Incentives and justice for sequencing problems.

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INCENTIVES AND JUSTICE FOR SEQUENCING PROBLEMS

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ABSTRACT. We address the mechanism design issue for the sequencing problem. We identify the just sequencing rule that serves the agents in the non-increasing order of their waiting costs and prove that it is a Rawlsian rule. We identify all rVCG mechanisms that implement the just sequencing rule. The other properties of the just sequencing rule that we identify are the following. It is an affine cost minimizer. It can be implemented with budget balanced rVCG mechanisms. Finally, when waiting cost and processing time are private information, we identify all generalized rVCG mechanisms that ex-post implement the just sequencing rule.

JEL Classifications: C72, D63, D82.

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1. INTRODUCTION

In this paper we address the mechanism design issue for the sequencing problem where agents have quasi-linear preferences. The setting comprises of a finite set of agents each of whom has one job to process using one facility. The facility can only handle one job at a time. No job can be interrupted once it starts processing. Each job is characterized by processing time and waiting cost. The latter represents the agent's disutility for waiting one unit of time. There is a well established literature in this direction (see Dolan [10], Duives, Heydenreich, Mishra, Muller and Uetz [11], Hain and Mitra [13], Mitra [25], Moulin [27] and Suijs [33]).

A well-known and well studied concept is the outcome efficient sequencing rule that minimizes the aggregate job completion cost of the agents. Outcome efficiency, as pointed out by Smith [32], requires that the jobs of the agents are processed in the non-increasing order of their urgency index where urgency index of any agent as the ratio of his waiting cost and his processing time. It is well-known that, as long as preferences are 'smoothly connected' (see Holmström [15]), outcome efficienct rules

can be implemented in dominant strategies if and only if the mechanism is a Vickrey-Clarke-Groves (VCG) mechanism (see Clarke [8], Groves [12] and Vickrey [35]). For the sequencing problem outcome efficiency was analyzed by Dolan [10], Mitra [25] and Suijs [33].

The main contribution of this paper is to address the implementability issue of the Rawlsian sequencing rule which is based on John Rawls' principle of distributive justice (see Rawls [30]). The Rawlsian sequencing rule first identifies the maximum agent specific job completion cost for each order of serving and then picks that order which minimizes this maximum agent specific job completion cost. We show that a sequencing rule for which agents are served in the non-increasing order of their waiting costs is a Rawlsian sequencing rule. We refer to this rule as the just sequencing rule and show that this Rawlsian rule is implementable in dominant strategies. We identify all '$rVCG$ mechanisms' that implements the just sequencing rule.

Our result on implementation of the just sequencing rule shows compatibility of incentives and justice. In the mechanism design literature without transfers where preferences of the agents are defined using distance from the bliss points, Chichilnisky and Heal [3] argued that Rawlsian rules are "locally dictatorial" and hence implementable. However, in the mechanism design literature with transfers, this compatibility of incentives and justice is indeed rare. Papers by Deb and Mishra [9] and Lavi, Mu'alem and Nisan, N. [22] show that the Rawlsian allocation is incompatible with implementability in dominant strategies. To the best of our knowledge, the only other paper that shows this compatibility between incentives and justice is by Velez [34] for the (house) allocation problems.[1]

In the mechanism design literature a classic contribution is due to Roberts [31] who proved that, with unrestricted domains, if an allocation rule is implementable in dominant strategies, then it must be an 'affine maximizer' allocation rule. For the sequencing problem the correct adoption of Roberts' affine maximizer allocation rules is the 'affine cost minimizer sequencing rules'. It is quite easy to verify that the outcome efficient sequencing rule is an affine cost minimizer. Interestingly, the just sequencing rule, in spite of being a Rawlsian sequencing rule, is an affine cost minimizer.

Consider the sequencing problem where the processing time of the agents are identical. Such situations are referred to as the queueing problem. Queueing problems has been analyzed extensively from both normative and strategic viewpoints (see Chun [4], Chun, Mitra and Mutuswami [5], Hashimoto and Saitoh [14], Kayi and Ramaekars

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[1]Velez [34] showed that the Generalized Money Rawlsian Fair solutions implements the no envy solution in Nash and Strong Nash equilibria.

[20], Maniquet [23], Mitra [24], Mitra and Mutuswami [29] and Mukherjee [28]). For the queueing problem, the outcome efficient sequencing rule implies that the rule is also the just sequencing rule. However, for sequencing problems with non-identical processing time across agents, outcome efficient sequencing rule is different from the just sequencing rule and hence such sequencing problems brings out the trade-off between outcome efficient sequencing rule and the just sequencing rule.

The importance of finding balanced VCG mechanisms to implement outcome efficiency for canonical allocation problems was highlighted by Zhou [37]. However, for many economic environments implementing outcome efficiency with balanced transfers is not possible (see Hurwicz [16], Hurwicz and Walker [17] and Walker [36]). For sequencing problem it is possible to get budget balanced (or first best) implementation with the outcome efficiency (see Mitra[25] and Suijs [33]). We show that we can also find rVCG mechanisms that implements the just sequencing rule with balanced transfers and identify the set of all such balanced rVCG mechanisms. Again, for the queueing problem, the set of all balanced rVCG mechanisms coincide with the set of all balanced VCG mechanisms.[2]

The just sequencing rule is independent of the processing time of the agents. Therefore, if we have a two-dimensional incentive problem, where waiting cost and processing time are private information, ex-post implementability of the just sequencing rule is possible. If processing times are private information, we have mechanism design problem under interdependent value, as the processing time generates interdependence across agents. Hence the correct notion of implementation is ex-post implementation. Specifically we show that the just sequencing rule is ex-post implementable by making some minor 'modification' in the rVCG mechanisms. Moreover, given the earlier results on implementability of the just sequencing rule with balanced rVCG mechanisms, it follows that ex-post implementability with balanced transfers is also a possibility.

Jehiel, Meyer-ter-Vehn, Moldovanu, and Zame [18] proved that the only deterministic social choice functions that are ex-post implementable in generic mechanism design frameworks with multidimensional signals and interdependent valuations are those rules for which the same alternative is chosen irrespective of agents' signals, that is, the outcome should be independent of the interdependent signals. In sequencing with two-dimensional incentive problem, one dimension is waiting cost which is the private value and the other dimension is processing time that generates interdependence in terms of cost of completion time. The just sequencing rule is non-trivial

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[2]The literature on balanced VCG mechanisms for the queueing problem includes paper like Chun , Mitra and Mutuswami [6], Kayi and Ramaekars [20] and Mitra [24].

in terms of the waiting cost or private value dimension and is independent of the interdependence inducing processing time (like the independence of the interdependent signal required by Jehiel, Meyer-ter-Vehn, Moldovanu, and Zame [18] for ex-post implementability) and hence, the just sequencing rule is a non-trivial rule which is ex-post implementable. Moreover, for the outcome efficient sequencing rule the profile contingent order is dependent on the processing time and hence it is not ex-post implementable under this two-dimensional incentive problem.[3]

The paper is organized in the following way. In Section 2, we provide the framework of the sequencing problems. In Section 3, we introduce and analyze the just sequencing rule. In Section 4 we deal with properties of the just sequencing rule. This is followed by Section 5 which is the conclusion.

2. THE FRAMEWORK

Consider a finite set of agents $N = \{1, 2, \ldots, n\}$ in need of a facility that can be used sequentially. Using this facility, the agents want to process their jobs. The job processing time can be different for different agents. Specifically, for each agent $i \in N$, the job processing time is given by $s_i > 0$. Let $\mathbb{R}_{++}$ be the positive orthant of the real line $\mathbb{R}$ and let $\theta_i S_i$ measure the cost of job completion for agent $i \in N$ where $S_i \in \mathbb{R}_{++}$ is the job completion time for this agent and $\theta_i \in \Theta := \mathbb{R}_{++}$ denotes his constant per-period waiting cost. Due to the sequential nature of providing the service, the job completion time S_i for agent i depends not only on his own processing time s_i but also on the processing time of the agents who precedes him in the order of service. By means of an order $\sigma = (\sigma_1, \ldots, \sigma_n)$ on N, one can describe the positions of each agent in the order. Specifically, $\sigma_i = k$ indicates that agent i has the k-th position in the order. Let $\Sigma(N)$ be the set of $n!$ possible orders on N. We define $P_i(\sigma) = \{j \in N \setminus \{i\} \mid \sigma_j < \sigma_i\}$ to be the predecessor set of i in the order σ, that is, set of agents served before agent i in the order σ. Similarly, $P'_i(\sigma) = \{j \in N \setminus \{i\} \mid \sigma_j > \sigma_i\}$ denotes the successor set of i in the order σ, that is, set of agents served after agent i in the order σ. Let $s = (s_1, \ldots, s_n) \in \mathcal{S} := \mathbb{R}^n_{++}$ denote the vector of processing time of the agents. Given a vector $s = (s_1, \ldots, s_n) \in \mathcal{S}$ and an order $\sigma \in \Sigma(N)$, the cost of job completion for agent $i \in N$ is $\theta_i S_i(\sigma)$, where the job completion time is $S_j(\sigma) = \sum_{j \in P_i(\sigma)} s_j + s_i$. The agents have quasi-linear utility of the form $v_i(\sigma, \tau_i; m_i = (\theta_i, s_i); s_{-i}) = -\theta_i S_i(\sigma) + \tau_i$ where σ is the order, $\tau_i \in \mathbb{R}$ is the transfer that he receives and the parameters of the

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[3]Ex-post implementability literature includes papers by Bergemann and Morris [1], Bikhchandani [2], Chung and Ely [7], Jehiel, Meyer-ter-Vehn, Moldovanu and Zame [18], Jehiel and Moldovanu [19], and, Karsten, Kittsteiner, and Benny Moldovanu [21]). For the sequencing problem with private information only in processing time, incentive issues were addressed by Hain and Mitra [13] and Moulin [27].

model are agents own parameter $m_i = (\theta_i, s_i)$ that constitutes of the waiting cost θ_i and the processing time s_i, and, more importantly, the processing time of the other agents that determines agent i's job completion time $S_i(\sigma)$. Specifically, given a commonly known job processing time vector $s = (s_1, \ldots, s_n) \in \mathcal{S}$ and an order $\sigma \in \Sigma(N)$, the utility of agent i, with just the waiting cost parameter θ_i, reduces to

$$v_i(\sigma, \tau_i; m_i = (\theta_i, s_i); s_{-i}) := U_i(\sigma, \tau_i; \theta_i) = -\theta_i S_i(\sigma) + \tau_i = -\theta_i \left(s_i + \sum_{j \in P_i(\sigma)} s_j \right) + \tau_i.$$

If we assume that both waiting cost and processing time are private information, then we have a general sequencing problem for N agents which we denote by $\Omega_N = (\Theta^n, \mathcal{S})$. In this context we associate the utility function $v_i(.)$ for each $i \in N$. If the processing time vector $s \in \mathcal{S}$ is given and waiting cost is private information, then we have a sequencing problem $\Omega_N^s = (\Theta^n, s)$ and in that case the utility function reduces to $U_i(.)$ (from $v_i(.)$) for each $i \in N$. Except for Subsection 4.3, we will deal with Ω_N^s. Hence our first objective is to design direct revelation mechanisms for any given Ω_N^s.

For any set X, let $|X|$ denote the cardinality of X. A typical profile of waiting costs is denoted by $\theta = (\theta_1, \ldots, \theta_n) \in \Theta^n$, and, for any $i \in N$, $\theta_{-i} \in \Theta^{|N \setminus \{i\}|}$ denotes the profile $(\theta_1 \ldots \theta_{i-1}, \theta_{i+1}, \ldots \theta_n)$ which is obtained from the profile θ by eliminating i's waiting cost. For a given sequencing problem Ω_N^s, a (direct revelation) mechanism is (σ, τ) that constitutes of a sequencing rule σ and a transfer rule τ. A *sequencing rule* is a function $\sigma : \Theta^n \to \Sigma(N)$ that specifies for each profile $\theta \in \Theta^n$ a unique order $\sigma(\theta) = (\sigma_1(\theta), \ldots, \sigma_n(\theta)) \in \Sigma(N)$.[4] A *transfer rule* is a function $\tau : \Theta^n \to \mathbb{R}^n$ that specifies for each profile $\theta \in \Theta^n$ a transfer vector $\tau(\theta) = (\tau_i(\theta), \ldots, \tau_n(\theta)) \in \mathbb{R}^n$. Specifically, given any sequencing problem Ω_N^s and given any mechanism (σ, τ), if (θ_i', θ_{-i}) is the announced profile when the true waiting cost of i is θ_i, then utility of i is $U_i(\sigma(\theta_i', \theta_{-i}), \tau_i(\theta_i', \theta_{-i}); \theta_i) = -\theta_i S_i(\sigma(\theta_i', \theta_{-i}) + \tau_i(\theta_i', \theta_{-i}).$

Definition 1. A mechanism (σ, τ) *implements* the sequencing rule σ in dominant strategies if the transfer rule $\tau : \Theta^n \to \mathbb{R}^n$ is such that for any $i \in N$, any $\theta_i, \theta_i' \in \Theta$ and any $\theta_{-i} \in \Theta^{|N \setminus \{i\}|}$,

$$(1) \qquad U_i(\sigma(\theta), \tau_i(\theta); \theta_i) \geq U_i(\sigma(\theta_i', \theta_{-i}), \tau_i(\theta_i', \theta_{-i}); \theta_i).$$

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[4]The sequencing rule is a function and not a correspondence. Hence, we will require tie-breaking rule to reduce a correspondence to a function which, unless explicitly discussed, will be fixed. We use the following tie-breaking rule. We fix any linear order $\succ$ on the set of agents N. For any sequencing rule σ and any profile $\theta \in \Theta^n$ with a tie situation between agents $i, j \in N$, we pick the order $\sigma(\theta)$ with $\sigma_i(\theta) < \sigma_j(\theta)$ if and only if $i \succ j$.

Implementation of a rule σ via a mechanism (σ, τ) requires that the transfer rule τ is such that truthful reporting for any agent weakly dominates false reporting irrespective of other agents' report.

2.1. **The outcome efficient sequencing rule.**

Definition 2. A sequencing rule σ^* is *outcome efficient* if for any $\theta \in \Theta^n$, $\sigma^*(\theta) \in \text{argmin}_{\sigma \in \Sigma(N)} \sum_{i \in N} \theta_i S_i(\sigma)$.

For each profile the outcome efficient sequencing rule selects an order to minimization the aggregate cost of completion time. Define $u_i := \theta_i / s_i$ as the urgency index of agent i which is the ratio of his waiting cost and his processing time. From Smith [32] we know that for any sequencing problem Ω_N^s a sequencing rule σ^* is outcome efficient if and only if for any profile θ, the selected order $\sigma^*(\theta)$ satisfies condition (OE): For any $i, j \in N, \theta_i / s_i \geq \theta_j / s_j \Leftrightarrow \sigma_i^*(\theta) \leq \sigma_j^*(\theta)$.[5] Therefore, outcome efficient sequencing rule requires that the agents are ordered in the non-increasing order of their urgency index. From (OE) it is clear that if $\theta_i / s_i \geq \theta_j / s_j$, then $S_i(\sigma^*(\theta)) \leq S_j(\sigma^*(\theta))$. Consider the outcome efficient sequencing rule σ^*. It is well-known that VCG mechanisms are the only mechanisms that implement σ^* (see Holmström [15]).

Definition 3. For the outcome efficient sequencing rule σ^*, a mechanism (σ^*, τ) is a *VCG mechanism* if the transfer rule is such that for all $\theta \in \Theta^n$ and all $i \in N$,

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(2) \qquad \tau_i^*(\theta) = \begin{cases} h_i(\theta_{-i}) & \text{if } P_i'(\sigma^*(\theta)) = \emptyset, \\ h_i(\theta_{-i}) - s_i \sum_{j \in P_i'(\sigma^*(\theta))} \theta_j & \text{otherwise.} \end{cases}
$$

where the function $h_i : \Theta^{|N \setminus \{i\}|} \to \mathbb{R}$ is arbitrary.

Given a sequencing problem Ω_N^s, for any profile $\theta \in \Theta^n$ and any $i \in N$ and any $j \in N \setminus \{i\}$, let $\theta_j s_i$ be the *pivotal cost* of agent i on agent j. We call this the pivotal cost because $\theta_j s_i$ is the incremental cost that agent j has to incur if agent i precedes agent j in any order. The VCG transfer in condition (2) specifies that for any $i \in N$ and any $\theta_{-i} \in \Theta^{N \setminus \{i\}}$, if θ_i is such that agent i is served last in the outcome efficient order $\sigma^*(\theta_i, \theta_{-i})$, that is, if $P_i'(\sigma^*(\theta_i, \theta_{-i})) = \emptyset$, then $\tau_i^*(\theta_i, \theta_{-i}) = h_i(\theta_{-i})$. If, however, θ_i' is such that agent i is not served last in the order $\sigma^*(\theta_i', \theta_{-i})$, that is, if $P_i'(\sigma^*(\theta_i', \theta_{-i}) \neq \emptyset$, then agent i's transfer $\tau_i^*(\theta_i', \theta_{-i})$ not only has $h_i(\theta_{-i})$ but he also has to pay the sum of the pivotal cost that agent i incurs on his followers in the order $\sigma^*(\theta_i', \theta_{-i})$ (that is, the waiting cost of all the agent(s) served after him times his own processing time).

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[5]Given the tie-breaking rule, this selection is unique.

The VCG transfer $\tau_i^*(\theta)$ (in condition (2)) for which the agent specific constant functions $h_i(.)$ are always zero for all agents gives us the pivotal mechanism for implementing the outcome efficient order σ^*. The first work that identified the pivotal mechanism for any sequencing problem is by Dolan [10]. It must be pointed out that the specification (2) of the VCG transfers is the pivotal based representation of the VCG transfers and is not its standard representation. We show that an appropriate transformation of the standard VCG transfers gives us (2). The standard way of specifying the VCG transfers is that for all θ and for all $i \in N$,

(3) $$\tau_i^*(\theta) = - \sum_{j \in N \setminus \{i\}} S_j(\sigma^*(\theta))\theta_j + g_i(\theta_{-i}).^6$$

Consider the outcome efficient order $\sigma^*(\theta)$ for the profile $\theta \in \Theta^n$ and suppose that agent i leaves. We define the "induced" order $\sigma^*(\theta_{-i})$ (of length $|N \setminus \{i\}|$) for the agents in $N \setminus \{i\}$ as follows:

(4) $$\sigma_j^*(\theta_{-i}) = \begin{cases} \sigma_j^*(\theta) - 1 & \text{if } j \in P_i'(\sigma^*(\theta)), \\ \sigma_j^*(\theta) & \text{if } j \in P_i(\sigma^*(\theta)). \end{cases}$$

In words, $\sigma^*(\theta_{-i})$ is the order formed by removing agent i and moving all agents behind him up by one position. Given the same tie-breaking rule for the economy with $N \setminus \{i\}$ agents, it is easy to see that if $\sigma^*(\theta)$ is outcome efficient for the profile θ, then $\sigma^*(\theta_{-i})$ is also outcome efficient in $N \setminus \{i\}$ for the profile θ_{-i}. Without loss of generality, we can write for all θ and all $i \in N$, (A) $g_i(\theta_{-i}) = \sum_{j \in N \setminus \{i\}} S_j(\sigma^*(\theta_{-i}))\theta_j + h_i(\theta_{-i})$. By substituting (A) in (3) we get

(5) $$\tau_i^*(\theta) = - \sum_{j \in N \setminus \{i\}} [S_j(\sigma^*(\theta)) - S_j(\sigma^*(\theta_{-i}))]\theta_j + h_i(\theta_{-i}).$$

If for a profile $\theta \in \Theta^n$, the outcome efficient order is $\sigma^*(\theta)$ and agent i leaves, then the order $\sigma^*(\theta_{-i})$ is such that if $j \in P_i(\sigma^*(\theta))$, then j's completion time remains unaltered and if $k \in P_i'(\sigma^*(\theta))$, then k's completion time reduces by s_i. Hence

(6) $$S_j(\sigma^*(\theta)) - S_j(\sigma^*(\theta_{-i})) = \begin{cases} s_i & \text{if } j \in P_i'(\sigma^*(\theta)), \\ 0 & \text{if } j \in P_i(\sigma^*(\theta)). \end{cases}$$

By substituting condition (6) in the transformed VCG transfer (5) and then simplifying it we get the VCG transfer (2).[7]

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[6] See Mitra [25] and Suijs [33].

[7] A similar argument for the pivotal based representation of VCG transfers (like condition (2)) for the queueing problem can be found in Chun, Mitra and Mutuswami [5].

3. An implementable Rawlsian sequencing rule

Definition 4. A sequencing rule σ' is *Rawlsian* if for each profile $\theta \in \Theta^n$, $\sigma'(\theta) \in \min_{\sigma \in \Sigma(N)} \max_{j \in N} S_j(\sigma)\theta_j$.

Given any profile $\theta \in \Theta^n$, for each order $\sigma \in \Sigma(N)$, let $M(\sigma) = \theta_j S_j(\sigma) \geq \theta_k S_k(\sigma)$ for all $k \in N$, that is, for the given profile θ and given the order σ, $M(\sigma)$ is the maximum value of the cost of completion time among all agents in N. The Rawlsian sequencing rule picks that order $\sigma' \in \Sigma(N)$ for which $M(\sigma')$ is minimum, that is $M(\sigma') \leq M(\sigma)$ for all $\sigma \in \Sigma(N)$.

Example 1. Consider the sequencing problem Ω_N^s such that $N = \{1, 2, 3\}$ and $s = (s_1 = 1, s_2 = 2, s_3 = 3)$. Let the waiting cost vector be $\theta = (\theta_1 = 100, \theta_2 = 5, \theta_3 = 3)$. For θ, outcome efficiency uniquely picks the order $\sigma^1 = (\sigma_1 = 1, \sigma_2 = 2, \sigma_3 = 3)$ since $u_1 > u_2 > u_2$. However, the Rawlsian selection is not unique. For profile θ we have the following: $M(\sigma^1 = (\sigma_1 = 1, \sigma_2 = 2, \sigma_3 = 3)) = \theta_1 s_1 = 100$, $M(\sigma^2 = (\sigma_1 = 1, \sigma_2 = 3, \sigma_3 = 2)) = \theta_1 s_1 = 100$, $M(\sigma^3 = (\sigma_1 = 2, \sigma_2 = 1, \sigma_3 = 3)) = \theta_1(s_2 + s_1) = 300$, $M(\sigma^4 = (\sigma_1 = 2, \sigma_2 = 3, \sigma_3 = 1)) = \theta_1(s_3 + s_1) = 400$, $M(\sigma^5 = (\sigma_1 = 3, \sigma_2 = 1, \sigma_3 = 2)) = \theta_1(s_2 + s_3 + s_1) = 600$, and $M(\sigma^6 = (\sigma_1 = 3, \sigma_2 = 2, \sigma_3 = 1)) = \theta_1(s_3 + s_2 + s_1) = 600$. Therefore, for the profile θ, the Rawlsian rule can either pick σ^1 or σ^2 implying that the Rawlsian rule does not guarantee state contingent unique order selection. Note that the order σ^1 also has the property that it serves the agents in the decreasing (hence non-increasing) order of their waiting cost, that is given $\theta_1 > \theta_2 > \theta_3$, agent 1 is served first followed by agent 2 and then by agent 3.

Let the waiting cost vector be $\theta' = (\theta_1' = 10, \theta_2 = 5, \theta_3 = 3)$. For profile θ' we have the following: $M(\sigma^1 = (\sigma_1 = 1, \sigma_2 = 2, \sigma_3 = 3)) = \theta_3(s_1 + s_2 + s_3) = 18$, $M(\sigma^2 = (\sigma_1 = 1, \sigma_2 = 3, \sigma_3 = 2)) = \theta_2(s_1 + s_3 + s_2) = 30$, $M(\sigma^3 = (\sigma_1 = 2, \sigma_2 = 1, \sigma_3 = 3)) = \theta_1'(s_2 + s_1) = 30$, $M(\sigma^4 = (\sigma_1 = 2, \sigma_2 = 3, \sigma_3 = 1)) = \theta_1'(s_3 + s_1) = 40$, $M(\sigma^5 = (\sigma_1 = 3, \sigma_2 = 1, \sigma_3 = 2)) = \theta_1'(s_2 + s_3 + s_1) = 60$, and $M(\sigma^6 = (\sigma_1 = 3, \sigma_2 = 2, \sigma_3 = 1)) = \theta_1'(s_3 + s_2 + s_1) = 60$. For the profile θ', $M(\sigma^1) < M(\sigma^k)$ for all $k \in \{2, \ldots, 6\}$ and hence any Rawlsian rule uniquely picks σ^1.

The above example suggests that we can have profiles θ for which more than one order is Rawlsian. It also seems likely that serving the agents in the non-increasing order of their waiting costs is always Rawlsian. In the next theorem we show that this is indeed the case.

Definition 5. A sequencing rule $\tilde{\sigma}$ is *just* if for each profile $\theta \in \Theta^n$, the chosen order $\tilde{\sigma}(\theta)$ satisfies the following property: for any $i, j \in N$ such that $\theta_i \geq \theta_j$, $\tilde{\sigma}_i(\theta) \leq \tilde{\sigma}_j(\theta)$.[8]

Theorem 1. For any Ω_N^s, the just sequencing rule $\tilde{\sigma}$ is Rawlsian.

Proof: To prove that the just sequencing rule $\tilde{\sigma}$ is Rawlsian, consider any profile $\theta \in \Theta^n$ and the just order $\tilde{\sigma}(\theta)$. Consider that agent $i \in N$ for whom the cost of completion time $\theta_i S_i(\tilde{\sigma}(\theta)) = \theta_i(\sum_{j \in P_i(\tilde{\sigma}(\theta))} s_j + s_i)$ is maximum under $\tilde{\sigma}(\theta)$, that is $M(\tilde{\sigma}(\theta)) = \theta_i S_i(\tilde{\sigma}(\theta))$.[9] Define $O := P_i(\tilde{\sigma}(\theta)) \cup \{i\}$ as the set that includes the set of predecessors of i under the order $\tilde{\sigma}(\theta)$ and that also includes agent i. Observe that from the definition of just sequencing rule it follows that $\theta_j \geq \theta_i$ for all $j \in O$. Consider any other order $\sigma \in \Sigma(N) \setminus \{\tilde{\sigma}(\theta)\}$. For this order σ, there is one agent in O who will be served last under σ relative to the other members of O, that is, there exists an agent $j \in O$ such that $\sigma_j > \sigma_k$ for all $k \in O \setminus \{j\}$. This means that $O \subseteq P_j(\sigma) \cup \{j\}$, and hence $S_j(\sigma) \geq S_i(\tilde{\sigma}(\theta))$, that is the completion time of agent j under the order σ is not less than the completion time of agent i under the order $\tilde{\sigma}(\theta)$. Therefore, the maximum cost of completion time $M(\sigma)$ under σ is at least as large as the maximum cost of completion time $M(\tilde{\sigma}(\theta)) = \theta_i S_i(\tilde{\sigma}(\theta))$ under $\tilde{\sigma}(\theta)$ since $M(\sigma) \geq \theta_j S_j(\sigma) \geq \theta_j S_i(\tilde{\sigma}(\theta)) \geq \theta_i S_i(\tilde{\sigma}(\theta)) = M(\tilde{\sigma}(\theta))$. Since the selection of σ was arbitrary the result follows. $\square$

Remark 1. Consider any sequencing problem $\Omega_N^{s^*}$ with $s^* = (s_1^*, \ldots, s_n^*) \in \mathcal{S}$ and $s_1^* = \ldots = s_n^*$ so that we have the queueing problem. Then the outcome efficient sequencing rule implies the just sequencing rule and hence a Rawlsian sequencing rule. For the queueing problem $\Omega_N^{s^*}$, for any profile $\theta \in \Theta^n$, the order of the urgency indexes and that of the waiting costs are identical and hence this implication.

Definition 6. For the just sequencing rule $\tilde{\sigma}$, a mechanism $(\tilde{\sigma}, \tilde{\tau})$ is an *rVCG mechanism* if the transfer rule is such that for all $\theta \in \Theta^n$ and all $i \in N$,

$$(7) \qquad \tilde{\tau}_i(\theta) = \begin{cases} h_i(\theta_{-i}) & \text{if } P_i'(\tilde{\sigma}(\theta)) = \emptyset, \\ h_i(\theta_{-i}) - \sum\limits_{j \in P_i'(\tilde{\sigma}(\theta))} \theta_j s_j & \text{otherwise.} \end{cases}$$

where the function $h_i : \Theta^{|N \setminus \{i\}|} \to \mathbb{R}$ is arbitrary.

Given a sequencing problem Ω_N^s, for any profile $\theta \in \Theta^n$ and any $j \in N$, let $\theta_j s_j$ be the *minimum cost* of agent j, that is the cost that agent j would have incurred if he was served first in any order. Like the VCG transfers (2), the rVCG transfers (7)

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[8]Given the tie-breaking rule, for any profile $\theta \in \Theta^n$, this selection $\tilde{\sigma}(\theta)$ is unique.

[9]If there are more than one agent for whom the cost is maximum, pick any one of them arbitrarily.

specifies that for any $i \in N$ and any $\theta_{-i} \in \Theta^{N \setminus \{i\}}$, if θ_i is such that agent i is served last in the just order $\tilde{\sigma}(\theta_i, \theta_{-i})$, then $\tilde{\tau}_i(\theta_i, \theta_{-i}) = h_i(\theta_{-i})$. If θ_i' is such that agent i is not served last in the order $\tilde{\sigma}(\theta_i', \theta_{-i})$, then agent i's transfer $\tilde{\tau}_i(\theta_i', \theta_{-i})$ not only has $h_i(\theta_{-i})$ but he also has to pay the sum of the minimum cost of his followers in the just order $\tilde{\sigma}(\theta_i', \theta_{-i})$. As long as we are in a sequencing problem Ω_N^s where the processing time of the agents are not identical, there will be some profile θ and some agent j for whom the minimum cost $\theta_j s_j$ will be different from the pivotal cost $\theta_j s_i$ of i on j for some $i \in N \setminus \{j\}$. Hence the payment amounts in the rVCG transfers are qualitatively different from the payment amounts under the VCG transfers (2) where, recall that, the payment amount is the sum of the pivotal cost of agent i on all his followers in the outcome efficient sequencing order. Therefore, it is easy to see that for the queueing problem Ω_N^{s*} we have the following: For any agent $j \in N$, any agent $i \in N \setminus \{i\}$ and any profile θ, the minimum cost $\theta_j s_j^*$ is identical to the pivotal cost $\theta_j s_i^*$ since $s_i^* = s_j^*$ and, given the same tie-breaking rule, $\sigma^*(\theta) = \tilde{\sigma}(\theta)$. Thus, the VCG-mechanisms and the rVCG mechanisms are identical for the queueing problem Ω_N^{s*}.

Theorem 2. The just sequencing rule $\tilde{\sigma}$ is implementable if and only if the mechanism $(\tilde{\sigma}, \tau)$ that implements it is an rVCG mechanism.

Proof: Consider the just sequencing rule $\tilde{\sigma}$. We first prove that if a mechanism $(\tilde{\sigma}, \tau)$ implements $\tilde{\sigma}$, then it is necessarily the rVCG mechanism. Consider any agent $i \in N$ and fix any profile $\theta_{-i} \in \Theta^{|N \setminus \{i\}|}$ of all but agent i. By taking any two types θ_i and θ_i' for agent i and applying the two implementation inequalities (1) $U_i(\tilde{\sigma}(\theta), \tau_i(\theta); \theta_i) \geq U_i(\tilde{\sigma}(\theta_i', \theta_{-i}), \tau_i(\theta_i', \theta_{-i}); \theta_i)$ and (2) $U_i(\tilde{\sigma}(\theta), \tau_i(\theta); \theta_i') \leq U_i(\tilde{\sigma}(\theta_i', \theta_{-i}), \tau_i(\theta_i', \theta_{-i}); \theta_i')$. In general, from inequalities (1) and (2) we get

$$(8) \quad [S_i(\tilde{\sigma}(\theta_i', \theta_i)) - S_i(\tilde{\sigma}(\theta))]\theta_i \geq \tau_i(\theta_i', \theta_{-i}) - \tau_i(\theta) \geq [S_i(\tilde{\sigma}(\theta_i', \theta_i)) - S_i(\tilde{\sigma}(\theta))]\theta_i'.$$

If θ_i and θ_i' are such that $\tilde{\sigma}(\theta) = \tilde{\sigma}(\theta_i', \theta_{-i})$, then $S_i(\tilde{\sigma}(\theta_i', \theta_i)) = S_i(\tilde{\sigma}(\theta))$ and inequality (8) gives $\tau_i(\theta) = \tau_i(\theta_i', \theta_{-i})$.

Let $\theta_{(1)} \geq \theta_{(2)} \geq \ldots \geq \theta_{(n-1)}$ be the decreasing order of the waiting cost for the fixed profile θ_{-i}. Consider any pair $(\theta_i^{t+1}, \theta_i^t) \in [\theta_{(t+1)}, \theta_{(t)}] \times [\theta_{(t)}, \theta_{(t-1)}]$. Using the just sequencing rule $\tilde{\sigma}$ and applying the implementability condition (8) when the actual profile is $(\theta_i^{t+1}, \theta_{-i})$ $((\theta_i^t, \theta_{-i}))$ and the misreport of agent i is θ_i^t (θ_i^{t+1}) we get

$$(9) \quad \theta_i^{t+1} s_{(t)} \leq \tau_i(\theta_i^{t+1}, \theta_{-i}) - \tau_i(\theta_i^t, \theta_{-i}) \leq \theta_i^t s_{(t)}.$$

Since (9) must hold for all $(\theta_i^{t+1}, \theta_i^t) \in [\theta_i^{(t+1)}, \theta_i^{(t)}] \times [\theta_i^{(t)}, \theta_i^{(t-1)}]$, it follows that

$$(10) \quad \tau_i(\theta_i^{t+1}, \theta_{-i}) - \tau_i(\theta_i^t, \theta_{-i}) = \theta_{(t)} s_{(t)}.$$

Condition (10) must hold for all $t \in \{1, \ldots, n-1\}$. By setting $\tau_i(\theta_i^n, \theta_{-i}) = h_i(\theta_{-i})$ for any $\theta_i^n \in (0, \theta_{(n-1)})$ and then solving condition (10) recursively we get the rVCG transfers (7).

For the converse consider any agent $i \in N$ and any profile θ_{-i}. Let θ_i be the true waiting cost of i and $B_i(\theta_i'; \theta_i) := U_i(\tilde{\sigma}(\theta_i', \theta_{-i}), \tau_i(\theta_i', \theta_{-i}); \theta_i) - U_i(\tilde{\sigma}(\theta_i, \theta_{-i}), \tau_i(\theta_i, \theta_{-i}); \theta_i)$ be the benefit of agent i from a misreport θ_i'.

(D1) If $\theta_i' > \theta_i$ and $P_i(\tilde{\sigma}(\theta_i, \theta_{-i})) \setminus P_i(\tilde{\sigma}(\theta_i', \theta_{-i})) \neq \emptyset$, then from the just sequencing rule we get $\theta_j \geq \theta_i$ for all $j \in P_i(\tilde{\sigma}(\theta_i, \theta_{-i})) \setminus P_i(\tilde{\sigma}(\theta_i', \theta_{-i}))$. Therefore, using any rVCG transfer we get $B_i(\theta_i'; \theta_i) = \sum_{j \in P_i(\tilde{\sigma}(\theta_i, \theta_{-i})) \setminus P_i(\tilde{\sigma}(\theta_i', \theta_{-i}))} (\theta_i - \theta_j) s_j \leq 0$.

(D2) If $\theta_i' < \theta_i$ and $P_i(\tilde{\sigma}(\theta_i', \theta_{-i})) \setminus P_i(\tilde{\sigma}(\theta_i, \theta_{-i})) \neq \emptyset$, then from the just sequencing rule we get $\theta_j \leq \theta_i$ for all $j \in P_i(\tilde{\sigma}(\theta_i', \theta_{-i})) \setminus P_i(\tilde{\sigma}(\theta_i, \theta_{-i}))$. Hence using any rVCG transfer we get $B_i(\theta_i'; \theta_i) = \sum_{j \in P_i(\tilde{\sigma}(\theta_i', \theta_{-i})) \setminus P_i(\tilde{\sigma}(\theta_i, \theta_{-i}))} (\theta_j - \theta_i) s_j \leq 0$.

(D3) Finally, if $\theta_i' \neq \theta_i$ and $P_i(\tilde{\sigma}(\theta_i', \theta_{-i})) = P_i(\tilde{\sigma}(\theta_i, \theta_{-i}))$, then $B_i(\theta_i'; \theta_i) = 0$.

Therefore, cases (D1)-(D3) show that agent i cannot benefit from any deviation. Since the selection of agent i was arbitrary, the result follows. $\qquad\square$

4. PROPERTIES OF THE JUST SEQUENCING RULE

In this section we focus on three nice properties of the just sequencing rule.

4.1. **The just sequencing rule as an affine cost minimizer.**
Roberts [31] defined affine maximizer allocation rules in a general mechanism design framework with quasi-linear preferences. However, sequencing problems are cost problems and hence the appropriate adoption of affine maximizer allocation rule of Roberts to the sequencing problem is the 'affine cost minimizer sequencing rules'.

Definition 7. A sequencing rule $\sigma^{w,\kappa} : \Theta^n \to \Sigma(N)$ is an *affine cost minimizer* if for each $\theta \in \Theta^n$, $\sigma^{w,\kappa}(\theta) \in \arg\min_{\sigma \in \Sigma(N)} \left\{ \kappa(\sigma) + \sum_{j \in N} w_j \theta_j S_j(\sigma) \right\}$, where $w_j \geq 0$ for all $j \in N$ with at least one $w_j > 0$ and the function $\kappa : \Sigma(N) \to \mathbb{R}$ is arbitrary.

The affine maximizer sequencing rule $\sigma^{\underline{1},\underline{0}}$ with $\kappa(\sigma) = 0$ for all $\sigma \in \Sigma(N)$ and $w_j = 1$ for all $j \in N$ gives the *outcome efficient sequencing rule*, that is $\sigma^{\underline{1},\underline{0}} = \sigma^*$.[10] Thus, the outcome efficient sequencing rule is a affine cost minimizer.

Proposition 1. The just sequencing rule $\tilde{\sigma}$ is an affine cost maximizer.

---

[10]The equality $\sigma^{\underline{1},\underline{0}} = \sigma^*$ is with the implicit assumption that the tie-breaking rule followed to obtain $\sigma^{\underline{1},\underline{0}}$ is identical to the tie-breaking rule of σ^*.

Proof: To prove that the just sequencing rule is a affine cost minimizer, we show that the affine cost minimizer sequencing rule $\sigma^{w^*,\underline{0}}$ with $\kappa(\sigma) = 0$ for all $\sigma \in \Sigma(N)$ and $w_j^* = 1/(\prod_{r \neq j} s_r) > 0$ for all $j \in N$ coincides with the just sequencing rule $\tilde{\sigma}$.

Consider any profile $\theta \in \Theta^n$ and any two agents $i, j \in N$ with $\theta_i \geq \theta_j$. Consider any order σ such that $k = \sigma_j = \sigma_i + 1$ for any given $k \in \{1, \ldots, n-1\}$. Given our objective is to minimize $\sum_{j \in N} w_j^* \theta_j S_j(\sigma)$, the sum of weighted completion costs of i and j for σ is

(11)

$$C_{ij}(\sigma) := w_i^* \theta_i S_i(\sigma) + w_j^* \theta_j S_j(\sigma) = \theta_i \left(\frac{s_i + \sum_{k \in P_i(\sigma)} s_k}{\prod_{r \neq i} s_r} \right) + \theta_j \left(\frac{s_i + s_j + \sum_{k \in P_i(\sigma)} s_k}{\prod_{r \neq j} s_r} \right).$$

Consider the order σ' obtained from σ by interchanging the positions of i and j only, that is $\sigma_j' = \sigma_i$, $\sigma_i' = \sigma_j$ and $\sigma_k' = \sigma_k$ for all $k \in N \setminus \{i, j\}$. The sum of weighted completion costs of i and j for σ' is

(12)

$$C_{ij}(\sigma') := w_i^* \theta_i S_i(\sigma') + w_j^* \theta_j S_j(\sigma') = \theta_i \left(\frac{s_i + s_j \sum_{k \in P_i(\sigma)} s_k}{\prod_{r \neq i} s_r} \right) + \theta_j \left(\frac{s_j + \sum_{k \in P_i(\sigma)} s_k}{\prod_{r \neq j} s_r} \right).$$

By taking the difference $C_{ij}(\sigma') - C_{ij}(\sigma)$ and using conditions (11) and (12) and making some obvious simplifications we get

(13) $$C_{ij}(\sigma') - C_{ij}(\sigma) = (\theta_i - \theta_j) \left(\frac{1}{\prod_{r \in N \setminus \{i, j\}} s_r} \right) \geq 0.$$

In condition (13), the inequality is strict if $\theta_i > \theta_j$. Condition (13) shows that if $\theta_i \geq \theta_j$, then for any two consecutive positions in an order it is not more costly to serve agents i before agent j and if $\theta_i > \theta_j$, then it is less costly to serve agent i before agent j. Notice that for this conclusion we neither required the profile of waiting costs of the agents $N \setminus \{i, j\}$ nor did we require the positions of the agent $N \setminus \{i, j\}$ in the order. Hence, if for any profile $\theta \in \Theta^n$, our objective is to select $\sigma^{w^*,\underline{0}}(\theta) \in \arg\min_{\sigma \in \Sigma(N)} \left\{ \sum_{j \in N} w_j^* \theta_j S_j(\sigma) \right\}$, then $\sigma^{w^*,\underline{0}}(\theta)$ must follow the following property: (Aff) For any $i, j \in N$ such that $\theta_i \geq \theta_j$, $\sigma_i^{w^*,\underline{0}}(\theta) \leq \sigma_j^{w^*,\underline{0}}(\theta)$. Moreover, since $N \setminus \{i, j\}$ did not matter in condition (13), the converse is also true, that is, given any profile θ, condition (Aff) gives $\sigma^{w^*,\underline{0}}(\theta) \in \arg\min_{\sigma \in \Sigma(N)} \left\{ \sum_{j \in N} w_j^* \theta_j S_j(\sigma) \right\}$. Given the tie breaking rule, the order obtained by following condition (Aff) is the same as the order obtained from the just sequencing rule $\tilde{\sigma}$. Hence the result follows. $\square$

Remark 2. In general, for any affine cost minimizer sequencing rule $\sigma^{w,\bar{\kappa}}$ with the property that $\kappa(\sigma) = \bar{\kappa}$ for all $\sigma \in \Sigma(N)$, urgency index can be replaced by weighted

urgency index $u_i^w = (w_i \theta_i)/s_i$ to determine the profile contingent order of serving the agents. Specifically, like Smith's [32] rule for outcome efficiency, for these affine cost minimizers $\sigma^{w,\bar{\kappa}}$, $\sigma^{w,\bar{\kappa}}(\theta)$ affine cost minimizes $\{\bar{\kappa} + \sum_{j \in N} w_j \theta_j S_j(\sigma)\}$ if and only if the selected order $\sigma^{w,\bar{\kappa}}(\theta)$ satisfies condition (ACM): For any $i, j \in N$, $(w_i \theta_i)/s_i \geq (w_j \theta_j)/s_j \Leftrightarrow \sigma_i^{w,\bar{\kappa}}(\theta) \leq \sigma_j^{w,\bar{\kappa}}(\theta)$.[11] More importantly, the affine cost minimizer sequencing rule $\sigma^{w^*,\underline{0}}$ that gives the just sequencing rule $\tilde{\sigma}$ has $w_j^* = (1/\prod_{k \in N \setminus \{j\}} s_k)$ as the agent specific weights and hence weighted urgency index is $u_i^{w^*} = (\theta_i / \prod_{k \in N} s_k)$. Therefore, for any $\theta \in \Theta^n$ and any $i, j \in N$, $u_i^{w^*} \geq u_j^{w^*}$ if and only if $\theta_i \geq \theta_j$. That is, for the affine cost minimizer sequencing rule $\sigma^{w^*,\underline{0}}$ that gives $\tilde{\sigma}$, comparison of weighted urgency index across agents reduces to the comparison of the waiting costs across agents.

4.2. **Budget balanced implementability of the just sequencing rule.** For implementation of outcome efficient allocation rules, Zhou [37] provided examples of both public and private good allocation problems for which no budget balancing VCG mechanisms exists and for which all VCG mechanisms are inferior to other 'reasonable' non-VCG mechanisms. Zhou [37] concluded that unless one can find a budget balancing VCG mechanism, one should limit its use. Implementation of outcome efficient rules with balanced transfers is difficult to get in many economic environments (see, for example, Hurwicz and Walker [17] and Walker [36]). That implementation of outcome efficiency is possible with balanced transfers for sequencing and queueing problems was established by Mitra [24], [25] and Suijs [33]. Mitra and Sen [26] characterized domains in a heterogenous objects model with private values where an outcome efficient rule can be implemented in dominant strategies with balanced transfers. They show that these domains are non-trivial and are 'closely' related to incentive problems in sequencing and queueing problems studied in Mitra [24], [25] and Suijs [33]. We prove that for sequencing problems, if we replace outcome efficient sequencing rule with the just sequencing rule, we get implementability with balanced transfers provided there are at least three agents.

Definition 8. A sequencing rule σ is *implementable with balanced transfers* if the mechanism (σ, τ) that implements it has a budget balanced transfer, that is, for all $\theta \in \Theta^n$, $\sum_{j \in N} \tau_j(\theta) = 0$.

Proposition 2. For any sequencing problem $\Omega_{N=\{1,2\}}^{(s_1, s_2)}$ with two agents, implementation of the just sequencing rule $\tilde{\sigma}$ with balanced transfers is not possible.

---

[11]Given the tie-breaking rule, this profile contingent selection $\sigma^{w,\kappa}(\theta)$ is unique.

Proof: Fix any processing time vector (s_1, s_2) and consider the sequencing problem $\Omega^s_{N=\{1,2\}}$. To implement the refined sequencing rule it is necessary that the mechanism (σ, τ) is an rVCG mechanism (Theorem 2). Pick any rVCG mechanism. Consider the waiting costs $\theta_1, \theta'_1, \theta_2, \theta'_2$ such that $\theta_1 > \theta'_2 > \theta'_1 > \theta_2$ and $\theta'_1 s_1 \neq \theta'_2 s_2$. Budget balance for the profile (θ_1, θ_2) gives (B1) $h_1(\theta_2) + h_2(\theta_1) - \theta_2 s_2 = 0$. Budget balance for (θ_1, θ'_2) gives (B2) $h_1(\theta'_2) + h_2(\theta_1) - \theta'_2 s_2 = 0$. Budget balance for (θ'_1, θ_2) gives (B3) $h_1(\theta_2) + h_2(\theta'_1) - \theta_2 s_2 = 0$. Finally, budget balance for the profile (θ'_1, θ'_2) gives (B4) $h_1(\theta'_2) + h_2(\theta'_1) - \theta'_1 s_1 = 0$. By adding (B1) and (B4) and subtracting both (B2) and (B3) from it we get $\theta'_2 s_2 - \theta'_1 s_1 = 0$ which contradicts the restriction that $\theta'_1 s_1 \neq \theta'_2 s_2$. $\qquad\square$

Definition 9. Consider any Ω^s_N with at least three agents. A mechanism $(\tilde{\sigma}, \tilde{\tau}^*)$ is a *balanced rVCG mechanism* if the transfer rule is such that for all $\theta \in \Theta^n$ and all $i \in N$,

(14)
$$
\tilde{\tau}^*_i(\theta) = \begin{cases}
\sum\limits_{j \in N \setminus \{i\}} \left(\frac{\tilde{\sigma}_j(\theta) - 1}{n-2} \right) \theta_j s_j + g_i(\theta_{-i}) & \text{if } P'_i(\tilde{\sigma}(\theta)) = \emptyset, \\[2ex]
- \sum\limits_{j \in N \setminus \{i\}} \left(\frac{n - \tilde{\sigma}_j(\theta)}{n-2} \right) \theta_j s_j + g_i(\theta_{-i}) & \text{if } P_i(\tilde{\sigma}(\theta)) = \emptyset, \\[2ex]
\sum\limits_{j \in P_i(\tilde{\sigma}(\theta))} \left(\frac{\tilde{\sigma}_j(\theta) - 1}{n-2} \right) \theta_j s_j - \sum\limits_{j \in P'_i(\tilde{\sigma}(\theta))} \left(\frac{n - \tilde{\sigma}_j(\theta)}{n-2} \right) \theta_j s_j + g_i(\theta_{-i}) & \text{otherwise.}
\end{cases}
$$

where $g_i : \Theta^{|N \setminus \{i\}|} \to \mathbb{R}$ for each $i \in N$ is such that for any $\theta \in \Theta^n$, $\sum_{i \in N} g_i(\theta_{-i}) = 0$.

The balanced rVCG transfer requires that for any profile $\theta \in \Theta^n$ and given any agent specific constant transfer $g_i(\theta_{-i})$ to agent i, each agent i, in addition, receives as reward a position specific weighted sum of the minimum cost of all agents served before him $(\sum_{j \in P_i(\tilde{\sigma}(\theta))} [(\tilde{\sigma}_j(\theta) - 1)/(n-2)] \theta_j s_j)$ under the refined Ralwsian sequencing order $\tilde{\sigma}(\theta)$ (provided $P_i(\tilde{\sigma}(\theta)) \neq \emptyset$) and agent i also pays a position specific weighted sum of the minimum cost of all agents served after him $(\sum_{j \in P'_i(\tilde{\sigma}(\theta))} [(n - \tilde{\sigma}_j(\theta))/(n-2)] \theta_j s_j)$ under the refined Ralwsian sequencing order $\tilde{\sigma}(\theta)$ (provided $P'_i(\tilde{\sigma}(\theta)) \neq \emptyset$). In addition, the balanced rVCG mechanism also requires that selection of the function $g_i : \Theta^{|N \setminus \{i\}|} \to \mathbb{R}$ for each agent $i \in N$ must be such that for any $\theta \in \Theta^n$, $\sum_{j \in N} g_j(\theta_{-j}) = 0$.

Theorem 3. Let Ω^s_N be a sequencing problem with at least three agents. The just sequencing rule is implementable with balanced transfers if and only if the mechanism $(\tilde{\sigma}, \tau)$ that implements it is a balanced rVCG mechanism.

Proof: We know that the just sequencing rule is implementable if and only if the mechanism is an rVCG mechanism (Theorem 2). Therefore, identifying the complete class of

mechanisms that implements the just sequencing rule with balanced transfers reduces to identifying the complete class of balanced rVCG mechanisms.

Suppose there is the refined Ralwsian order $\tilde{\sigma}(\theta)$ for the profile $\theta \in \Theta^n$ and agent i leaves. We define the "induced" order $\tilde{\sigma}(\theta_{-i})$ (of length $|N \setminus \{i\}|$) for the agents in $N \setminus \{i\}$ as follows:

$$
(15) \qquad \tilde{\sigma}_j(\theta_{-i}) = \begin{cases} \tilde{\sigma}_j(\theta) - 1 & \text{if } j \in P_i'(\tilde{\sigma}(\theta)), \\ \tilde{\sigma}_j(\theta) & \text{if } j \in P_i(\tilde{\sigma}(\theta)). \end{cases}
$$

In words, $\tilde{\sigma}(\theta_{-i})$ is the order formed by removing agent i and moving all agents behind him up by one position. Given the same tie-breaking rule for the economy with $N \setminus \{i\}$ agents, it is easy to see that $\tilde{\sigma}(\theta_{-i})$ is also just in $N \setminus \{i\}$ for the profile θ_{-i} if $\tilde{\sigma}(\theta)$ is just for the profile θ.

Given that the sequencing problem Ω_N^s has at least three agents, without loss of generality, we redefine the rVCG mechanisms by setting for each agent $i \in N$ and each profile θ_{-i},

$$
(16) \qquad h_i(\theta_{-i}) = \sum_{j \in N \setminus \{i\}} \left(\frac{\tilde{\sigma}_j(\theta_{-i}) - 1}{n - 2} \right) \theta_j s_j + g_i(\theta_{-i}),
$$

where $g_i : \Theta^{|N \setminus \{i\}|} \to \mathbb{R}$ is arbitrary. By substituting (16) in the rVCG transfer (7) we get the following for any $i \in N$ and any $\theta \in \Theta^n$.

(A) If $P_i'(\tilde{\sigma}(\theta)) = \emptyset$, then $\tilde{\sigma}_j(\theta_{-i}) = \tilde{\sigma}_j(\theta)$ for all $j \in N \setminus \{i\}$ and from (7) we get
$\tilde{\tau}_i(\theta) = h_i(\theta_{-i}) = \sum_{j \in N \setminus \{i\}} \left(\frac{\tilde{\sigma}_j(\theta) - 1}{n - 2} \right) \theta_j s_j + g_i(\theta_{-i})$.

(B) If $P_i(\tilde{\sigma}(\theta)) = \emptyset$, then $\tilde{\sigma}_j(\theta_{-i}) = \tilde{\sigma}_j(\theta) - 1$ for all $j \in N \setminus \{i\}$ and from (7) we get
$\tilde{\tau}_i(\theta) = -\sum_{j \in N \setminus \{i\}} \theta_j s_j + h_i(\theta_{-i}) = -\sum_{j \in N \setminus \{i\}} \theta_j s_j + \sum_{j \in N \setminus \{i\}} \left(\frac{\tilde{\sigma}_j(\theta) - 2}{n - 2} \right) \theta_j s_j + g_i(\theta_{-i}) = -\sum_{j \in N \setminus \{i\}} \left(\frac{n - \tilde{\sigma}_j(\theta)}{n - 2} \right) \theta_j s_j + g_i(\theta_{-i})$.

(C) If $P_i'(\tilde{\sigma}(\theta)) \neq \emptyset$ and $P_i(\tilde{\sigma}(\theta)) \neq \emptyset$, then $\tilde{\sigma}_j(\theta_{-i}) = \tilde{\sigma}_j(\theta)$ for all $j \in P_i(\tilde{\sigma}(\theta)) \neq \emptyset$, $\tilde{\sigma}_j(\theta_{-i}) = \tilde{\sigma}_j(\theta) - 1$ for all $j \in P_i'(\tilde{\sigma}(\theta)) \neq \emptyset$ and from (7) we get
$\tilde{\tau}_i(\theta) = -\sum_{j \in P_i'(\tilde{\sigma}(\theta))} \theta_j s_j + h_i(\theta_{-i})$

$= -\sum_{j \in P_i'(\tilde{\sigma}(\theta))} \theta_j s_j + \sum_{j \in P_i(\tilde{\sigma}(\theta))} \left(\frac{\tilde{\sigma}_j(\theta) - 1}{n - 2} \right) \theta_j s_j + \sum_{j \in P_i'(\tilde{\sigma}(\theta))} \left(\frac{\tilde{\sigma}_j(\theta) - 2}{n - 2} \right) \theta_j s_j + g_i(\theta_{-i})$

$= \sum_{j \in P_i(\tilde{\sigma}(\theta))} \left(\frac{\tilde{\sigma}_j(\theta) - 1}{n - 2} \right) \theta_j s_j - \sum_{j \in P_i'(\tilde{\sigma}(\theta))} \left(\frac{n - \tilde{\sigma}_j(\theta)}{n - 2} \right) \theta_j s_j + g_i(\theta_{-i})$.

Conditions (A)-(C) show that when the sequencing problem Ω_N^s has at least three agents, then an equivalent representation of the rVCG transfers is that for any $i \in N$ and any $\theta \in \Theta^n$,

(17)
$$\tilde{\tau}_i'(\theta) = \begin{cases} \displaystyle\sum_{j\in N\setminus\{i\}} \left(\frac{\tilde{\sigma}_j(\theta)-1}{n-2}\right)\theta_j s_j + g_i(\theta_{-i}) & \text{if } P_i'(\tilde{\sigma}(\theta)) = \emptyset, \\[2ex] \displaystyle -\sum_{j\in N\setminus\{i\}} \left(\frac{n-\tilde{\sigma}_j(\theta)}{n-2}\right)\theta_j s_j + g_i(\theta_{-i}) & \text{if } P_i(\tilde{\sigma}(\theta)) = \emptyset, \\[2ex] \displaystyle\sum_{j\in P_i(\tilde{\sigma}(\theta))} \left(\frac{\tilde{\sigma}_j(\theta)-1}{n-2}\right)\theta_j s_j - \sum_{j\in P_i'(\tilde{\sigma}(\theta))} \left(\frac{n-\tilde{\sigma}_j(\theta)}{n-2}\right)\theta_j s_j + g_i(\theta_{-i}) & \text{otherwise,} \end{cases}$$

where for each $i \in N$, the function $g_i : \Theta^{|N\setminus\{i\}|} \to \mathbb{R}$ is arbitrary. By taking the representation (17) of the rVCG transfer we get for any $\theta \in \Theta^n$,

$$\sum_{k\in N} \tilde{\tau}_k'(\theta) = \sum_{k\in N}\left\{\sum_{j\in P_k(\tilde{\sigma}(\theta))}\left(\frac{\tilde{\sigma}_j(\theta)-1}{n-2}\right)\theta_j s_j - \sum_{j\in P_k'(\tilde{\sigma}(\theta))}\left(\frac{n-\tilde{\sigma}_j(\theta)}{n-2}\right)\theta_j s_j\right\} + \sum_{k\in N} g_k(\theta_{-k})$$

$$= \sum_{k\in N}\sum_{j\in P_k(\tilde{\sigma}(\theta))}\left(\frac{\tilde{\sigma}_j(\theta)-1}{n-2}\right)\theta_j s_j - \sum_{k\in N}\sum_{j\in P_k'(\tilde{\sigma}(\theta))}\left(\frac{n-\tilde{\sigma}_j(\theta)}{n-2}\right)\theta_j s_j + \sum_{k\in N} g_k(\theta_{-k})$$

$$= \sum_{j\in N}\left\{|P_j'(\tilde{\sigma}_j(\theta))|\left(\frac{\tilde{\sigma}_j(\theta)-1}{n-2}\right)\right\}\theta_j s_j - \sum_{j\in N}\left\{|P_j(\tilde{\sigma}_j(\theta))|\left(\frac{n-\tilde{\sigma}_j(\theta)}{n-2}\right)\right\}\theta_j s_j + \sum_{k\in N} g_k(\theta_{-k})$$

$$= \sum_{j\in N}\left\{(n-\tilde{\sigma}_j(\theta))\left(\frac{\tilde{\sigma}_j(\theta)-1}{n-2}\right)\right\}\theta_j s_j - \sum_{j\in N}\left\{(\tilde{\sigma}_j(\theta)-1)\left(\frac{n-\tilde{\sigma}_j(\theta)}{n-2}\right)\right\}\theta_j s_j + \sum_{k\in N} g_k(\theta_{-k})$$

$$= \sum_{k\in N} g_k(\theta_{-k}).$$

Therefore, by taking the representation (17) of the rVCG transfer we have proved that for any $\theta \in \Theta^n$, $\sum_{k\in N}\tilde{\tau}_k'(\theta) = \sum_{k\in N} g_k(\theta_{-k})$. Therefore, an rVCG mechanism represented by (17) is budget balanced if and only if (I) for all $\theta \in \Theta^n$, $\sum_{k\in N} g_k(\theta_{-k}) = 0$. From the representation (17) of the rVCG transfers and from condition (I), the result follows. □

Remark 3. There are papers that show that for the family of sequencing problems Ω_N^s with three or more agents we can find budget balanced VCG mechanisms and hence implementation with balanced transfer is also possible with the outcome efficient sequencing rule (see Mitra [25] and Suijs [33]). From Remark 1 it follows that for the queueing problem Ω_N^{s*}, the set of all balanced rVCG mechanism coincides with the set of all VCG mechanisms that first best implements the outcome efficient sequencing rule.

4.3. **Two-dimensional incentives and the just sequencing rule.** If we assume that both waiting cost and the processing time are agent specific private information, then we have the mechanism design problem for the general sequencing problem $\Omega_N =$

$(\Theta^n, \mathcal{S})$. Specifically, we have an interdependent value situation and therefore the correct notion is ex-post implementability. The type of any agent $i \in N$ is $m_i = (\theta_i, s_i) \in \Theta \times \mathbb{R}_{++}$ that constitutes of his waiting cost as well as his processing time. A profile is $m = (m_1, \ldots, m_n) \in \Theta^n \times \mathcal{S}$. For any $i \in N$, let m_{-i}, denote the profile $(m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_n) \in \Theta^{|N \setminus \{i\}|} \times \mathbb{R}_{++}^{|N \setminus \{i\}|}$ which is obtained from the profile m by eliminating i's type. For the general sequencing problem Ω_N, a (direct revelation) mechanism is (σ^g, τ) that constitutes of a general sequencing rule σ^g and a transfer rule τ. A *general sequencing rule* is a function $\sigma^g : \Theta^n \times \mathcal{S} \to \Sigma(N)$ that specifies for each profile $m \in \Theta^n \times \mathcal{S}$, a unique order $\sigma^g(m) = (\sigma_1^g(m), \ldots, \sigma_n^g(m)) \in \Sigma(N)$. A *transfer rule* is a function $\tau : \Theta^n \times \mathcal{S} \to \mathbb{R}^n$ that specifies for each profile $m \in \Theta^n \times \mathcal{S}$ a transfer vector $\tau(m) = (\tau_1(m), \ldots, \tau_n(m)) \in \mathbb{R}^n$. Specifically, for Ω_N and given any mechanism (σ^g, τ), if m_i' is the announced type of agent i when his true type is m_i and m_{-i} is the true profile for agents $N \setminus \{i\}$, then the utility of i is given by $v_i(\sigma^g(m_i', m_{-i}), \tau_i(m_i', m_{-i}); m_i; s_{-i}(m_{-i})) = -\theta_i \left(s_i + \sum_{j \in P_i(\sigma^g(m_i', m_{-i}))} s_j \right) + \tau_i(m_i', m_{-i})$.

Definition 10. A mechanism (σ^g, τ) *ex-post implements* the general sequencing rule σ^g if the transfer rule $\tau : \Theta^n \times \mathcal{S} \to \mathbb{R}^n$ is such that for any $i \in N$, any m_i, m_i' and any true profile m_{-i},

$$(18) \qquad v_i(\sigma^g(m), \tau_i(m); m_i; s_{-i}(m_{-i})) \geq v_i(\sigma^g(m_i', m_{-i}), \tau_i(m_i', m_{-i}); m_i; s_{-i}(m_{-i})).$$

Ex-post implementability requires truth-telling is a Nash equilibrium for any agent and for every true type profile m.

Definition 11. A mechanism (σ^g, τ) *ex-post implements with balanced transfers* the general sequencing rule σ^g if the transfer rule $\tau : \Theta^n \times \mathcal{S} \to \mathbb{R}^n$ satisfies ex-post implementability condition (18) and is also budget balanced, that is for all $m \in \Theta^n \times \mathcal{S} \to \mathbb{R}^n$, $\sum_{j \in N} t_j(m) = 0$.

Definition 12. A general sequencing rule $\tilde{\sigma}^g$ is *just* if for each profile $m \in \Theta^n \times \mathcal{S}$, the chosen order $\tilde{\sigma}^g(m)$ satisfies the following property: for any $i, j \in N$ such that $\theta_i \geq \theta_j$, $\tilde{\sigma}_i^g(m) \leq \tilde{\sigma}_j^g(m)$.[12]

For any true $m = (m_1 = (\theta_1, s_1), \ldots, m_n = (\theta_n, s_n))$ we say θ *is obtained from* m if it is a collection of the first element from $m_j = (\theta_j, s_j)$ for each $j \in N$ and we say s *is obtained from* m if it is a collection of the second element from $m_j = (\theta_j, s_j)$ for each $j \in N$. Similarly, for any $i \in N$ and any true $m_{-i} = (m_1 = (\theta_1, s_1), \ldots, m_{i-1} = $

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[12]The tie-breaking rule guarantees state contingent uniqueness of $\tilde{\sigma}^g$.

$(\theta_{i-1}, s_{i-1}), m_{i+1} = (\theta_{i+1}, s_{i+1}), \ldots, m_n = (\theta_n, s_n))$ we say θ_{-i} is obtained from m_{-i} if it is a collection of the first element from $m_j = (\theta_j, s_j)$ for each $j \in N \setminus \{i\}$ and we say s_{-i} is obtained from m_{-i} if it is a collection of the second element from $m_j = (\theta_j, s_j)$ for each $j \in N \setminus \{i\}$. For Ω_N, the just general sequencing rule satisfies the following: For any $m = (m_1, \ldots, m_n) \in \Theta^n \times \mathcal{S}$, $\tilde{\sigma}^g(m) = \tilde{\sigma}(\theta)$ where θ is obtained from m.

Definition 13. For $\tilde{\sigma}^g$, a mechanism $(\tilde{\sigma}^g, \tilde{\tau})$ is a *generalized rVCG mechanism* if the transfer rule is such that for all $m \in \Theta^n \times \mathcal{S}$ and all $i \in N$,

$$(19) \qquad \tilde{\tau}_i^g(\theta_i, m_{-i}) = \begin{cases} h_i^g(m_{-i}) & \text{if } P_i'(\tilde{\sigma}(\theta_i, \theta_{-i})) = \emptyset, \\ h_i^g(m_{-i}) - \displaystyle\sum_{j \in P_i'(\tilde{\sigma}(\theta_i, \theta_{-i}))} \theta_j s_j & \text{otherwise.} \end{cases}$$

where θ_{-i} is obtained from m_{-i} and $h_i^g : \Theta^{|N \setminus \{i\}|} \times \mathbb{R}_{++}^{|N \setminus \{i\}|} \to \mathbb{R}$ is arbitrary.

Proposition 3. The just general sequencing rule $\tilde{\sigma}^g$ is ex-post implementable if and only if the mechanism $(\tilde{\sigma}^g, \tau)$ that implements it is a generalized rVCG mechanism.

Proof: We only prove that if the just general sequencing rule $\tilde{\sigma}^g$ is ex-post implementable via a mechanism $(\tilde{\sigma}^g, \tau)$, then the mechanism is a generalized rVCG mechanism. The other part is easy and hence omitted.

Consider any $i \in N$ and fix any true type m_{-i} for all agents other than i. For any true type $m_i = (\theta_i, s_i)$ of i and any misreport $m_i' = (\theta_i', s_i')$ by i, ex-post implementability of the just general sequencing rule $\tilde{\sigma}^g$ requires $v_i(\tilde{\sigma}^g(m), \tau_i(m); m_i; s_{-i}(m_{-i})) \geq v_i(\tilde{\sigma}^g(m_i', m_{-i}), \tau_i(m_i', m_{-i}); m_i; s_{-i}(m_{-i}))$ which is the same as requiring

$$(20)$$
$$v_i(\tilde{\sigma}(\theta_i, \theta_{-i}), \tau_i(m_i, m_{-i}); m_i; s_{-i}(m_{-i})) \geq v_i(\tilde{\sigma}(\theta_i', \theta_{-i}), \tau_i(m_i', m_{-i}); m_i, s_{-i}(m_{-i})).$$

Moreover, given the true type m_{-i} of all agents and given any pair of types $(m_i' = (\theta_i', s_i'), m_i'' = (\theta_i', s_i''))$ for i with the property the waiting cost is identical in m_i' and m_i'', $\tilde{\sigma}^g(m_i', m_{-i}) = \tilde{\sigma}(\theta_i', \theta_i) = \tilde{\sigma}^g(m_i'', m_{-i})$ since just sequencing rule ignores the processing time. Hence, using ex-post implementability, we get the following inequalities:

(1) $-\theta_i'(s_i' + \displaystyle\sum_{j \in P_i(\tilde{\sigma}(\theta_i', \theta_{-i}))} s_j) + \tau_i(m_i', m_{-i}) \geq -\theta_i'(s_i' + \displaystyle\sum_{j \in P_i(\tilde{\sigma}(\theta_i', \theta_{-i}))} s_j) + \tau_i(m_i'', m_{-i})$.

(2) $-\theta_i'(s_i'' + \displaystyle\sum_{j \in P_i(\tilde{\sigma}(\theta_i', \theta_{-i}))} s_j) + \tau_i(m_i', m_{-i}) \leq -\theta_i'(s_i'' + \displaystyle\sum_{j \in P_i(\tilde{\sigma}(\theta_i', \theta_{-i}))} s_j) + \tau_i(m_i'', m_{-i})$.

Inequality (1) gives $\tau_i(m_i', m_{-i}) \geq \tau_i(m_i'', m_{-i})$ and inequality (2) gives $\tau_i(m_i', m_{-i}) \leq \tau_i(m_i'', m_{-i})$ and therefore, $\tau_i(m_i', m_{-i}) = \tau_i(m_i'', m_{-i})$. Hence we have

$$(21) \qquad\qquad \tau_i(m_i', m_{-i}) = \tau_i(m_i'', m_{-i}) := \tau_i(\theta_i', m_{-i}).$$

Using (21) in (20) we get

(22) $\quad v_i(\tilde{\sigma}(\theta_i, \theta_{-i}), \tau_i(\theta_i, m_{-i}); m_i; s_{-i}(m_{-i})) \geq v_i(\tilde{\sigma}(\theta'_i, \theta_{-i}), \tau_i(\theta'_i, m_{-i}); m_i, s_{-i}(m_{-i})).$

As long as the true type m_{-i} of agents $N \setminus \{i\}$ is known, the processing time of these agents s_{-i} obtained from m_{-i} is known and θ_{-i} obtained from m_{-i} is also known and hence the waiting cost cut-off vector $(\theta_{\mu(1)}, \ldots, \theta_{\mu(R)})$ for agent i with the just sequencing rule $\tilde{\sigma}$ is also known and the calculation of the transfer for agent i is independent of the processing time of agent i. Therefore, from inequality (22) we get that for any given $s_i > 0$, the following inequality must be true.

(23) $\qquad U_i(\tilde{\sigma}(\theta_i, \theta_{-i}), \tau_i(\theta_i, m_{-i}); \theta_i) \geq U_i(\tilde{\sigma}(\theta'_i, \theta_{-i}), \tau_i(\theta'_i, m_{-i}); \theta_i).$

Note that if inequality (23) holds for some given $s_i > 0$ and for all $\theta_i \in \Theta$, then for any other $s'_i > 0$ with the same transfer, inequality (23) holds for any $\theta_i \in \Theta$ since s_i (either actual or misreported) does not enter the calculation of the transfer $\tau_i(\theta_i, m_{-i})$. The true processing time s_i only enters in the calculation of the completion time $S_i(\tilde{\sigma})$.

Inequality (23) is similar to the implementability inequality (1) with $\tau_i(\theta_i, \theta_{-i})$ of inequality (1) replaced by $\tau_i(\theta_i, m_{-i})$ in (23) where θ_{-i} is the waiting cost vector obtained from m_{-i}. Moreover, the waiting cost cut-off vector $(\theta_{\mu(1)}, \ldots, \theta_{\mu(R)})$ for the just sequencing rule is same for both m_{-i} and θ_{-i} when θ_{-i} is obtained from m_{-i}. Hence for any true m_{-i}, true θ_{-i} obtained from m_{-i} and any $\theta_i, \theta'_i \in \Theta$, we must have (B) $\tau_i(\theta_i, \theta_{-i}) - \tau_i(\theta'_i, \theta_{-i}) = \tau_i(\theta_i, m_{-i}) - \tau_i(\theta'_i, m_{-i})$. From (B) it follows that for any $\theta_i \in \Theta$, any true m_{-i} and θ_{-i} obtained from m_{-i}, (C) $\tau_i(\theta_i, m_{-i}) - \tau_i(\theta_i, \theta_{-i}) := g_i(m_{-i})$. Recall that implementability of the just sequencing rule $\tilde{\sigma}$ requires that the transfer must be an rVCG transfer, that is for any θ_{-i}, and any $\theta_i \in \Theta$, the transfer $\tau_i(\theta_i, \theta_{-i})$ in (C) must be equal to the rVCG transfer $\tilde{\tau}_i(\theta_i, \theta_{-i})$ given by (7). Hence from (C) we get

(24) $\quad \tau_i(\theta_i, m_{-i}) = \begin{cases} h_i(\theta_{-i}) + g_i(m_{-i}) & \text{if } P'_i(\tilde{\sigma}(\theta_i, \theta_{-i})) = \emptyset, \\ h_i(\theta_{-i}) + g_i(m_{-i}) - \sum\limits_{j \in P'_i(\tilde{\sigma}(\theta_i, \theta_{-i}))} \theta_j s_j & \text{otherwise.} \end{cases}$

where the functions $h_i : \Theta^{|N \setminus \{i\}|} \to \mathbb{R}$ and $g_i : \Theta^{|N \setminus \{i\}|} \times \mathbb{R}_{++}^{|N \setminus \{i\}|} \to \mathbb{R}$ are arbitrary. Without loss of generality, by setting $h_i(\theta_{-i}) + g_i(m_{-i}) = h_i^g(m_{-i})$ in (24) we get $\tau_i(\theta_i, m_{-i}) := \tilde{\tau}_i^g(\theta_i, m_{-i})$ and the result follows. $\qquad \square$

That ex-post implementable with balanced transfers is not possible for $\Omega_{N=\{1,2\}}$ with two agents is a natural extension of the arguments in Proposition 2. By applying arguments similar to that in Theorem 3, it is easy to see that the just general sequencing rule $\tilde{\sigma}^g$ is ex-post implementable with balanced transfers.

Definition 14. Consider the general sequencing problem Ω_N with at least three agents. For $\tilde{\sigma}^g$, a mechanism $(\tilde{\sigma}^g, \tilde{\tau}^{g*})$ is a *balanced generalized rVCG mechanism* if the transfer rule is such that for all $m \in \Theta^n \times \mathcal{S}$ and all $i \in N$,

(25)

$$\tilde{\tau}_i^{g*}(\theta_i, m_{-i}) = \begin{cases} \sum\limits_{j \in N\backslash\{i\}} A_j(\theta_i)\theta_j s_j + g_i^g(m_{-i}) & \text{if } P_i' = \emptyset, \\ -\sum\limits_{j \in N\backslash\{i\}} B_j(\theta_i)\theta_j s_j + g_i^g(m_{-i}) & \text{if } P_i = \emptyset, \\ \sum\limits_{j \in P_i(\tilde{\sigma}(\theta_i, \theta_{-i}))} A_j(\theta_i)\theta_j s_j - \sum\limits_{j \in P_i'(\tilde{\sigma}(\theta_i, \theta_{-i}))} B_j(\theta_i)\theta_j s_j + g_i^g(m_{-i}) & \text{otherwise.} \end{cases}$$

where θ_{-i} is obtained from m_{-i}, $A_j(\theta_i) := (\tilde{\sigma}_j(\theta_i, \theta_{-i}) - 1)/(n-2)$, $B_j(\theta_i) := (n - \tilde{\sigma}_j(\theta_i, \theta_{-i}))/(n-2)$ and for any $i \in N$, $g_i^g : \Theta^{|N\backslash\{i\}|} \times \mathbb{R}_{++}^{|N\backslash\{i\}|} \to \mathbb{R}$ is such that for any $m \in \Theta^n \times \mathcal{S}$, $\sum_{i \in N} g_i^g(m_{-i}) = 0$.

Proposition 4. Let Ω_N be the general sequencing problem with at least three agents. The just general sequencing rule $\tilde{\sigma}^g$ is ex-post implementable with balanced transfers if and only if the mechanism $(\tilde{\sigma}^g, \tau)$ that implements it is a balanced generalized rVCG mechanism.

5. CONCLUSIONS

We have argued that the nice properties of the just sequencing rule from a mechanism design perspective are the following.

(1) It is a Rawlsian sequencing rule which is implementable in dominant strategies.
(2) It is an affine cost minimizer.
(3) Its implementation is costless, that is, it is implementable with balanced transfers.
(4) Unlike outcome efficiency, even when both waiting cost and processing time are private information it is ex-post implementable with balanced transfers.

The cost of selecting the just sequencing rule is the efficiency loss, that is, the profile contingent difference in aggregate completion cost between the just sequencing rule and the outcome efficient sequencing rule. When the processing time of the agents are non-identical, this loss is positive for profiles where the order under the just sequencing rule is different from that of the outcome efficient sequencing rule.

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