) domain experts (e.g. interviews, circulating questionnaires, example problem-solving sessions etc.)
(3) experimental data (e.g. numerical or laboratory experiments);
(4) input data for the particular problem.

Again the knowledge acquired can be *domain knowledge* (i.e. the knowledge related to the specific area of application e.g. finite element structural analysis) or *problem knowledge* (i.e. knowledge related to a particular problem in the domain e.g. the dimensions of a structure). However, they can be either deep or surface and declarative or procedural.

Domain knowledge is normally independent of the particular problems of application in the domain. Since such knowledge does not vary from problem to problem it is often called *static knowledge*. For the same reason, this can be in-built in the system permanently in the form of rules, nets or frames so that the end-user need not have any control on it. In general, the first three sources mentioned above are exploited to acquire such knowledge. Some of the examples relevant to the finite element analysis are rules for selection of appropriate theoretical bases for structural modelling, proper finite element mesh, type of analysis and solution etc.

On the other hand, problem knowledge varies from problem to problem (hence often called *dynamic knowledge*) and it is generated from the input data for a particular problem in the domain in the user-interactive mode. This class of knowledge gains particular importance in the present context. Some of the examples are whether the structure is 'thin' or 'thick', flat or curved, isotropic or anisotropic, whether a plane stress or a plane strain model is required etc. It can be observed that input data regarding the structural topology, material constitution and the loading environment are pre-requisites for generating such knowledge. This can be achieved through an efficient user interface. A typical flow of logic in modelling a complex structure with simple hexahedral substructures and determining the relative characteristic dimensions of each of the substructures is shown in figure 5. Such knowledge is essential while interpreting domain knowledge (e.g. selecting the appropriate theoretical basis and type of elements) later.

3.1b *Knowledge representation*: An equally important task is to represent the knowledge that is acquired in an efficient form that can easily be addressed by the inference mechanism. Knowledge can be represented in a program in two ways –

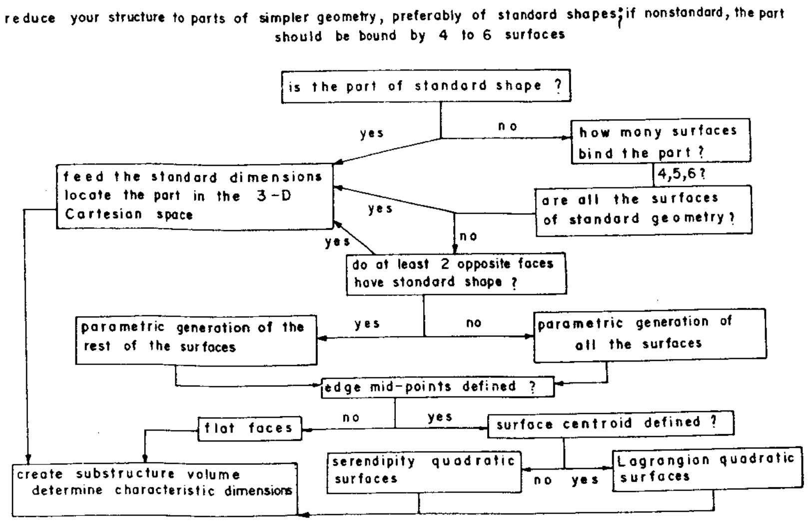


Figure 5. Flow of logic for geometric data generation.

procedural and *declarative*. Procedural representation is normally used in conventional algorithmic programming and is very efficient. But the knowledge involved is highly context-dependent and embedded in the code, thus making the knowledge opaque – unintelligible and difficult to modify. On the other hand, declarative representation encodes knowledge as data so that it is context-independent and is therefore easier to comprehend and modify. While semantics are distributed over the code in procedural representation, they are collected in one place in declarative representation. Several approaches of declarative knowledge representation are available in the AI literature (Hayes-Roth *et al* 1983; O'Shea & Eisenstadt 1984; Brachman & Levesque 1985). The major ones are briefly described below.

Rule-based production system – This has been the most favoured representational approach for building expert systems. A production system (Brownston *et al* 1985; O'Shea & Eisenstadt 1984; Van Melle 1979, pp. 923–925) is a collection of several rules each of which consists of an ordered pair of symbols (IF and THEN). The first constitutes the premises or the conditions while the second constitutes the actions. This is the most popular and simplest method. The problem reduction method (Barr & Feigenbaum 1981–82; Cohen & Feigenbaum 1982; Ishizuka *et al* 1983; Rich 1983; Winston 1984; Charniak & McDermott 1985) is often useful for such knowledge representation so that a complex problem can be subdivided into simpler problems, described hierarchically, each having a subgoal.

Logic-predicate calculus – This is basically an extension of the so-called proposition logic where a proposition can be either true or false. A first order predicate calculus (Ishizuka *et al* 1983; Ogawa *et al* 1984) consists of a number of components or propositions (like predicate symbols, variable symbols, function symbols and constant symbols) which are interconnected through the logical connectives (like AND, OR, NOT, EQUIVALENT, IMPLIES etc.). This type of knowledge representation is very convenient, in particular, with declarative languages like PROLOG.

Semantic networks and triplets – Semantic network, normally, involves *nodes* and *arcs* (*links*). Nodes represent objects, concepts or situations, their attributes and values. The arcs link these nodes and define the *nodal heirarchy*. If the net consists of only one object, and its attribute and value, the three nodes form the so-called *semantic triplet* or OAV triplet (Adeli 1988).

Frames – Sometimes frame-like structures are found to be convenient for knowledge representation, in particular for representing sequences of events in expert systems (Minsky 1975; Aikins 1983; Yao & Fu 1985). A frame is composed of its name and several slots. Each slot may have a number of facets. A slot in a frame may be an attribute of the object or it may be a relation. A relation slot is used to link different frames together.

In the present context, for such a dominatingly analytical exercise, a rule-based production system appears to be most promising.

3.1c *Representing uncertain knowledge – interval modelling*: Often the available knowledge about the domain and the problems of interest in the domain are uncertain and incomplete. Usually, such knowledge is qualitative e.g. strength of a singularity is *high* or plate thickness ratio is *low* etc. In fact, the intelligence of an expert (and hence of an expert system) is reflected in his ability to handle such knowledge. In such cases the so-called *approximate reasoning* or *inexact reasoning* or *reasoning under uncertainty* (Zadeh 1974, pp. 591–594; Blockley 1980; Ruspini 1982; Ishizuka *et al* 1983; Negoita 1985; Lecot & Parker 1986) should be employed for representing and processing the available uncertain knowledge. Interval modelling (Wong 1986) is one of the most popular mathematical tools used for handling uncertainty. Different mathematical measures which use interval modelling concept are illustrated in Wong (1986). Some of them, which are useful for the present exercise, are briefly discussed here.

(a) *Simple intervals* – This needs minimum information as seen in figure 6a. The value of a parameter is estimated to be within an interval $[a, b]$ and nothing more is known.

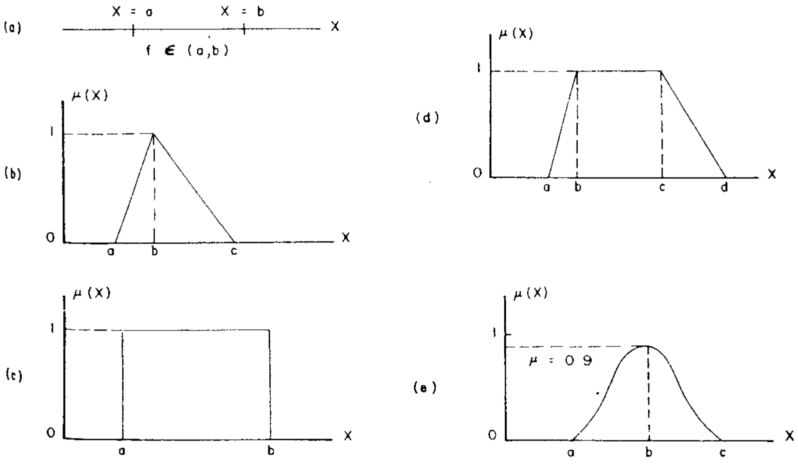


Figure 6. Different types of fuzzy knowledge representation: (**a**) simple interval, (**b**) triangular set, (**c**) rectangular set, (**d**) trapezoidal set, (**e**) *normal* set.

(b) *Probability measures* – Probability measure implies exact knowledge of the uncertainty. Although the value of the parameter is uncertain, there is certainty about the character of its randomness (e.g. *normal curve*). The probability measure is built on a simple interval and the probability of an event A, say $p(A)$ is known at any point in the interval. It should be noted that $p(A)$ implies that probability of the event (not A) is also certain and $p(\text{not } A) = 1 - p(A)$.

(c) *Possibility measures and fuzzy sets* – Possibility measures are again built upon a combination of simple intervals as seen in figures 6b to 6e. The possibility of an event $\mu(A)$ is known at any point in these interval and the resulting set is generally called *fuzzy set*. Here, the stringent presumption of probability theory is relaxed by introducing the concept of ignorance (Shafer 1976) that the probability of event A should not say anything about the probability of (not A) unless there is explicit evidence supporting the latter i.e. $\mu(\text{not } A)$ is independent of $\mu(A)$. This measure is often identified with the so-called *evidence measure*. Depending on the variation of the possibility factor $\mu(A)$ associated with the event/object A the fuzzy set can be of any shape. For example, a triangular set (figure 6b) can be used for a pessimistic representation of the event A that at only one point $\mu(A) = 1$. On the other hand, a rectangular set (figure 6c) can be used for optimistic representation such that $\mu(A) = 1$ at all points except at the two end points. Again the possibility function can be a continuous function (figure 6e). But a trapezoidal set (figure 6d) is seen to be the most practical choice for representing fuzzy knowledge.

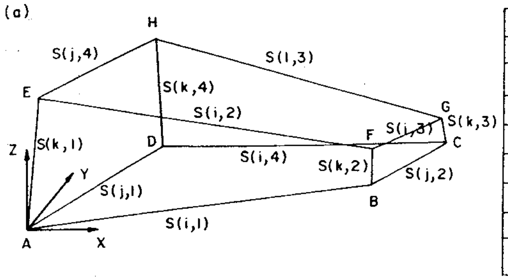
There are several instances in finite element structural analysis where the knowledge representation becomes fuzzy. The most common one is of determining the dimension of analysis for the structure, in particular, if the structural geometry is *irregular* and *distorted*. Figure 7 shows how the *thickness* ratio for a general hexahedral structural domain can be represented using the fuzzy set theory (also see §4·1). Some other instances where fuzzy knowledge representation is required in finite element structural analysis are – the intensity of mesh refinement required in an adaptive finite element analysis are – the intensity of mesh refinement required in an adaptive finite element approach, the strength of a structural singularity, strength of the inter-laminar bond in a composite structure etc., where the knowledge is often heuristic and verbal.

3.2 *Inference mechanism and expert system architecture*

An inference mechanism is the part of an expert system that deduces new facts from known facts of the knowledge-base. This mainly constitutes the problem-solving strategy for the required task. Problem solving can be considered as a search for solution through a *state space* by the application of operators, where the space (the possible states in a problem solution) consists of an initial state, a goal state and an intermediate state. The solution path consists of all states that lead from an initial state to a goal state. A number of different problem-solving paradigms are available in the AI literature – the describe-and-match paradigm, the goal-reduction paradigm, generate-and-test systems, means–ends analysis, rule-based (production) systems etc. (Barr & Feigenbaum 1981–82; Cohen & Feigenbaum 1982; Ishizuka *et al* 1983; O'Shea & Eisenstadt 1984; Winston 1984; Brownston *et al* 1985; Charniak & McDermott 1985). Many of these approaches overlap or can be used, in conjunction with each other.

However, it is the production system which is popularly used for developing most of the knowledge-based expert systems so far. This has additional advantages over other strategies, being simple to code, with improved clarity, and being suitable for

(a)



point	X	Y	Z
A	0	0	0
B	21	3	0·4
C	20	5	0·6
D	1	9	0
E	1	0	2·2
F	20	4	0·9
G	19	6	0·8
H	−1	11	1·8

	m = 1	m = 2	m = 3	m = 4	mean	dimn.
S(i,m)	10·863	13·928	10·198	7·071	10·515	l
S(j,m)	5·0	8·544	9·434	13·675	9·163	b
S(k,m)	5·0	8·544	10·630	9·950	8·531	t

min.(t/l) = 0·067
max.(t/l) = 0·173
mean(t/l) = 0·108

min(b/l) = 0·105
max(b/l) = 0·576
mean(b/l) = 0·307

min(t/b) = 0·128
max(t/b) = 1·498
mean(t/b) = 0·594

(b)

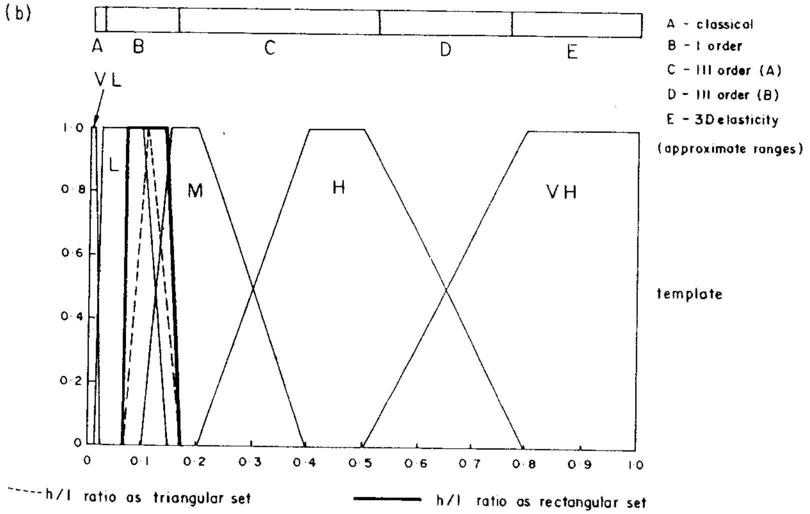


Figure 7. Fuzzy sets for the dimension ratios: (**a**) generation of dimension ratios; (**b**) comparison of fuzzy sets.

parallel computations and for designing an explanation facility for the system. In particular a rule-based system combined with the goal-reduction paradigm can form a very efficient and powerful model of the human information processing and problem solving abilities. A production system normally consists of three main elements – a set of IF-THEN rules (or knowledge base), a global database or working memory or the context and an inference mechanism.

3.2a *Inference mechanism – problem solving strategies*: The next question is, how are the rules determined from the knowledge base and which ones should be used at a certain state of the solution path. For this one has to choose an inference mechanism or a control strategy which constitutes the heart of a knowledge-based expert system. Inference mechanism fires the rules in accordance with its built-in reasoning process. The two major types of reasoning are antecedent reasoning and consequent reasoning.

(a) *Antecedent reasoning (forward-chaining, data-driven control strategy)* – In this type of inference mechanism, the rules are scanned until one is found whose antecedents (left-hand sides) match the information for the problem entered in the working memory. Then the rule is fired and the working memory is appropriately updated. This process is repeated until the goal state is achieved or no useable rule is found. This strategy is particularly useful if the goal state is not known or if there are too many possible outcomes. Complex planning problems can be tackled through this approach. The reasoning involved in the finite element structural analysis predominantly falls into this category e.g. determination of the dimension of analysis and the theoretical support required, given the structural topology, material constitution and loading environment, error prediction from finite element analysis and heuristic expertise etc.

(b) *Consequent reasoning (backward-chaining, goal-driven control strategy)* – In this type of inference mechanism, rules are scanned and those whose consequent actions can lead to the desired goal are determined. The associated antecedents are tested to see whether they match the working memory information. If they all match, the rules are fired and the problem is solved. If the goal states are known and their number is small then this reasoning process seems to be advantageous. This is popular in the diagnostic type of expert system. Sometimes it may be convenient to use backward chaining in finite element structural analysis e.g. selection of appropriate elements and design of the required finite element mesh based on the knowledge of the dimension of analysis and the theoretical supports that are already established, adaptive refinement of the finite element mesh based on error predictions etc.

(c) *Opportunistic reasoning (hybrid or mixed chaining)* – This is a combination of both forward and backward chaining. It follows that this becomes mandatory, in particular, for an expert system for finite element structural analysis. This approach can be efficiently utilised through the use of the so-called 'Blackboard Environment' (Nii 1986). The blackboard will keep track of the simultaneous application of backward and forward reasoning chains which are activated at the most 'opportune' time.

3.2b *Blackboard architecture*: The concept of blackboard architecture was first implemented in HEARSAY-II, a speech understanding system (Reddy *et al* 1973, pp. 185–193). The control of the program can be made flexible and efficient and the restricted accessibility of routines could be eliminated by isolating the routines (or modules), but allowing all the subroutines to make use of the common data structure. The blackboard model of problem solving is highly structured and essentially a special case of opportunistic reasoning. The knowledge necessary for solving the problem is divided into independent groups of rules called knowledge sources. A blackboard plays the part of a communication vehicle among knowledge sources and keeps track of the incremental changes made in the problem state until a solution is found.

In the present context of expert systems for finite element structural analysis such an architecture appears to be the most suitable since all the information and the associated rules necessary to model the structural domain geometry, boundary conditions, material constitution and loading conditions (acquired/generated as mentioned in the previous section) can be stored in the respective independent knowledge sources. Figure 4 shows a schematic diagram of the blackboard model for the system which acts as a global database in controlling the above-mentioned logical knowledge sources as well as the computational knowledge sources.

3.2c *User interface*: The primary motivation for expert-system development is to make the expert knowledge, in a specific area of interest, available to the amateur user of the system. Thus inclusion of an exhaustive user interface in the system becomes mandatory. This module generally includes Advice/Help, Explanation and Learning facilities. This monitors functions like modelling the problem (e.g. structural topology and material constitution and the loading environment), selection of appropriate tools (e.g. finite elements, mesh size, type of analysis and solution routines etc.) through a live conversation with the user.

At any state in the problem space the system should be able to advise the user with proper verbal/graphic support, if asked for, about taking decisions appropriately depending upon the problem environment. Also capability of self-justification is desirable so that the user should finally know why and how the system arrived at the suggested goal/advice. In fact the main purpose of an expert system, that transfer of the experts' knowledge to the novice users of the system be simple and flexible, cannot be fulfilled without such an explanation facility. For example, if the system decides that 1-D analysis is enough for a structure, it should be able to educate the user about why such a decision was taken and what are the preceding factors that lead to such a decision, thus enabling him to examine the reasoning process.

Finally, the system should update its knowledge, as its human counterpart would normally do, from time to time. For example, if the user creates a new geometrical shape which is unknown to the system and if he feels that it invites frequent usage, the system should be able to 'learn' the necessary details and store them in its library with the help of the user. A more powerful example is of self learning. For example, if the system reaches the same goal (e.g. nonlinear analysis for a structure) during several runs, appropriate chunks of knowledge (e.g. geometry, material, boundary conditions and loading) involved in all such runs may be compared to generate, probably, a new heuristic rule which can be used in future under similar conditions without doing the actual computations required. Building such automated learning systems is quite a difficult challenge and the expected efficiency and effectiveness are yet to be achieved.

4. A case study – an expert system for finite element analysis of structural problems using 3-dimensional elements

A project on developing an *expert system* for control of modelling of the structural problems using finite elements (only 3-D elements at present) is under progress at the National Aeronautical Laboratory in Bangalore (Prathap & Naganarayana 1991). Mesh refinement is automated using an *h*-type strategy based on strain energy density error norms. In this section, we take this up as a case study to demonstrate some of the issues involved in such developmental work. The inexact representation of certain problem knowledge using fuzzy sets is first discussed. Later some rules related to the present system are briefly listed below (the knowledge-base, however, is built for using 1-D, 2-D as well as 3-D elements).

4.1 *Inexact representation of knowledge*

The major parameters influencing the selection of the theoretical bases to be used, as we already discussed, are the domain dimensions, material constitution and presence of singularities. Usually it is not possible to represent the knowledge about these conditions in an exact sense. The fuzzy set representation of these can streamline the problem-solving process for elegance and efficiency.

4.1a *Fuzzy sets for ratios of characteristic structural dimensions* – $\langle D_{hl} \rangle$ *and* $\langle D_{hb} \rangle$: First, the major sets of dimensions are identified and their mean values are calculated. Then the ratios of the smallest mean dimension with the rest are calculated. For example, consider a hexahedral domain (figure 7) which has 6 surfaces, and 3 sets of dimensions, each set having 4 values. Let $S_i(m)$, $S_j(m)$ and $S_k(m)$ (for $m = 1,4$) represent the three sets and let L_i, L_j ana L_k be their corresponding means. Let l and h represent the length and thickness dimensions corresponding to the maximum and minimum mean dimensions and let b represent the remaining (breadth) dimension. Let $\langle D_{hl} \rangle$, $\langle D_{hb} \rangle$ and $\langle D_{bl} \rangle$ be the fuzzy sets representing the ratios h to l, h to b and b to l. These sets can be constructed as follows (also refer to figure 7): Let,

$$X_{hl} = (S_h/S_l)_{\min};\ Y_{hl} = (S_h/S_l)_{\max} \text{ and } Z_{hl} = (S_h/S_l)_{\text{mean}}.$$

Using the values X and Y for the two points (A and B) of zero possibility factor and Z for the point (C) of unit possibility factor a triangular fuzzy set can be formed (figure 7b), Trapezoidal sets can be constructed as follows. Let,

$$H_{hl1} = \text{Abs}(Z_{hl} - X_{hl}) \text{ and } H_{hl2} = \text{Abs}(Z_{hl} - Y_{hl})$$

and

$$H_{hl} = \text{Min}(H_{hl1}, H_{hl2}) \times f_{hl},$$

where f_{hl} can be a factor of ignorance (normally ranging from 0·9 to 1·0). Similarly H_{hb} and H_{bl} can be calculated. The two points (C and D) of the unit possibility factor for the trapezoidal fuzzy sets $\langle D_{hl} \rangle$, $\langle D_{hb} \rangle$ and $\langle D_{bl} \rangle$ are then given by

$$(Z_{hl} \pm H_{hl});\ (Z_{hb} \pm H_{hb}) \text{ and } (Z_{bl} \pm H_{bl}),$$

respectively, (figure 7b). From a comparison of these fuzzy sets with a template of standard sets (figure 7b) one can decide the dimension and theoretical requirements for the analysis of a structure (as will be seen in the next subsection).

Note: If fuzzy sets are not used, then, the scalars,

$$D_{hl} = L_h/L_l \text{ and } D_{hb} = L_h/L_b,$$

calculated based on the mean dimensions, can be used for comparison as practised conventionally.

4.1b *Fuzzy sets for singularities* – $\langle S_i \rangle$: Let there be n singularities in the problem domain. Associated with each, there are two parameters that can influence theoretical support – stress concentration factor or criticality of the singularities $\langle SC_i \rangle$ $(i = 1, n)$ and the accuracy of stress recovery demanded by the user $\langle SA_i \rangle$. SA can be used as a weighting factor for deriving the resultant fuzzy set $\langle S_i \rangle$ for ith singularity.

4.1c *Fuzzy set for material structure* – $\langle C \rangle$: As discussed earlier, the laminated structure may require the use of higher order theories, especially if high accuracy is demanded for inter-laminar stresses. If the inter-laminar bonds are weak, failure normally occurs across the laminae. In this case, higher order theories are essential for meaningful stress calculations. Let $\langle SL \rangle$ and $\langle BW \rangle$ represent the fuzzy sets for the accuracy of laminar stress recovery demanded by the user and the interlaminar bond weakness, respectively. Using SL as a weighting factor, the resultant fuzzy set for lamination effects $\langle C \rangle$ can be constructed.

4.2 *Some typical rules*

Some of the typical rules that can be used for modelling a structure using finite-elements and for remeshing the mesh adaptively, are listed here.

4.2a *Rules for 3-D analysis*:

(1) *Global 3-D analysis if*,
{Number of the binding surfaces $\leqslant 4$} *or*
{3 or more adjacent surfaces are fully or partially supported/loaded} *or*
{Both $\langle D_{hl} \rangle$ and $\langle D_{hb} \rangle$ are very high} *or*
{Both $\langle D \rangle$'s are high or very high and $\langle S \rangle$ and/or $\langle C \rangle$ are very high}.
(2) *Local 3-D analysis around ith singularity if*
{When $\langle D_{hl} \rangle$ and/or $\langle D_{hb} \rangle$ are small} *and*
{singularity $\langle S_i \rangle$ is very high}.

4.2b *Rules for 2-D analysis*

(3) *2-D analysis if*,
{any of $\langle D \rangle$'s, $\langle S \rangle$ and $\langle C \rangle$ is not *very high*}.

Theoretical basis:

(4) *Third-order plate/shell theory–II if*,
{all $\langle D \rangle$'s, $\langle S \rangle$ and/or $\langle C \rangle$ are *high*}.
(5) *Third-order plate/shell theory – I if*,
{both $\langle D \rangle$'s are *high* or *moderate* and $\langle S \rangle$ and/or $\langle C \rangle$ are *moderate* (or vice versa)}.
(6) *First-order plate/shell theory if*,
{both $\langle D \rangle$'s, $\langle S \rangle$ and/or $\langle C \rangle$ are *moderate* or *low* or *very low* but not $\langle D \rangle$'s as well as $\langle S \rangle$ and/or $\langle C \rangle$ are *very low*}.
(7) *Elementary plate/shell theory if*,
{all $\langle D \rangle$'s and $\langle S \rangle$ and/or $\langle C \rangle$ are *very low*).

Geometric basis:

Now construct the mid-surface in the l–b curvilinear 'plane' by considering the midpoints of the edges in the h-direction and find out the curvatures $k1$ and $k2$ in the l–h and b–h planes respectively. This is done if standard domain shapes are not chosen. Find the Gaussian curvature $K = k1 \times k2$.

(8) *Plates if,*
{$k1 = k2 = 0$}.
(9) *Plane stress model if*
{(8) and (6) or (7) are satisfied}.
(10) *Plane strain model if,*
{both $\langle D \rangle$'s are *high* or *very high* and the *h*-, *b*- dimensions, loads and boundary conditions do not vary with the *l*-dimension}.
(11) *Zero Gaussian curvature shells if,*
{either of $k1$ and $k2$ are non-zero but $K = 0$}.
(12) *Positive Gaussian curvature shells if,*
{$K > 0$}.
(13) *Negative Gaussian curvature shells if,*
{$K < 0$}.
(14) *Shallow shells if,*
$k1$ and $k2$ are very small.

4.2c *Rules for 1-D analysis*:

(15) *1-D analysis if,*
{$\langle D_{hb} \rangle$ is very high but $\langle D_{hl} \rangle$ is not *high* or *very high*}.
(16) *Higher order beam theory if,*
{$\langle D_{hl} \rangle$ is *moderate*}.
(17) *First-order beam theory if,*
{$\langle D_{hl} \rangle$ is *low*}.
(18) *Elementary beam theory if,*
{$\langle D_{hl} \rangle$ is *very low*}.

Since both *b*- and *h*- dimensions are comparable, the line passing through the centroids of the $b - h$ planes forms the 1-D arc for which the curvature k is calculated if non-standard domain shape was selected. If load is acting in the *b*-direction D_{bl} should be used in place of D_{hl}.

(19) *Straight beam if,*
{$k = 0$}.
(20) *Curved beam if.*
{k is not zero}.
(21) *Warping corrections are necessary if,*
{$b - h$ planes are non-circular}.

4.2d *Rules for computation*:

(22) If the structure is a plate/straight-beam or shallow shell/curved-beam without singularies,
{use linear elements in the coarse mesh}.
(23) If the structure is a deep shell/curved-beam and
if $K = 0$, (use linear elements),
if $K \neq 0$, (use quadratic elements).
(24) If the structure has singularities,
{use quadratic elements around them}.

(25) If the structure is a plate and load is acting only in the $l - b$ plane,
{calculate energy and the stiffness corresponding to inplane normal strain components only} *and*
{suppress all the degrees of freedom other than the inplane displacements before assembling the stiffness}.
(26) If the structure is a plate and load is acting only normal to the plate
{calculate only transverse shear and flexural strain energies and the corresponding stiffness matrices} *and*
{suppress all the degrees of freedom corresponding to inplane displacements before assembling the stiffness}
(27) Calculate the laminate elastic matrices and carry out the related analysis only if
{the structure is laminated}.
(28) If the laminate is symmetric and the structure is a plate
{calculate only $[A]$ and/or $[D]$ matrices and carry out analysis as applicable according to rules (25) and (26)}.
(29) If the material is nonlinearly elastic or visco-elastic
{use the material nonlinear capabilities of the code}.

4.2e *Initial mesh strategy*:

(30) If a surface is quadrilateral,
{use initial mesh strategy as shown in figure 8a}.
(31) If a surface is triangular,
{use initial mesh strategy as shown in figure 8b}.
(32) If a continuous surface of a domain has discontinous boundary conditions (e.g. cracks) leading to very critical singularities
{appropriately bias the initial mesh}.

4.2f *Rules for mesh refinement*:

(33) If the $\langle$*discretisation error*$\rangle$ is *high* for a zone
{use *hp*-refinement}.
(34) If the $\langle$discretisation error$\rangle$ is *moderate* for a zone
{use *h*-refinement}.
(35) If the $\langle$discretisation error$\rangle$ is *low* for a zone
{use *p*-refinement}.

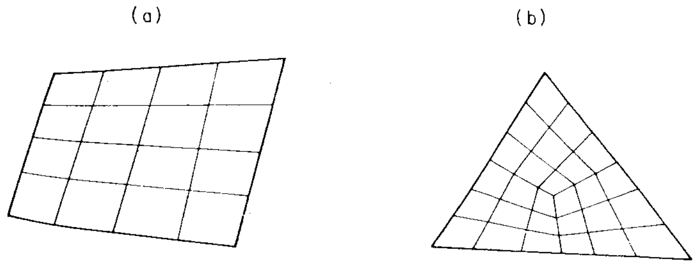


Figure 8. Initial mesh patterns: (**a**) quadrilateral surface, (**b**) triangular surface.

(36) If the structure is 3-dimensional
{refine the element (3-D elements) mesh in all the three principal directions (*l*-, *b*- and *t*- directions)}.

(37) If the structure is 2-dimensional
{refine the element mesh only in *l*- and *b*- directions}.

(38) If the structure is 1-dimensional
{refine the element mesh only in the *l*- directions}.

4.2g *Some diagnostics rules based on a posteriori knowledge*:

(39) If the deflection at any point of the structure is comparable with its minimum characteristic dimension (e.g. thickness of a plate/shell)
{repeat the exercise with the geometric nonlinear capabilities of the code}.

(40) If the stress at any point of the structure is comparable with the corresponding design stress (design stress is normally taken as equal to yield-strength × factor-of-safety)
{repeat the exercise with the material nonlinear capabilities of the code}.

It can be noted that both backward-chaining (e.g. rules for selection of dimension and theoretical basis) and forward-chaining (e.g. rules for computation and mesh generation/refinement) are used. Thus an expert system for the present task requires preferably an opportunistic strategy of problem solving.

5. Conclusions

In this paper, several aspects of the finite element analysis of problems in structural mechanics, which require high levels of expertise, are discussed. It is obvious that there is a need for knowledge-based expert system control in modelling complex structures with appropriate finite elements and proper mesh for optimal accuracy at minimum computational cost. A brief review of the ideas and concepts that are emerging in the field and of several efforts in this direction are given. A comprehensive set of rules which can constitute the skeletion of a full-fledged expert system for finite element analysis of a large scale structure is illustrated.

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Approximate factorization scheme for transonic potential flow computations*

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Abstract. The implicit approximate factorization scheme known as AF2 is investigated here for the purpose of application to the solution of two- and three-dimensional transonic full potential equations in conservative form. The artificial viscosity used by different authors has been deduced, and is discussed in detail. A second-order correction to the implicit artificial viscosity is tested for transonic flow past a Korn aerofoil at both design and off-design conditions. The inviscid transonic flow past different aerofoils, wings and wing–body configurations has been computed using the AF2 scheme and the solutions are compared with experimental and other numerical results. It is shown that the AF2 scheme is fast, and is not sensitive to grid stretching.

Keywords. Implicit schemes; approximate factorization schemes; inviscid flows; transonic flows; computational fluid dynamics.

1. Introduction

The computation of transonic potential flow past aerofoils, wings and bodies has reached a state of relative maturity due to the development of fast algorithms, such as the Alternating Direction Implicit (ADI) and Approximate Factorization (AF) schemes and multigrid techniques, complemented by the availability of high speed computers with vast memories. The inaccuracy introduced by the assumption of irrotationality (which is justified only for weak shocks) in potential flow computation has been accepted in order to save on the computer storage and time that would be required to obtain solutions of the more accurate Euler and Navier–Stokes equations.

For many years, Successive Line Over-Relaxation (SLOR) has proved to be a reliable but slow method for solving the compressible potential equation. The use of successive mesh refinement does hasten convergence but demands a considerable number of

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