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ORONCE FINE'S DE SPECULO USTORIO: A HERETOFORE IGNORED EARLY FRENCH RENAISSANCE PRINTED TREATISE ON MATHEMATICAL OPTICS

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SUMMARIES

This paper describes the contents and sources of De Speculo Ustorio Liber Unicus *(1st printing Paris, 1551), and the role of this and other works of Oronce Fine (1494-1555) in the revival of the mathematical arts in Renaissance France.*

In hoc opusculo quae scripsevit in de speculo ustorio libro unico (editio princeps Lutetiae, 1551) Orontius Fineus (1494-1555) et quibus fontibus usus sit explicavi. Praeterea quales partes hic liber et alia Finei opera eberint in mathematicis artibus renovandis in Gallia saeculo renacentiae demonstravi.

מאמר זה מתאר את תוכן העינינים ואת מקורותיו של De Speculo Ustorio Liber Unicus (מהדורה ראשונה בפריז, 1551) מאת Orontius Finaeus (1494-1555) וגם את תרומתו של הספר הזה ושל יתר עבודתו לחידוש המדעים המתמטיים בצרפת ברנסנס.

Only recently has Oronce Fine (1494-1555) received attention for his role in early French Renaissance mathematics. [1] One of Fine's works that has yet to be considered is *De Speculo Ustorio Liber Unicus*, first printed at Paris in Latin in 1551 and reprinted at Venice in an Italian translation in 1587 and 1670 [2] I have been unable to find any reference, other than bibliographical, to this work in the standard sources on the history of mathematical optics [e.g., Kaestner 1797; Ronchi 1970]. From one viewpoint this is understandable, since *De Speculo Ustorio* made no advances in the science of mathematical optics. From an historical viewpoint, however, it is unfortunate, because this and other work by Fine made a significant contribution to the revival of the mathematical arts in early Renaissance France.

In the present study, Part I summarizes the contents of *De Speculo Ustorio,* Part II discusses its sources, and Part III is an appreciation of this and Fine's other efforts for the rebirth of French mathematics.

I. CONTENTS

In a preface to the work proper, Fine makes a number of general observations on the nature and value of burning mirrors and touches briefly on two earlier works on this subject (see Part II below). Fine ends his preface by stating that the two objects of his treatise are the mathematical demonstration of the superiority of the parabolic burning mirror to all others and the description of various methods for actually fabricating such a mirror. It is his boast that in regard to the latter object his own work surpasses that of Witelo.

The work proper is for the most part in the form of a mathematical argument. It begins with twelve definitions on the parts of the right cone and the associated parabolic curve. It next presents four postulates on catoptrics relating to burning mirrors whose purpose is to justify the geometrical treatment of catoptrical phenomena. These are: (Post. I) all solar rays falling on a mirror surface may be treated as straight lines; (Post. II) for any solar ray falling on a flat mirror, the angle of incidence equals the angle of reflection; (Post. III) for any solar ray falling on a concave or convex mirror, the angle of incidence equals the angle of reflection, where these angles are defined with respect to the flat mirror tangent to the given concave or convex mirror at the ray's point of incidence; (Post. IV) it is possible to generate a fire at that point alone at which solar rays, having been reflected from a mirror surface, fall.

There follows a series of geometric propositions, proven rigourously with Euclid's *Elements* serving as the authoritative reference, and culminating in Proposition VII: the angle formed at a given point on the parabolic curve by the tangent to and in the plane of this curve at this point and by a straight line parallel to the curve's axis (Fine terms this axis the *sagitta)* equals the angle formed by the tangent and by a straight line drawn from the given point to tha axis' midpoint (Fine does not assign any term to the axis' midpoint, which is generally called the *focus*). Corollary I to Proposition VII states that the same result will hold also for a parabolic surface produced by revolving the parabolic curve about its *sagitta*. Corollary II asserts that a burning mirror in the shape of a parabolic surfact directly facing the sun will therefore reflect all solar rays to exactly one point. This is the case, Fine explains, because the sun is so much larger than the earth and so far removed from it that the solar rays falling directly on a mirror may be treated as straight lines all of which are parallel to the mirror's *sagitta*. The authority cited in support of these assertions is Alphraganus, according to whom, Fine informs us, the sun is 166 times larger than the earth and at a distance from it equal to 1070 terrestrial radii. But, Fine continues, by Postulate III the angle of incidence of these rays equals their angle of reflection, so that, by Corollary I of Postulate VII, all such incoming rays will fall on one point, namely the midpoint of the *sagitta*. Fine concludes this line of argument in Corollary III, which states that the parabolic mirror is therefore the best possible.

Proposition VIII sets forth various methods for drawing a parabolic line, The most interesting method from the viewpoint of the mathematician (but not an original one -- see Part II) is based on Proposition IV, in which it was proven that any straight line perpendicularly intersecting the *sagitta* of a parabolic curve and bounded by the curve (Fine calls such a line a *linea ordinis sagittae*) is the mean between the *sagitta* itself and the distance from the point of intersection to the parabola's *vertex*. But, Fine observes, the straight line perpendicular to the diameter of a semicircle and bounded by the diameter and the semicircular curve is also a mean, in this case between the two lengths into which the perpendicular splits the diameter. Therefore, the following procedure may be used to construct an approximation to a parabolic curve of given sagittal length, say $\overline{AB}$: extend straight line AB through B to a point C so that $\overline{AC}$ is double $\overline{AB}$; mark off lengths $\overline{BX}$, $\overline{BY}$,..., $\overline{BZ}=\overline{BC}$ along BC from B to C; draw semicircles with diameters AX, AY,...,AZ; draw a straight line through B to AC, and mark off the points F, G,..., H at which the semicircles with diameters (respectively) AX, AY,..., AZ intersect this straight line; the resulting lengths $\overline{BF}$, $\overline{BG}$,..., $\overline{BH}$, being the means between (respectively) $\overline{AB}$ and $\overline{BX}$, $\overline{AB}$ and $\overline{BY}$,..., $\overline{AB}$ and $\overline{BZ}$, when doubled will serve therefore as *lineae ordinis sagittae* of a parabolic curve with *sagitta* AB when spaced along AB at distances $\overline{BX}$, $\overline{BY}$,..., $\overline{BZ}$ from A to B. Fine adds that the smaller the intervals $\overline{BX}$, $\overline{BY}$, ..., $\overline{BZ}$ are made, the better the approximation becomes.

Proposition IX draws on the results of Proposition VIII in order to present methods for fabricating a parabolic mirror. One way to do this, Fine observes, is to make a cutting instrument whose edge is in the shape of the parabolic curve found by Proposition VIII and then to apply this instrument in a revolving motion to a block of metal so as to hollow out from the block a parabolic surface. This surface may then be polished into a mirror by rubbing it with pulverized stone.

Proposition X is not concerned with burning mirrors but with the properties of certain conic sections. It states that on a conic surface may be described two lines whose mutual distance decreases as the lines are extended yet which never touch. One of these lines is the straight line produced by intersecting the right cone with a plane parallel to and including the cone's axis, while the other is the curved line produced by intersecting the cone with a plane parallel to but not including the cone's axis. Fine uses neither the term *hyperbola* to describe the latter, nor the term *asymptoticitas* to describe its relation to the former.

II. SOURCES

Among the sources for *De Speculo Ustorio* the most important, as Fine stated in the work's preface, is Witelo's *Perspectiva,* Book IX, of which Propositions 33-44 were the close models for Fine's Propositions I-VII and IX. Witelo's *Perspectiva* first appeared in print in 1535 at Nuremberg. However, Fine's familiarity with the work clearly predated its first printing, because he referred to it in Book II, Chapter 10, of his *De Geometria Libri II,* which was written in 1530 and first printed at Paris in 1532. The other source on burning mirrors cited in the preface as an unspecified poorly translated treatise by an anonymous Arab author was very likely *De Sectione Conica... Quae Parabola Dicitur, deque Speculo Ustorio Libelli Duo,* printed at Louvain in 1548, whose title-page describes it as the work of an anonymous Arab scientist. It is in fact an edited version of Alhazen's *De Speculis Comburentibus*. Apollonius' *Conics*, also mentioned in the preface and the source for Fine's Proposition X, was perhaps known to Fine in its first printing at Venice in 1537 in a Latin translation of Books I-IV by Joannes Baptista Memus. Fine probably knew it through other works, however, including Witelo's *Perspectiva*, Book I, and also through two works not referred to in *De Speculo Ustorio,* namely Georgius Valla's *Libri VI de Geometria,* printed in 1501 at Venice as part of the same author's *De Expetendis et Fugiendis Rebus Opus,* and Johann Werner's *Libellus super 22 Elementis Conicis* (Nuremberg, 1522). Fine's familiarity with Valla's work is attested by the fact that Valla's treatment of the quadrature of the circle appearing in *Libri VI de Geometria* was extracted by Fine for inclusion in the appendix to his edition of Reisch's *Margarita Philosophica* (1st printing Basel, 1535). Valla's work not only gave the first printed summary of Books I-III of Apollonius' *Conics* but also a brief treatment of the principles of optics and catoptrics. There is no mention of Werner's *Libellus* in any of Fine's works, but Fine's method for drawing a parabolic line set forth in Proposition VII of *De Speculo Ustorio* is taken directly from Werner's *Libellus,* Book IX, where this particular construction appeared for the first time [Coolidge 1945, 26-27]. [3] Therefore Fine's boast in the preface that his own work surpasses that of Witelo's by offering a method for actually drawing parabolic lines, thereby making possible the fabrication of a parabolic mirror, becomes rather hollow.

III. ROLE

It is clear from the above that *De Speculo Ustorio* was not an original contribution to mathematical optics. It may be said in its defence that the first half of the sixteenth century was not a creative period in this area of mathematical science [Ronchi 1960, 49]. But it is more to the point to observe that *De Speculo Ustorio* is best understood and evaluated not as a contribution to the mathematical arts but to their revival in France.

Quadrivial studies were in an extremely backward state at Paris as well as elsewhere in France in the early years of the sixteenth century. [4] A turning point came when Francis I, in furtherance of the efforts of the humanists in behalf of the restoration of the full cycle of the liberal arts, established the Royal College, and, in 1532, appointed Oronce Fine to its chair of mathematics. Fine had already made a reputation for himself as a leader in the revival of French mathematics. Following in the steps of Lefèvre D'Etaples, the outstanding figure in the very earliest phase of the rebirth [5], Fine had brought out editions of standard quadrivial textbooks, for example, Peuerbach's *Theoricae Novae Planetarum* (1st printing Paris, 1525), and Sacrobosco's *Mundialis Sphaerae Opusculum* (1st printing Paris, 1527). With the appointment to the royal professorship, Fine was able to expand his activities and to give them more authority. His work as editor consequently became more substantial. In his years as royal professor he prepared an *editio princeps* of Roger Bacon's *De Mirabili Potestate Artis et Naturae (= De Philosophorum Lapide Libellus)* (1st printing Paris, 1542), and a new edition of Gregor Reisch's *Maragarita Philosophica* (1st printing Basel, 1535), whose appendix was enriched for students of the mathematical arts by the inclusion of quadrivial works (e.g., excerpts from Valla's *Libri VI de Geometria*, Bovillus' *De Quadratura Circuli* and *Introductio in Scientiam Perspectivam*, and Nicolas of Cusa's *De Quadratura Circuli)*. His finest achievement in this area, however, was his edition of Euclid's *Elements*, Books I-VI (1st printing Paris, 1536), with text not only in Latin but also in Greek (drawn from Symon Grynaeus' *editio princeps*, published in Basel in 1533), to which Fine added proofs and commentary.

Fine also composed works of his own in each of the principal divisions of the quadrivial curriculum -- geometry, arithmetic, astronomy, cosmography, and music. These works were read throughout Western Europe into the seventeenth century, going through numerous printings in France, England, Spain, Germany, and Italy, in Latin, French, English, and Italian. They owed their popularity in large measure to the need they met among university students for sound, up-to-date quadrivial textbooks. On this point, the following data are relevant: Fine's *De Sinibus Libri II* (1st printing Paris, 1542) was the first trigonometry printed in France; his *Epithoma Musicae Instrumentalis ad Omnimodam Hemispherii seu Luthina et Theoreticam et Practicam* (Paris, 1530) was the first work printed in France on instrumental music; *De Speculo Ustorio* was the earliest printed catoptrical treatise, and one of the very first printed optical treatises, in France (see below). Fine's textbooks owed their popularity in part to the excellence of their execution. Generally speaking, they were written in simple and clear Latin or French, printed in readable type, and made additionally useful by many excellent diagrams done by Fine himself, who was distinguished in his time for his work as a book illustrator [Mortimer 1964, *passim.*]. Finally, Fine's textbooks were praiseworthy for the logic of their arrangement and the rigor of their demonstrations -- in this, his model and guide was Euclid's *Elements*.

Fine was not merely a textbook writer, however. In one area in particular, cartography, he ranked among the outstanding figures of his age, in the estimate not only of his contemporaries but now also of modern historians of science. His contributions to cartography included many fine maps of France and of the world (the latter incorporating the latest information from the overseas explorations of the Portuguese and Spanish, and set apart by their novel projections, e.g., cordiform and double cordiform), and a number of terrestrial and celestial globes. This work influenced activity in cartography throughout Western Europe. John Dee, Elizabethan England's leader in this study until the time of Harriot, traveled to Paris in order to study with Fine, and one of Mercator's early works was a copy of a globe made by Fine.

Fine not only encouraged the mathematical arts by means of his many excellent textbooks and by virtue of his reputation as one of Europe's outstanding cartographers, but also by linking his work to one of the principal intellectual movements of the age, humanism. His Greek-Latin edition of Euclid's *Elements* may be viewed from this perspective. So may his musical treatise *Epithoma Musicae Instrumentalis,* whose subject matter was the theory and practice of playing the lute, the instrument the French humanists considered the most appropriate for realizing compositions in the antique style [Heartz 1967, 1-204; Carpenter 1958, 140-153]. Fine's efforts in behalf of the revival in France of the liberal arts bearing on mathematics did not go unnoticed by leading early French humanists. Dorat, for example, composed a poem in Greek to serve as the dedicatory introduction to the posthumous edition of Fine's *De Rebus Mathematicis, hactenus Desideratis* (Paris, 1556). Finally, the appointment to the royal professorship made Fine's ties to humanism and to its leadership official.

For all these reasons Fine was able to stimulate to a significant degree the revival of interest in the mathematical arts among university students at Paris and throughout France. Contemporary biographers of Fine [Thevet 1584; Mizauld 1556] tell us that his lectures were unusually well attended, not only by students but by foreign and native dignitaries and scholars. We are further told by these biographers that Fine's efforts even extended to remaking his private apartment into an informal center for mathematical studies at which any student, dignitary, scholar, or navigator could talk with Fine or use his library, maps and globes, and various scientific instruments. Here it should be mentioned that in addition to his other talents Fine was a skilled instrument-maker. One of his products, a planetary clock (that is, a moving model of the heavens), is extant and on display at the Bibliothèque de Sainte-Geneviève in Paris.

The above remarks should help to put Fine's *De Speculo Ustorio* in proper perspective. The work was unoriginal, not because Fine was incapable of creativity but because it was written with the intention of rounding out the series of quadrivial textbooks for university students he had been engaged in composing since his appointment to the royal professorship. Such a work was certainly needed in France at this time. My own researches have uncovered only four treatises on mathematical optics printed in France prior to *De Speculo Ustorio*. Of these only one was by a Frenchman, Carolus Bovillus' (= Charles de Boulles) *Introductio in Scientiam Perspectivam* (1st printing Paris, 1503). It has already been mentioned that Fine renewed this work's availability by incorporating it into the appendix of his edition of Reisch's *Margarita Philosophica*. The other three works were Peckham's *Perspectiva* (undated, but an incunabulum), Viatoris' *De Artificiali Perspectiva* (1509), and Alberti's *De Re Aedificatoria* (1512). All four of these works were on perspective, thus making Fine's *De Speculo Ustorio* the first book on catoptrics to be printed in France.

NOTES

1. For a bibliography of works on Oronce Fine see Ross 1975, fn. 1. Except for a few additional works referred to in the present paper, that bibliography covers all the aspect of Fine's career touched on by this paper.
2. The title-page to the first printing (Paris, 1551) of *De Speculo Ustorio* reads as follows: *De Speculo Ustorio, Ignem ad Propositam Distantiam Generante, Liber Unicus. Ex quo Duarum Linearum semper Approprinquantium, & nunquam Concurrentium Colligitur Demonstratio. Orontio Finaeo Delphinate, Regio Mathematico Authore. Lutetiae, ex officina Michaelis Vascosani, Via Iacobaea, ad Insigne Fontis. M.D.LI. Cum Privilegio.* This edition, a quarto in 50 pages, was published bound with a reprint of another work by Fine, *Sphaera Mundi*. The title-page to the 1587 Italian translation, printed in Venice, reads: *Opere di Orontio Fineo dal Delfinato divise in cinque parti; arithmetica, geometria, cosmografia, e orivoli, tradotte da Cosimo Bartoli... Et gli specchi tradotti dal Cavalier Ercole Bottrigaro... Nuovamente poste in luce... Venetia, Presso Francesco Franceschi Senese, 1587.* This Italian translation was reprinted in 1670, with the same title-page as the 1587 printing, by two different publishers, Combi & La Noù and G. G. Hertz, in Venice. The Italian version is

identical in content to the original Latin version. In the present paper all references are to the original Latin version.

3. This was not the only time Fine copied Werner without giving credit. Another such instance is given in Nordenskiöld [1889, 90].
4. For a bibliography of works on French Renaissance mathematics consult May [1973, 642-644].
5. The one great exception to the low level of mathematical activity in Renaissance France was Chuquet's *Triparty*, composed in the 1480's. But this work was not printed until the 19th century and had no effect on the course of development of French mathematics.

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