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LIMITS OF PERFECTION

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Abstract—This personal narrative is an introduction to a collective effort by a number of scientists and artists to examine the role and significance of symmetry in the most diverse domains of nature and human activity. Material symmetry, devoid of the rigor of geometrical symmetry, is viewed applicable to material objects as well as abstractions with limitless implications.

To mark the 350th anniversary of Johannes Kepler's death, the Hungarian Post Office issued a beautiful memorial stamp (Fig. 1). Next to Kepler's portrait his famous model of the planetary system is shown. This was fitting since of all of Kepler's discoveries this is the best known to the general public, although it is viewed by some as his most spectacular failure[1].

A closer look at Kepler's activities, however, justifies the selection of the Hungarian Post Office. Although he is most famous for his three laws of heavenly mechanics, there is another piece of work that was also a milestone in a different branch of science, crystallography. If one is astonished by the depth of his understanding of the physics of the sky with the then available data, it is not less astonishing that Kepler could discuss the "atomic" arrangement in crystals two hundred years before Dalton and three hundred years before X-ray crystallography began. In his new year's gift of the hexagonal snowflake Kepler[2] not only examines the hexagonal symmetry of the snow crystals but lays the foundation of the principle of densest packing in crystal structures. Densest packing is then, of course, the key to the symmetry of crystal habit. The planetary model from the regular solids is also a densest packing model. Kepler's search for harmony was the bridge between his two lines of activities. Although the snowflake paper seems to be almost an accident on the background of his astronomy, the Hungarian stamp gave more credit to the *complete* Kepler than was probably envisioned by the planners of the stamp themselves.

According to the regular solids model, taking the *six* planets, known to Kepler, in order, the greatest distance of one planet from the Sun stands in a fixed ratio to the least distance of the next outer planet from the Sun. There are, of course, conveniently five ratios for the six planets. A regular solid can be interposed between two adjacent planets so that the inner planet, when at its greatest distance from the Sun, lies on the inscribed sphere of the solid, while the outer planet, when at its least distance, lays on the circumscribed sphere.

There are molecular structures which can be best described by polyhedra enveloping other polyhedra. The structure of $[Co_6(CO)_{14}]^{4-}$ is shown is Fig. 2: an omnicapped cube of carbonyl oxygens envelopes an octahedron formed by cobalt atoms[3].

One of today's most successful models in structural chemistry is based on extremely simple considerations of space distribution. The valence shell electron pair repulsion (VSEPR) model[4] postulates that the geometry of the molecule is determined by the space requirements of the electron pairs in the valence shell of its central atom. The bond configuration around atom A in an AX_n molecule is such that the electron pairs in the valence shell be at maximum distances from each other. Thus the arrangement may be visualized so that the electron pairs occupy well-defined parts of the space about the central atom. In a different concept, these space segments are called localized molecular orbitals.

It is easy to demonstrate the three-dimensional consequences of the VSEPR model. Only a few balloons have to be blown up and connected at their narrowing ends in groups of two, three or four[5]. The linear, equilateral triangular, and tetrahedral arrangements of these assemblies are what the VSEPR model predicts for the electron pairs. Another beautiful analogy is found on walnut trees[6]. As two, three, or four walnuts grow sometimes together, the above

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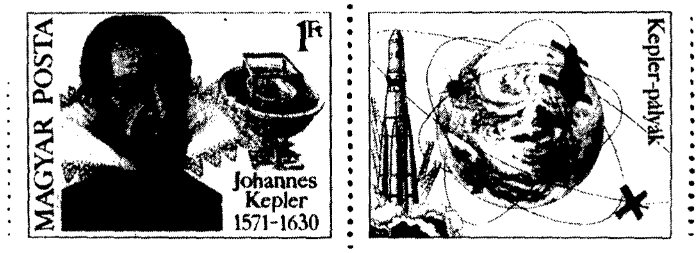


Fig. 1. Kepler memorial stamp. (Hungarian Post Office, 1980.)

arrangements occur unfailingly (see Fig. 3). The soft balloons and hard walnuts may even be viewed as representing weaker and stronger interactions and thereby represent even more subtle analogies for molecular structure. Can molecular geometry be so simple? There is obvious oversimplification in the model. Of the many effects determining molecular structure, one is taken into account and all the others are ignored. The model is applicable where this particular effect, to wit the repulsions of the electron pairs due to their space requirements, is dominant. The VSEPR model is successful because this effect is important enough in extensive classes of compounds.

In comparing the complexity of planetary motion and molecular structure, the real analogy is in the possibility of selecting a dominant effect and ignoring the others. This approach works much better for the planetary motion where the dominant effect is the gravitational attraction of the Sun while the others are perturbations. In the world of molecules, the dominant effect may change from one compound to another. In spite of the tremendous amount of accumulated knowledge about molecular structure, its basic principles are still being clarified. One of the characteristics of the forces keeping the molecule together is that they are very strong, whereas the gravitational forces are very weak. To discover the law of gravitational interactions, the observations had to be made on a large scale. The laboratory was the planetary system itself.

As we compare the symmetries of molecules and crystals, a striking difference is that there are no limitations for molecules and there are well-defined limitations for crystals. A consequence is the finite number (32) of symmetry classes for crystal habit with no such limits for molecules.

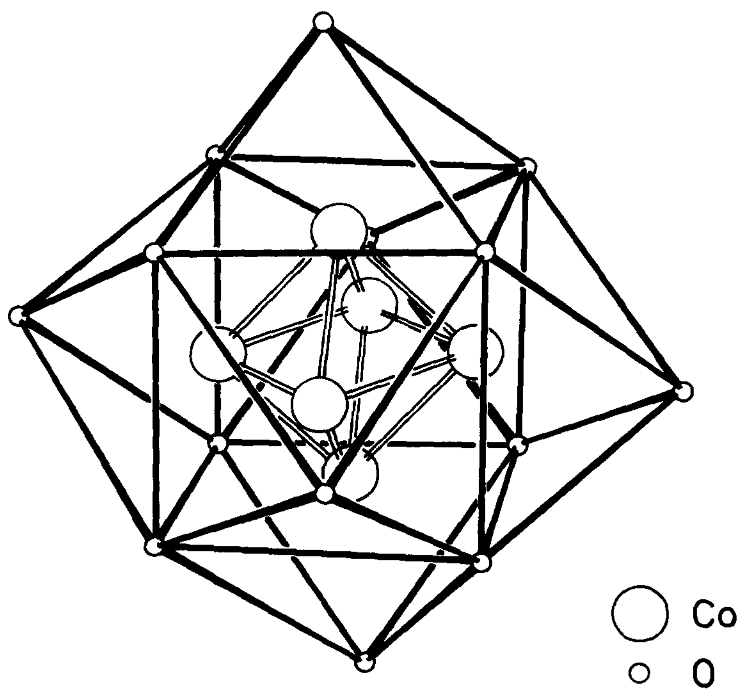


Fig. 2. The structure of $[Co_6(CO)_{14}]^{4-}$, the omnicapped cube of the carbonyl oxygens envelopes the cobalt octahedron. (After[3].)

Fig. 3. Walnut clusters with two, three, and four walnuts with linear, equilateral triangular, and tetrahedral arrangements. (Photographs by the author following the idea of Niac and Florea[6].)

The molecules are a more fundamental building unit in the hierarchy of structures than the crystals and many crystals themselves are built from molecules. Unfortunately, molecules are usually not to be seen by the naked eye whereas the crystals are. They are so appealingly symmetrical that they have become a sort of idol for symmetry.

For the Béla Bartók centenary a couple of years ago (1981), Victor Vasarely produced a limited edition of ten serigraphs created for ten of Bartók's musical pieces. Each serigraph was accompanied by a poem, each written by a different living Hungarian poet. More than once the word *crystal* was related to Bartók's music. If one considers their symmetry properties in a strictly technical sense, however, they could not be farther from each other. Bartók's music is interweaved throughout by the Fibonacci numbers and the golden section. These symmetries characterize for example the scattered leaf arrangements of many plants and are omnipresent in other domains of the animate world as well. On the other hand, "crystallization is death", as crystallographers themselves like to point out. Incidentally, pentagonal symmetry is conspicuously present in primitive organisms and crystallographer Nikolai Belov[7] suggested that it was their means of self-defense against crystallization.

The restrictions on crystal symmetry start with the fact that, strictly speaking, it exists only theoretically. Its main characteristic is translational symmetry, i.e. *infinite* periodic repetition, and in reality, of course, crystals always end somewhere; but apart from this, there is no five-fold symmetry axis, nor are there axes with higher order than six. Considering the more easily visualized two-dimensional surface of the regular polygons, only the equilateral triangle, the square and the regular hexagon can be used to completely cover an area without gaps, whereas the regular pentagon, heptagon, etc., can not. Similar limitations extend to the third dimension.

In spite of the limitations in crystal symmetry, the crystals have a unique appeal and are widely used also in analogies. Quote the Czech writer Karel Čapek[8] on his visit to the mineral collection of the British Museum

> . . .But I must speak again about crystals, shapes, colors. There are crystals as huge as the collonade of a cathedral, soft as mould, prickly as thorns; pure, azure, green, like nothing else in the world, fiery, black; mathematically exact, complete, like constructions by crazy, capricious scientists, or reminiscent of the liver, the heart. . . There are crystal grottos, monstrous bubbles of mineral mass, there is fermentation, fusion, growth of minerals, architecture and engineering art. . . Egypt crystallizes in pyramids and obelisks, Greece in

Fig. 4. Illustration for phyllotaxis. (Photograph by the author.)

> columns; the middle ages in gilly-flowers; London in grimy cubes. . . Like secret mathematical flashes of lightning the countless laws of construction penetrate the matter. To equal nature it is necessary to be mathematically and geometrically exact. Number and phantasy, law and abundance—these are the living, creative strengths of nature; not to sit under a green tree but to create crystals and to form ideas, that is what it means to be at one with nature!

This is then exactly the point where Bartók and the crystals meet. While the composer invariably refused to discuss the technicalities of his work, he liked to state "We create after Nature" and he meant it literally.

As a crystal is being built from molecules, and the energetically most favorable arrangement is being achieved, the molecules come into close, touching range with each other. Compared with the free molecules, their interactions may have perturbing effects on their structures. One of the simplest consequences may be the lowering of their original symmetry. Discussing the structural consequences of densest packing in molecular crystals, the following explanation is attributed to crystallographer Aleksandr Kitaigorodskii, "The molecule also has a body. When this body is hit, the molecule feels hurt all over." This analogy emphasizes the importance of spatial requirements of the molecules in building the molecular crystal rather than the peculiarities of its electronic structure which would be of greater importance in a more chemical behavior as in a chemical reaction. Personifying the molecule obviously had an appeal to the scientist, and it also has an appeal for children who generally like to do so with various objects. When some years ago the author's daughter was asked in the kindergarten about her father's occupation, she said he cured sick molecules. The tendency to make metaphors, however, seems to diminish with coming of age. Some valuable things may be lost in the educational process. It is much easier to get through to children the notion that there is much more to symmetry than what we call bilateral symmetry. Adults seem to be more narrow-minded and more indoctrinated.

Not everything is perfect, however, even with the most symmetrical molecules. Concerning their symmetry, that is. When the symmetry of a molecule is described, it is usually the motionless, frozen molecule that is meant. This structure would correspond to the minimum energy. The molecules are never motionless, however. Even if they could be cooled to the absolute coldest temperature of 0 K or -273.16°C, they would not come to a standstill.

The molecular vibrations often lead to some instantaneous distortion or lowering of the molecular symmetry. This is true for the relatively rigid molecules and even more so for the very flexible molecules. Intramolecular motion may even lead to a continuous permutation of the atoms in the molecule.

Imagine a ring molecule of five atoms with four being in one plane and the fifth atom sticking out of the plane. This arrangement lowers the symmetry of the five-member ring to a

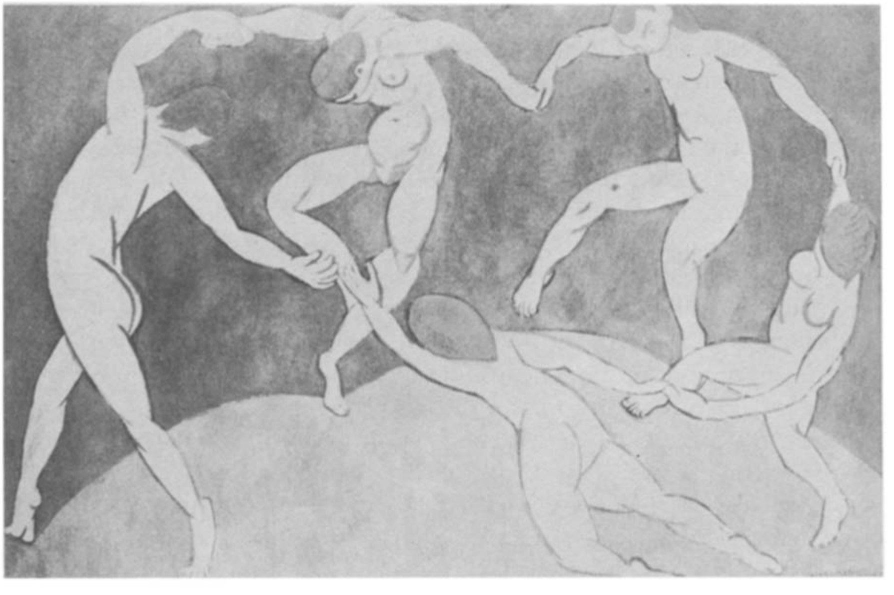


Fig. 5. Henri Matisse: Dance. (The Hermitage, Leningrad.) Reproduced by kind permission from The Hermitage.

mere symmetry plane. The point is that this molecule may be performing such intramolecular motion that during every second a million times or more the sticking out position switches from one atom to the next and to the next and so on.

Consider now five dancers in a circle instead of the five atoms (see Fig. 5). Let them make a jump one after the other in a quick succession. If we take photographs with very short exposures, we can catch various configurations, including even the most symmetrical one in which all the five dancers are on the ground. On the other hand, a longer exposure leads to a blurred picture all around the circle. The apparent symmetry of the dancing group obviously depends on the length of the exposure used, and also, of course on the speed of their movement.

The molecular structures and crystal structures represent two well-separated cases from the point of view of their symmetry properties. The molecule is characterized by point-group symmetry as it has at least one unique point in the whole structure. Crystal structures are characterized by space-group symmetry or translational symmetry as they have no unique point in their structure. When point-group and space-group symmetries are compared, it is not obvious how to distinguish between higher and lower symmetries. Within each domain, however, and the molecules and crystals are merely examples, there is a hierarchy of symmetries.

Increasing the symmetry beyond some limits may lead, however, to sterility and certainly diminishes the information content. Scientific instruments with ever increasing perfection may filter out important peculiarities which do not conform with the general pattern. Perfect symmetry may be aimless, and it irritates many. Perhaps symmetry considerations could facilitate relating the perception of structures and the world of emotions? On the level of analogies this seems to be possible as is illustrated by a poem by Ann Wickham[9]:

GIFT TO A JADE

For love he offered me his perfect world.
This world was so constricted and so small
It had no loveliness at all,
And I flung back the little silly ball.
At that cold moralist I hotly hurled
His perfect, pure, symmetrical, small world.

Geometrical symmetry is strict: it allows for no "degrees" of symmetry. Something is either symmetrical or not. What may be called *material symmetry*, on the other hand, implies a continuous spectrum of the degrees of symmetry. The term material symmetry here refers generally to non-geometrical symmetry and may be applied to real material objects as much as to any abstraction.

The human face is an obvious example of bilateral symmetry. However, none of us has a perfectly symmetrical face (Fig. 6). It may be a matter of flattery on the part of the painter to show more perfect symmetry than there is, or this may even be demanded by the paintee. Old religious paintings or contemporary political portraits may show more facial symmetry than there is. Any personality cult produces very symmetrical face images. However, minor asymmetries may have a strong appeal and the notion about the beauty of a face may also be changing. There was a minor uproar some time ago when a Budapest theater critic praised the acting of an actress and remarked also on her pretty, *modern* face. In fact, hers was conspicuously asymmetric, so people were wondering whether their faces were modern or old-fashioned. There is considerable interest currently in the origin and meaning of facial asymmetry.

Another fascinating symmetry/asymmetry relationship exists between our hands (Fig. 7). Distinguishing between left and right has definite connotations in almost all fields of human activity. The importance of chirality is ever growing in the sciences. Even human attitude toward handedness is evolving. Figure 8 shows two pictures from classrooms of the University of Connecticut. The older classrooms of the Chemistry Department have homochiral chairs only, designed for right-handed students. The more contemporary classrooms of the Mathematics Department are furnished with heterochiral chairs to accommodate both the right-handed and the left-handed students.

Fig. 6. Eszter Hargittai in front of a shop-window, 1984. (Photograph by the author.)

Fig. 7. Tomb in the Jewish cemetery, Prague. (Photograph by the author.)

Fig. 8(a). Classroom in the Chemistry Department, University of Connecticut, 1984.

Fig. 8(b). Classroom in the Mathematics Department, University of Connecticut, 1984. (Photographs by the author.)

Artistic creation as reflection may vary in a limitless range. The two photographs in Fig. 9 illustrate this point. Construction machinery on the bank of the Danube is shown in one of the photographs. Its reflection in the Danube is captured in the other. Imagine a mirror-perfect water surface with much more likeness, or gale conditions destroying any trace of resemblance. The origin of reflection itself as seen by Jean Effel is depicted in Fig. 10.

The sciences of the 20th century have opened up a new world, one which had previously been inaccessible to man's instruments, let alone his senses. To parallel this development, artistic expression is coping, or is at least attempting to do so, in reflecting our world on an

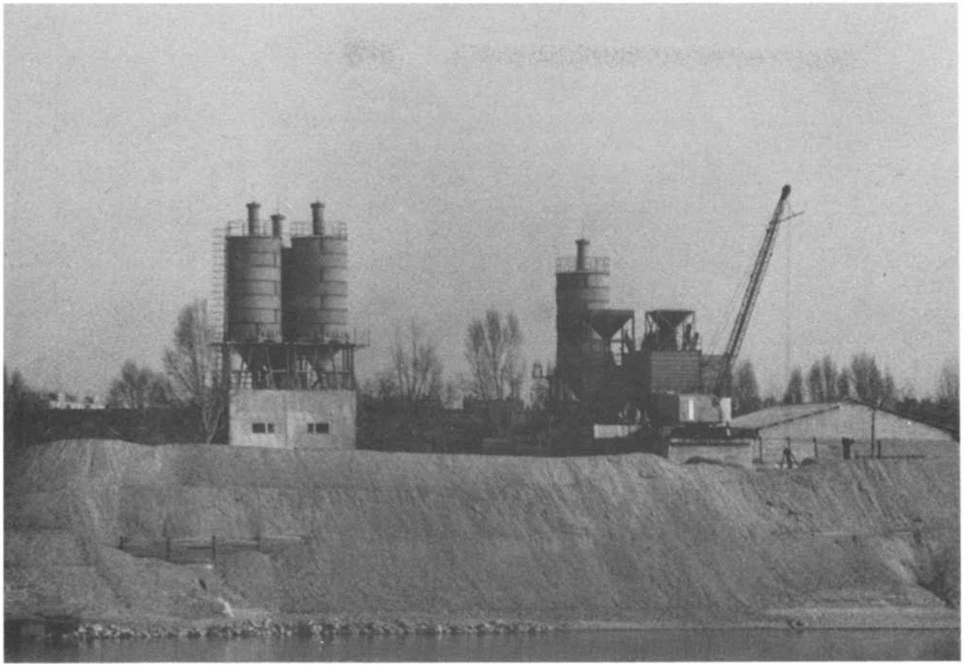


Fig. 9(a). Building construction machinery on the bank of the Danube.

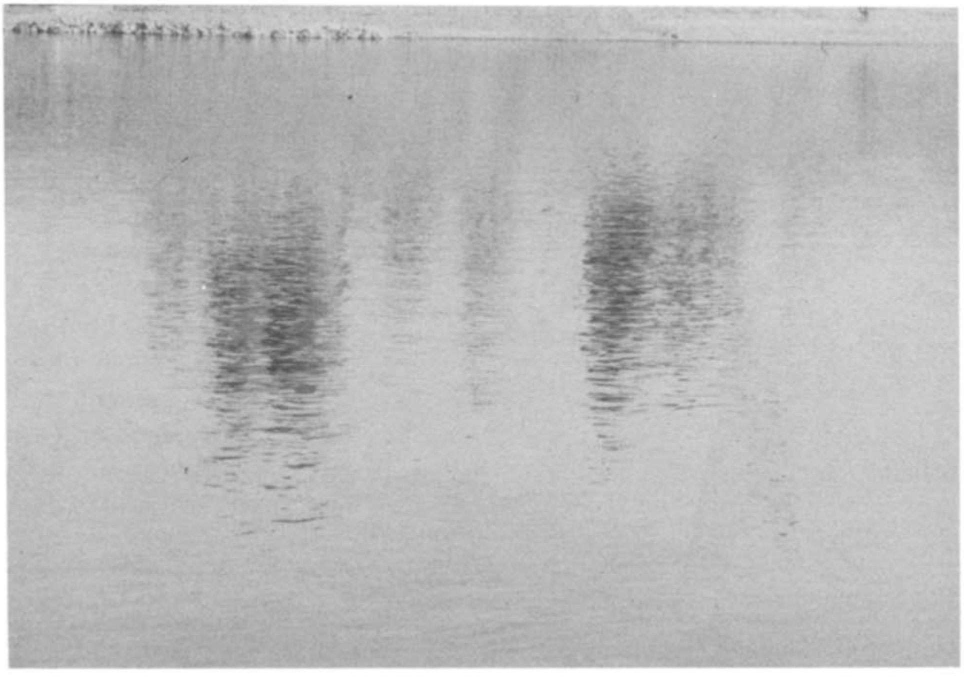


Fig. 9(b). Reflection of the building construction machinery in the Danube. (Photographs by the author.)

entirely new level. It is not that the artist is expected to bend over the screen of an electron microscope and paint "after nature", but the newly discovered domains and phenomena must and do find their reflections in artistic expression. It may be considered symbolic that X-ray crystallography and the Black Square of Kazimir Malevich were born about the same time. The picture from 1915 is reproduced in Fig. 11 along with the title page of a later Malevich work.

Current progress mandates the expansion of the well-established frameworks of the symmetry concepts. One of the cradles of modern symmetrology, crystallography is transforming itself to embrace all structural science on the atomic level[10]. Liquids, colloids, amorphous

Institution du reflet

— Les riverains auront, gracieusement, leur portrait à l'aquarelle...

Fig. 10. The creation of reflection, by Jean Effel. [La Creation du Monde]. ("Those who dwell by the river will have their portrait, gracefully, in water colors. . ."). (Reproduced by kind permission from Mme. Jean Effel.)

solids cannot be put into the existing "perfect" systems even though they are not without structure. We are witnessing their emancipation. We quote Ann Wickman[9] again:

THE WOMAN AND HER INITIATIVE

Give me a deed, and I will give a quality.
Compel this colloid with your crystalline.
Show clear the difference between you and me
By some plane symmetry, some clear stated line.
These bubblings, these half-actions, my revolt from unity.
Give me a deed, and I will show my quality.

John Bernal was the pioneer of generalized crystallography, and Belov noted in his obituary ". . .his last enthusiasm was for the laws of lawlessness." Did Ann Wickham's metaphors parallel Bernal's discoveries? Although several disciplines apart, geographically and chronologically they operated in close range (London).

In any case the non-classical, irregular, unstable, unusual, unexpected are gaining impor-

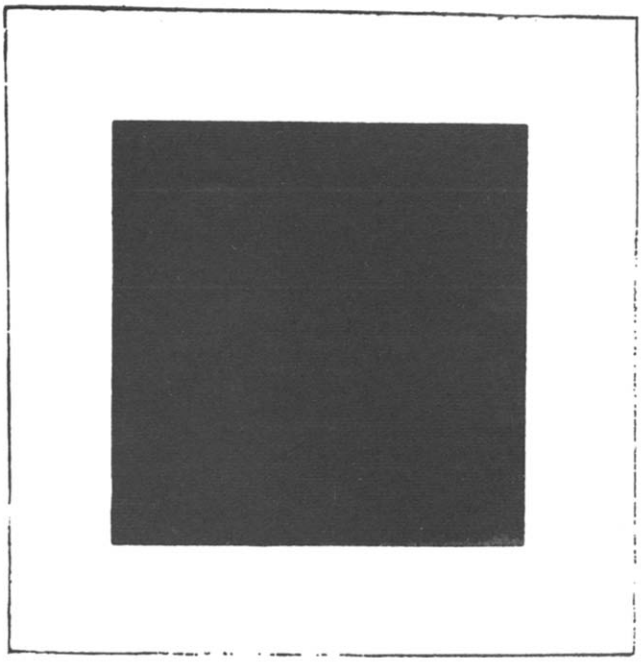


Fig. 11(a). Kazimir Malevich: Black square (1915?).

Fig. 11(b). Kazimir Malevich: Suprematism, 34 drawings, UNOVIS, Vitebsk, 1920, Title page.

tance in the sciences. The "morphology of the amorphous" is being investigated and "mapped"[11]. The 1978 Nobel laureate physicist Philip Anderson stated that "the next decade is very likely to be the most 'disordered' decade in theoretical physics."[12]

The increasingly recognized importance of non-periodic structures makes even more conspicuous the absence or near-absence of the three-dimensional space groups outside the world of crystals. The honeycomb built by the bees is a notable exception. Its regular hexagonal structure is then partially copied by the construction of the concrete base of off-shore oil platforms. There is unmistakable similarity to the honeycomb in the hexagonal joints of basaltic sheets resulting from contraction during cooling. These are examples that meet the eye and there may be others hidden.

How much visible and how much less visible symmetry is there in the arts? There is a lot of visible symmetry in Bach's music and there is also a lot in Bartók's—not so visible but well-established by research. There is a wealth of symmetry phenomena in other music as well as in other arts, not only in paintings and sculptures whose symmetry properties are most commonly considered.

One of the less frequently perceived symmetries is inversion, a combination of applying a two-fold rotation axis and a symmetry plane. Here is an example from Hungarian author Frigyes Karinthy's short story entitled "The same in man"[13] represented by some edited fragments. There are three characters: Bella the beloved lady, Fox the employee and Bella's suitor Sándor who is also Fox's boss. The editing means to present the two meeting in a parallel way rather than consecutively.

BELLA	FOX
Sometimes I just gaze before me without thinking of anything. . .	Sometimes I just gaze before me without thinking of anything. . .
SÁNDOR/BOSS	
Bella! If only you knew how beautifully you expressed yourself. . .	On my money? Then you'd better go to a lunatic asylum, that's where cases like you are treated. . .
BELLA	FOX
Sometimes I have the feeling that I'd like to be somewhere else than I am. I can't say where, somewhere I haven't been before.	I often have the feeling that I'd like to be somewhere else than I am. I don't know where, anywhere, somewhere, I haven't been before.
SÁNDOR/BOSS	
Bella, how true, how wonderful. . . How did you put it? Let me engrave it in the records of my mind. . .	The nuthouse, man, the nuthouse. That's where you belong.

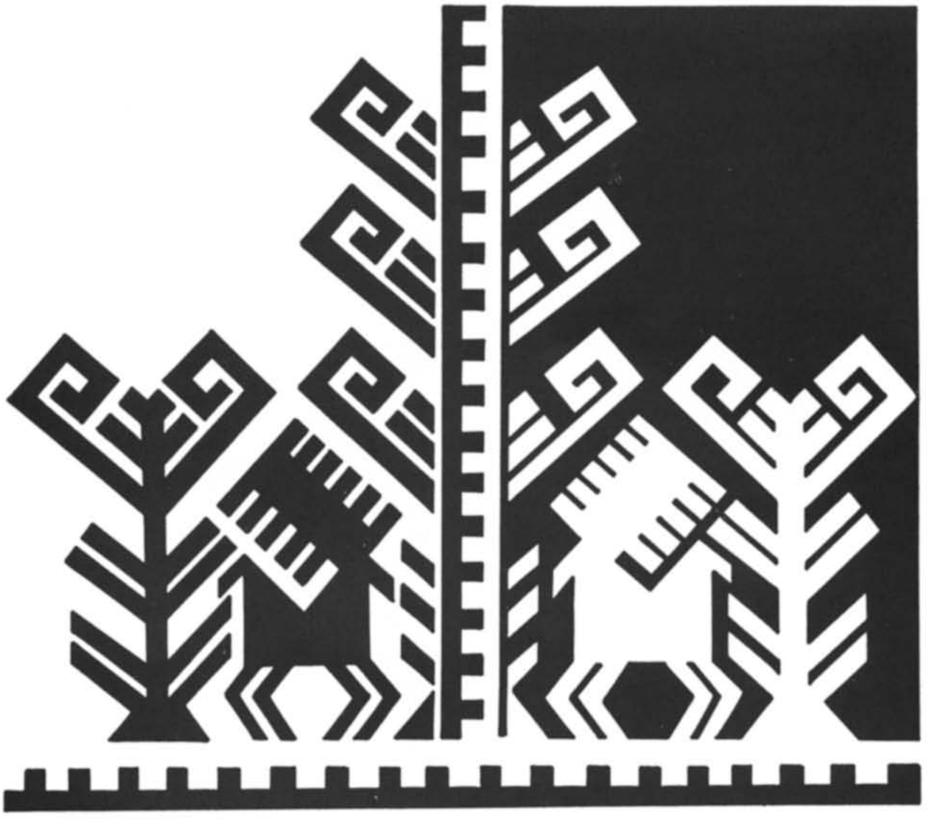
There is inversion in James Reston's description of New Zealand in his Letter from Wellington, Search for End of the Rainbow[14]:

> Nothing is quite the same here. Summer is from December to March. It is warmer in the North Island and colder in the South Island. The people drive on the left rather than on the right. Even the sky is different—dark blue velvet with stars of the Southern Cross—and the fish love the hooks.

Wellington and Madrid are approximately connected by a straight line going through the center of the Earth which is then the inversion center for them. It is too bad that the journalist did not date his letter from Madrid.

The black-and-white variation is the simplest case of color symmetry and it is also the simplest example for antisymmetry (see Fig. 12). The relationship between matter and antimatter is another example. "Operations of antisymmetry transform objects possessing two possible values of a given property from one value to the other"[15]. According to this general definition antisymmetry can be given broad interpretation and application. Geometrically less strict but in their atmosphere truly black and white antisymmetries are presented in Fig. 13.

Another literary example is taken from Karinthy's writing, this time to illustrate antisymmetry. It is edited from a short story entitled "Two Diagnoses"[16]. The same person Dr. Same goes to see a physician at two different places. At the recruiting station he would obviously



(a)

(b)

Fig. 12. Antisymmetries. (a) Hungarian motif after [17]. (b) Zagorsk. (Photograph courtesy of Dr. A. A. Ivanov, Moscow.)

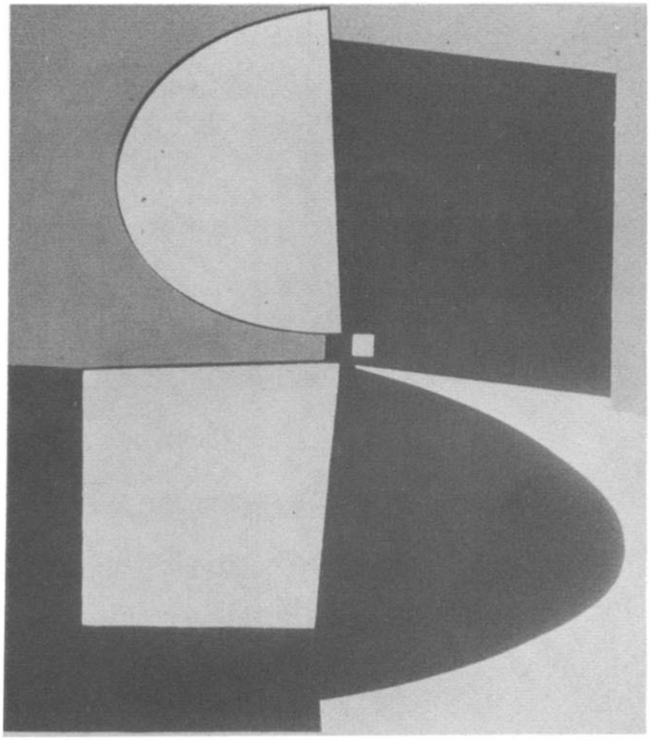


Fig. 13(a). Victor Vasarely: "P62-Basilan", 1951. (Reproduced by kind permission from the artist.)

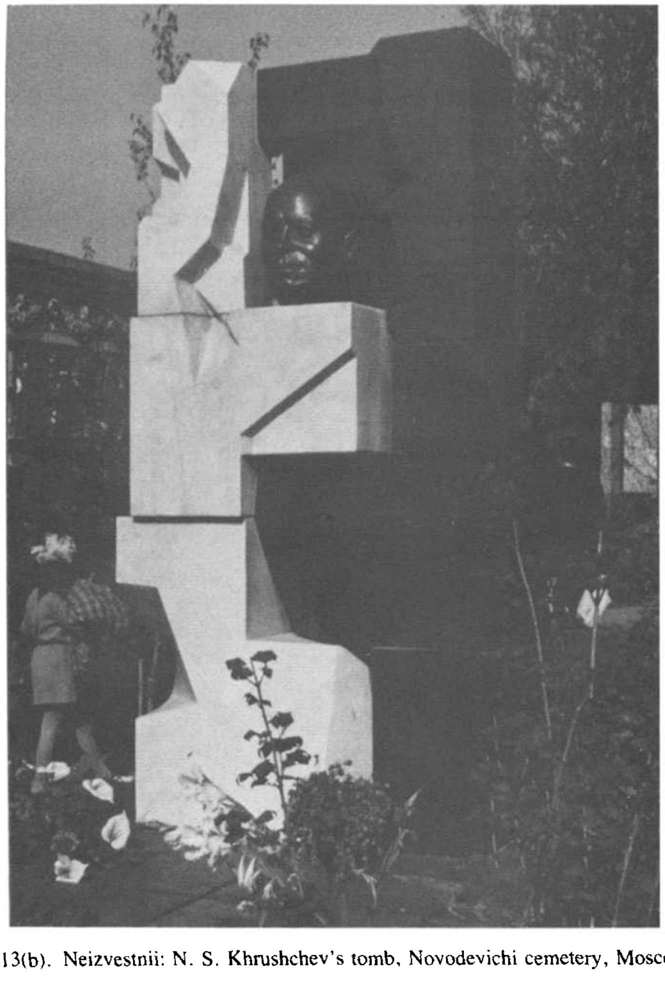


Fig. 13(b). Neizvestnii: N. S. Khrushchev's tomb, Novodevichi cemetery, Moscow.

Fig. 14. Two restaurants in downtown Washington, D. C.: *the* Sans Souci and *a* McDonald's. (Photograph by the author.)

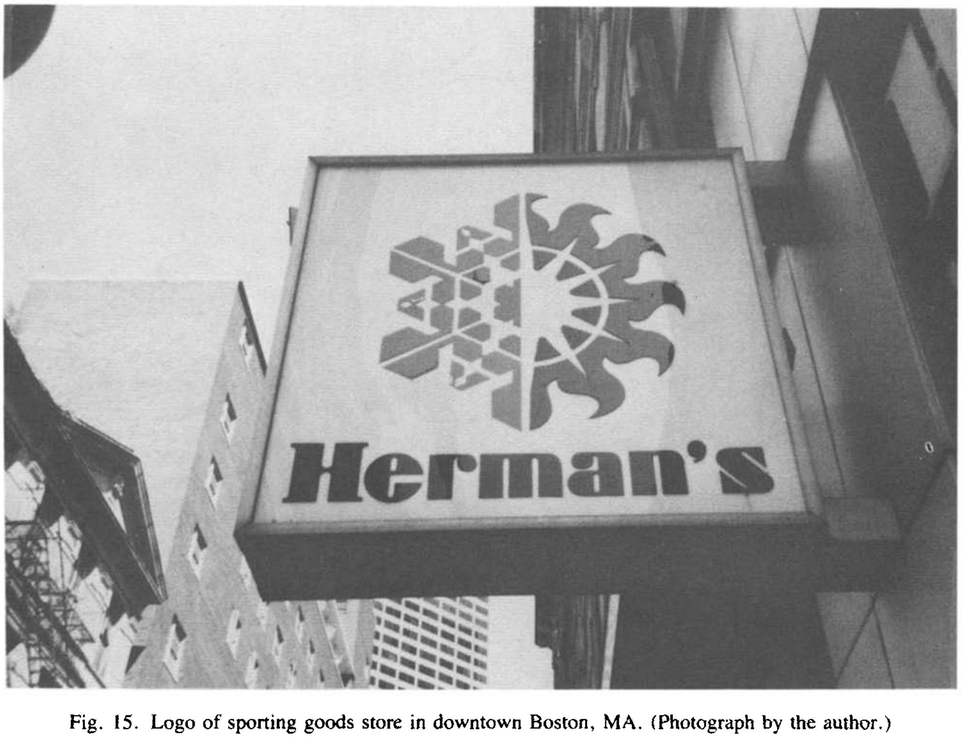


Fig. 15. Logo of sporting goods store in downtown Boston, MA. (Photograph by the author.)

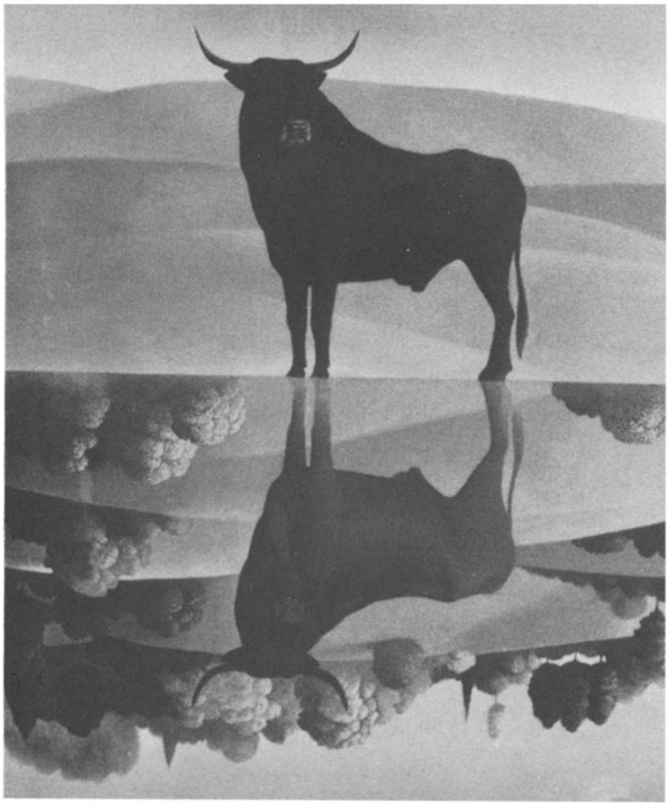


Fig. 16. A breed apart. Poster. (Reproduced by kind permission from Merrill Lynch Company.)

Fig. 17. Military jets and a sea gull, off Bodø, Norway. (Photograph by the author.)

like to avoid being drafted, while at the insurance society he would like to acquire the best possible terms for his policy. His answers to the identical questions of the two physicians are related by antisymmetry.

DR. SAME	PHYSICIAN	DR. SAME
At the recruiting station		At the insurance society
Broken-looking, sad, ruined human wreckage, feeble masculinity, haggard eyes, wavering movement.		*Young athlete with straightened back, flashing eyes.*
	How old are you?	
Old. . . very old, indeed.		*Coyly*. On, my gosh, I'm almost ashamed of it. . . I'm so silly. . .
	Your I.D. says you're thirty two.	
With pain. To be old is not to be far from the cradle—but near the coffin.		To be young is not to be near the cradle, but far from the coffin.
	Are you ever dizzy?	
Don't mention dizziness, please, Doctor, or else I'll collapse at once. I always have to walk in the middle of the street, because if I look down from the curb, I become dizzy at once.		Quite often, sorry to say. Every time I'm aboard an airplane and it's up-side-down, and breaking to pieces. Otherwise, not. . .

Two restaurants stand side by side in downtown Washington, D. C. One is the one-of-a-kind exclusive *San Souci*, the other is a *McDonald's* of the famous fast-food chain. The antisymmetry plane appears physically as a vertical wall between the two restaurants (Fig. 14).

Antisymmetry may be powerful in focusing attention, showing contrast, emphasizing a point. Figure 15 shows the logo of a sporting goods store in downtown Boston. The antisymmetry plane emphasizes that both winter *and* summer sport fans are welcome. A poster of the Merrill Lynch investment company is reproduced in Fig. 16. There is a horizontal antisymmetry plane in ''a breed apart''. Military jets and a sea gull fly on the two sides of an imaginary antisymmetry plane in our ultimate example (Fig. 17).

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