THE PARADOX OF THE PREFACE*

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In a number of recent papers I have been developing the theory of "nomic probability," which is supposed to be the kind of probability involved in statistical laws of nature. One of the main principles of this theory is an acceptance rule explicitly designed to handle the lottery paradox. This paper shows that the rule can also handle the paradox of the preface. The solution proceeds in part by pointing out a surprising connection between the paradox of the preface and the gambler's fallacy.

1. Introduction. There once was a man who wrote a book. He was very careful in his reasoning, and was confident of each claim that he made. With some display of pride, he showed the book to a friend (who happened to be a probability theorist). He was dismayed when the friend observed that any book that long and that interesting was almost certain to contain at least one falsehood. Thus it was not reasonable to believe that all of the claims made in the book were true. If it were reasonable to believe each claim, then it would be reasonable to believe that the book contained no falsehoods; so it could not be reasonable to believe each claim. Furthermore, because there was no way to pick out some of the claims as being more problematic than others, there could be no reasonable way of withholding assent to some but not others. "Therefore," concluded his friend, "you are not justified in believing anything you asserted in the book."

This is the paradox of the preface (so named because in the original version the author confesses in the preface that his book probably contains a falsehood).[1] The paradox of the preface is more than a curiosity. It has been used by some philosophers to argue that the set of one's warranted beliefs need not be deductively consistent, and by others to argue that you should not befriend probability theorists. Any account of probabilistic acceptance rules must be capable of explaining what is involved in the paradox of the preface. I have recently defended an acceptance rule that was motivated by a different but related paradox—the lottery paradox.[2]

*Received October 1984; revised January 1985.

[1]The paradox of the preface originated with D. C. Makinson (1965).
[2]The lottery paradox is due to Henry E. Kyburg, Jr. (1961).

Philosophy of Science, 53 (1986) pp. 246–258.

If that acceptance rule is correct it should also lead to a resolution of the paradox of the preface. The purpose of this paper is to show that it does. Along the way we will learn as much about the structure of reasoning as about acceptance rules.

The lottery paradox and the paradox of the preface seem superficially similar, so it might be supposed that a resolution of one will automatically generate a resolution of the other. But in fact, the opposite is true. We will find that the epistemic principle that makes possible the resolution of the lottery paradox is actually the principle responsible for the creation of the paradox of the preface. The resolution of the paradox of the preface requires an interesting amount of additional machinery over and above what is required for the lottery paradox.

2. An Acceptance Rule. Nomic probability is the kind of probability involved in statistical laws of nature. Where A and B are properties, prob(A/B) can be regarded, at least heuristically, as a measure of the proportion of physically possible B's that would be A's.[3] I have proposed (1983 and 1984) the following acceptance rule:

(A1) If F is projectible with respect to G then $\ulcorner Gc$ & $\mathrm{prob}(F/G) \geq r\urcorner$ is a prima facie reason for $\ulcorner Fc\urcorner$, the strength of the reason depending upon the value of r.

I will explain various parts of this rule and then show how it handles the lottery paradox.

The first thing to note is that the rule provides us with only a prima facie reason for believing $\ulcorner Gc\urcorner$. Prima facie reasons are "defeasible." That is, if we have no other relevant information, then they can justify us in believing their conclusions, but if certain kinds of additional information are acquired ("defeaters") then we may no longer be justified in believing the conclusion even though we are still justified in believing the original reason. Enumerative induction provides the most obvious example of a prima facie reason. Observation of a sample of A's all of which are B's may justify us in believing that all A's are B's. If we subsequently observe an A that is not a B then we cease to be justified in believing that all A's are B's despite the fact that we still have our original evidence. That original evidence provides us with a prima facie reason for believing the generalization, but the new observation provides us with a defeater. Other prima facie reasons are common throughout epistemology. For example, something's looking red to me provides me with a prima facie reason for thinking it is red.[4]

[3]For a more detailed discussion of nomic probability, see Pollock (1984b).

[4]See Pollock (1974) for a detailed discussion of prima facie reasons and their role in epistemology.

There are two kinds of defeaters for prima facie reasons. *Rebutting defeaters* are simply reasons for denying the conclusion. In induction, observation of a counterexample constitutes a rebutting defeater. There are also *undercutting defeaters,* which attack the connection between the reason and the conclusion rather than just attacking the conclusion. For example, we may object to a piece of inductive reasoning on the grounds that the sample was not chosen at random. This is not a reason for thinking that the conclusion of the inductive reasoning is false, but it does defeat the reasoning. What it does is attack the connection between the prima facie reason and the conclusion. In general, if P is a prima facie reason for Q, an undercutting defeater is any reason for denying that P would not be true unless Q were true.

The reason provided by (A1) is only a prima facie reason, and as such it is defeasible. As with any prima facie reason, it can be defeated by having a reason for denying the conclusion. The reason for denying the conclusion constitutes a rebutting defeater. But there is also an important kind of undercutting defeater for (A1). In (A1), we infer the truth of $\ulcorner Fc \urcorner$ on the basis of probabilities conditional on a limited set of facts about c (that is, the facts expressed by $\ulcorner Gc \urcorner$). But if we know additional facts about c that lower the probability, that defeats the prima facie reason:

(D1) If F is projectible with respect to H then $\ulcorner Hc \;\&\; \text{prob}(F/G\&H) < \text{prob}(F/G) \urcorner$ is an undercutting defeater for (A1).

This amounts to a kind of "total evidence requirement." It requires us to make our inference on the basis of the most comprehensive facts regarding which we know the requisite probabilities.[5] For example, suppose that I read a putative theorem in a topology textbook written by an eminent topologist. It is highly probable that such a putative theorem is true, and that gives me a strong prima facie reason for believing it. But suppose the theorem occurs in chapter 5, and I read a review of the book in which it is observed that a subtle error was made early in chapter 5 and it affects many of the subsequent theorems. The probability of a putative theorem being true under those circumstances is much lower and that correspondingly weakens any reason I may have for believing it.

One of the most puzzling features of (A1) is the projectibility constraint, but that turns out not to be relevant to the concerns of the present

[5]Notice that according to (D1), the use of (A1) is defeated even if prob(F/G & H) is just minutely smaller than prob(F/G). That might seem unreasonable. But this does not mean that you cannot use (A1) to infer $\ulcorner Fc \urcorner$—it just means that you cannot infer $\ulcorner Fc \urcorner$ from $\ulcorner Gc \;\&\; \text{prob}(F/G) \geq r \urcorner$. If prob($F/G$ & H) is only minutely smaller than prob(F/G), what you can do instead is infer $\ulcorner Fc \urcorner$ from $\ulcorner Gc \;\&\; Hc \;\&\; \text{prob}(F/G) \geq s \urcorner$ (for an appropriately smaller s). This still gives you a prima facie reason for $\ulcorner Fc \urcorner$, albeit a slightly weaker one.

paper, and as I have discussed it at length elsewhere (1983; 1984a), I will not discuss it further here. Let us turn instead to the question of how (A1) handles the lottery paradox. Suppose you hold one ticket in a fair lottery consisting of one million tickets, and suppose it is known that one and only one ticket will be drawn. Observing that the probability that your own ticket will be drawn is only .000001, it seems reasonable to accept the conclusion that your ticket will not win. But by the same reasoning, it will be reasonable to believe, for each ticket, that it will not win. This will make the set of warranted propositions deductively inconsistent. Assuming that to be impossible, it follows that we are not warranted in believing of each ticket that it will not win. Henry Kyburg opts instead for allowing the set of warranted propositions to be deductively inconsistent.[6] I discussed that position at length (1983) and rejected it on the grounds that it robs deductive reasoning of any significant role in the acquisition of warranted belief. I will not discuss Kyburg's position further here, but will just assume that the set of warranted propositions must be consistent.

The lottery paradox is resolved by appealing to a general epistemic principle that I call 'the principle of collective defeat'. Suppose we have equally good prima facie reasons for believing each member of a set of conclusions, and no defeaters for any of these prima facie reasons. But suppose the set of conclusions is a minimal set inconsistent with propositions we are independently warranted in believing. Then we cannot consistently accept the entire set of conclusions, and as we have no reason to prefer some of the conclusions to others, we cannot reasonably accept any of the conclusions. The conclusions are "collectively defeated." Applying this to the lottery, (A1) provides us with equally good prima facie reasons for believing of each ticket that it will not be drawn, but those conclusions are collectively inconsistent with something we are independently warranted in believing, namely, that some ticket will be drawn. Thus we have a case of collective defeat and are not warranted in concluding of any ticket that it will not be drawn.

The principle of collective defeat is not a primitive epistemic principle. It is a consequence of the way in which possibly conflicting prima facie reasons interact to determine what we are warranted in believing. Starting from propositions we are initially warranted in believing, we can construct arguments supporting other propositions. But that does not automatically make those additional propositions warranted, because some propositions supported in that way may be defeaters for steps of some of the other arguments. That is what happens in cases of collective defeat. Suppose we are warranted in believing a proposition R, and suppose we

[6]See particularly Kyburg (1970) and (1974).

have equally good prima facie reasons for each of $P_1, \ldots, P_n$, where $\{P_1, \ldots, P_n\}$ is a minimal set deductively inconsistent with R (that is, it is deductively inconsistent with R and has no proper subset deductively inconsistent with R). Then for each i, the conjunction $\ulcorner R \mathbin{\&} P_1 \mathbin{\&} \ldots \mathbin{\&} P_{i-1} \mathbin{\&} P_{i+1} \mathbin{\&} \ldots \mathbin{\&} P_n \urcorner$ entails $\sim P_i$. Thus by combining this entailment with the arguments for $P_1, \ldots, P_{i-1}, P_{i+1}, \ldots, P_n$ we obtain an argument for $\sim P_i$.[7] An argument is as good as its weakest link, so this argument for $\sim P_i$ is as good as the argument for P_i.[8] Thus we have equally strong support for both P_i and $\sim P_i$, and hence the reasons cancel one another. Under the circumstances we could not reasonably believe either P_i or $\sim P_i$, that is, neither is warranted. This holds for each i, so none of the P_i is warranted. They collectively defeat one another. Thus the principle of collective defeat can be formulated as follows:

(CD) If we are warranted in believing R and we have equally good prima facie reasons for each member of a minimal set of propositions deductively inconsistent with R, and none of these prima facie reasons is defeated in any other way, then none of the propositions in the set is warranted on the basis of these prima facie reasons.

The principle of collective defeat resolves the lottery paradox, but it will turn out that this same principle is actually responsible for the creation of the paradox of the preface.

3. The Gambler's Fallacy. I am going to argue that the paradoxical reasoning involved in the paradox of the preface is similar in important respects to a well-known fallacy in probabilistic reasoning—the gambler's fallacy. Let us begin by considering the latter. Suppose a fair coin is tossed six times and the first five tosses are heads.[9] We observe that it is highly improbable for six consecutive tosses to be heads, and this tempts us to conclude that the last toss will not be heads. Our intuitive reasoning consists of noting that

$$\begin{aligned}&\mathrm{prob}(Hx_1 \mathbin{\&} \ldots \mathbin{\&} Hx_6 / Tx_I \mathbin{\&} \ldots \mathbin{\&} Tx_6 \mathbin{\&} x_1, \ldots,\\ &x_6 \text{ distinct})\\ &= {}^1\!/_{64}.\end{aligned} \tag{3.1}$$

and using (A1) and the fact that $t_1, \ldots, t_6$ are distinct tosses to conclude $\ulcorner\sim(Ht_1 \mathbin{\&} \ldots \mathbin{\&} Ht_6)\urcorner$. Then because we know $\ulcorner Ht_1 \mathbin{\&} \ldots \mathbin{\&} Ht_5\urcorner$ we infer $\ulcorner\sim Ht_6\urcorner$.

[7]If $\{P_1, \ldots, P_n\}$ were not a *minimal* inconsistent set, then the antecedent of this entailment would be inconsistent and hence the entailment would not automatically constitute a reason for $\sim P_i$.

[8]The "weakest link" principle is defended in Pollock (1983).

[9]I will take a coin to be fair iff the probability of heads is $^1\!/_2$ and getting heads on one toss is statistically independent of the outcomes of any other finite set of tosses.

There is actually a very simple objection to this reasoning. It is not licensed by (A1) because of the projectibility constraint. Although $\ulcorner Hx_1 \,\&\, \ldots \,\&\, Hx_6 \urcorner$ is projectible (because projectibility is closed under conjunction), its negation is not. Disjunctions of projectible properties are not generally projectible, and $\ulcorner \sim(Hx_1 \,\&\, \ldots \,\&\, Hx_6) \urcorner$ is equivalent to the unprojectible property $\ulcorner \sim Hx_1 \vee \ldots \vee \sim Hx_6 \urcorner$.[10] But this is not the objection to the gambler's fallacy that I want to press. There is a more standard objection that is customarily levied at it. Because the coin is fair, we know that the tosses are independent, and hence:

$$\begin{aligned}&\mathrm{prob}(Hx_6/Hx_1 \,\&\, \ldots \,\&\, Hx_5 \,\&\, Tx_1 \,\&\, \ldots \,\&\, Tx_6 \,\&\, x_1, \ldots, \\ &x_6 \text{ distinct}) \\ &= {}^1\!/_2.\end{aligned} \tag{3.2}$$

By the probability calculus:

$$\begin{aligned}&\mathrm{prob}(Hx_1 \,\&\, \ldots \,\&\, Hx_6/Hx_1 \,\&\, \ldots \,\&\, Hx_5 \,\&\, \\ &Tx_1 \,\&\, \ldots \,\&\, Tx_6 \,\&\, x_1, \ldots, x_6 \text{ distinct}) \\ &= \mathrm{prob}(Hx_6/Hx_1 \,\&\, \ldots \,\&\, Hx_5 \,\&\, Tx_1 \,\&\, \ldots \,\&\, Tx_6 \,\&\, x_1, \\ &\ldots, x_6 \text{ distinct})\end{aligned} \tag{3.3}$$

Therefore,

$$\begin{aligned}&\mathrm{prob}(Hx_1 \,\&\, \ldots \,\&\, Hx_6/Hx_1 \,\&\, \ldots \,\&\, Hx_5 \,\&\, \\ &Tx_1 \,\&\, \ldots \,\&\, Tx_6 \,\&\, x_1, \ldots, x_6 \text{ distinct}) \\ &= {}^1\!/_2.\end{aligned} \tag{3.4}$$

Because ${}^1\!/_2 < {}^{63}\!/_{64}$ and we are warranted in believing $\ulcorner Ht_1 \,\&\, \ldots \,\&\, Ht_5 \urcorner$, (D1) provides us with a defeater for any inference from (3.1) in accordance with (A1). This is a reconstruction of the standard objection to the gambler's fallacy, and it is illuminating to see how it proceeds by appealing to (D1).

Next notice that we can reformulate the gambler's fallacy in a way that makes its resolution more difficult. Suppose that $t_1, \ldots, t_6$ are tosses of a particular fair coin c that is tossed just these six times and then melted down. Taking $\ulcorner Txy \urcorner$ to be $\ulcorner y$ is a toss of coin $x \urcorner$ and $\ulcorner Fx \urcorner$ to be $\ulcorner x$ is a fair coin$\urcorner$, we have:

$$\begin{aligned}&\mathrm{prob}[\![(\exists y)(Txy \,\&\, \sim Hy) \;/\; Fx \,\&\, (\exists y_1) \ldots (\exists y_6)\{y_1, \ldots, y_6 \\ &\text{are distinct} \,\&\, (\forall z)[Tzx \equiv (z = y_1 \vee \ldots \vee z = y_6)]\}]\!] = {}^{63}\!/_{64}.\end{aligned} \tag{3.5}$$

Because we also know

$$\begin{aligned}&Fc \,\&\, t_1, \ldots, t_6 \text{ are distinct} \,\&\, (\forall z) \\ &[Tzc \equiv (z = t_1 \vee \ldots \vee z = t_6)],\end{aligned} \tag{3.6}$$

[10]More accurately, the class of projectible properties is not closed under disjunction. I first pointed this out in Pollock (1974, pp. 233ff.). The arguments are repeated in Pollock (1983) and (1984a). Isaac Levi makes the same point in his (1967).

we are warranted in believing

$$Fc \,\&\, (\exists y_1) \ldots (\exists y_6)\{y_1, \ldots, y_6 \text{ are distinct } \& \\ (\forall z)[Tzc \equiv (z = y_1 \vee \ldots \vee z = y_6)]\}. \tag{3.7}$$

Therefore, by (A1), we have a prima facie reason for believing

$$(\exists y)(Tcy \,\&\, {\sim}Hy). \tag{3.8}$$

Having observed that $t_1, \ldots, t_5$ were heads, we can infer from (3.6) and (3.8) that t_6 is not heads. Thus we have the gambler's fallacy again in a more sophisticated guise.

I remarked above that an inference from (3.1) could be blocked by the projectibility constraint in (A1). That is at least not clearly true for an inference from (3.5). It is unclear what effect existential quantifiers have on projectibility so it is best not to try to resolve the problem in that way. Instead it seems intuitive that the same considerations that blocked the first version of the gambler's fallacy should also block this version. Different tosses of coin c are still independent of one another, so just as we had (3.2) before we now have:

$$\text{prob}\{Hy_6/Fx \,\&\, Hy_1 \,\&\, \ldots \,\&\, Hy_5 \,\&\, y_1, \ldots, y_6 \text{ are distinct} \\ \& \,(\forall z)[Tzx \equiv (z = y_1 \vee \ldots \vee z = y_6)]\} = {}^1\!/_2. \tag{3.9}$$

Intuitively, this ought to provide us with a defeater for the inference from (3.5). There is, however, a problem in explaining how this can be the case. Let

$$s = \text{prob}[\![(\exists y)(Txy \,\&\, {\sim}Hy)/Fx \,\&\, (\exists y_1) \ldots (\exists y_6) \\ \{y_1, \ldots, y_6 \text{ are distinct } \& \, Hy_1 \,\&\, \ldots \,\&\, Hy_5 \,\&\, (\forall z) \\ [Tzx \equiv (z = y_1 \vee \ldots \vee z = y_6)]\}]\!]. \tag{3.10}$$

This probability takes account of more information than does (3.5), so if it can be established that $s < {}^{63}\!/_{64}$ this provides us with a defeater of the form (D1) for an inference from (3.5). By the classical probability calculus and (3.9),

$$\begin{aligned} &\text{prob}\{(\exists y)(Txy \,\&\, {\sim}Hy)/Fx \,\&\, y_1, \ldots, y_6 \text{ are distinct } \& \\ &\quad Hy_1 \,\&\, \ldots \,\&\, Hy_5 \,\&\, (\forall z)[Tzx \equiv (z = y_1 \vee \ldots \vee z = y_6)]\} \\ &= \text{prob}\{{\sim}Hy_1 \vee \ldots \vee {\sim}Hy_6/Fx \,\&\, y_1, \ldots, y_6 \text{ are distinct } \& \\ &\quad Hy_1 \,\&\, \ldots \,\&\, Hy_5 \,\&\, (\forall z)[Tzx \equiv (z = y_1 \vee \ldots \vee z = y_6)]\} \\ &= \text{prob}\{{\sim}Hy_6/Fx \,\&\, y_1, \ldots, y_6 \text{ are distinct } \& \\ &\quad Hy_1 \,\&\, \ldots \,\&\, Hy_5 \,\&\, (\forall z)[Tzx \equiv (z = y_1 \vee \ldots \vee z = y_6)]\} \\ &= {}^1\!/_2. \end{aligned} \tag{3.11}$$

Note, however, that the first formula in (3.11) is not the same as (3.10).

The difference lies in the quantifiers; (3.10) is of the general form ⌜prob $(Ax/(\exists y)Rxy)$⌝ while the first formula in (3.11) is of the form ⌜prob (Ax/Rxy)⌝. What is the connection between these probabilities?

There is an important distinction between two kinds of probability. The probability that a particular proposition is true or that a particular state of affairs obtains is what I call a *definite probability*. Nomic probabilities are not definite probabilities. For example, the probability of a toss of coin c landing heads is not about any particular toss—it is about the property of being a toss of coin c and its relationship to the property of landing heads. Such probabilities are *indefinite probabilities*. The standard probability calculus originates with Kolmogoroff (1933) and was designed with definite probabilities in mind. When we turn to indefinite probabilities we encounter logical relationships pertaining to quantifiers and relations that are not relevant to the calculus of definite probabilities. The relationship between the calculus of definite probabilities and the calculus of indefinite probabilities is roughly analogous to the relationship between the propositional calculus and the predicate calculus. There is every reason to expect that the calculus of indefinite probabilities will contain additional theorems not present in the standard probability calculus.[11]

This suggests that there may be important connections between prob(Ax/Rxy) and prob$(Ax/(\exists y)Rxy)$ that are not recognized by the standard probability calculus. When I first started thinking about this it seemed to me that these probabilities should be equal, but there are simple counterexamples to that. For example, let Rxy be the relation ⌜$(x = 1 \;\&\; y = 1) \vee (x = 2 \;\&\; [y = 2 \vee y = 3])$⌝ and let Ax be ⌜x is even⌝. Then there are three ordered pairs satisfying Rxy; namely, $\langle 1,1\rangle$, $\langle 2,2\rangle$, and $\langle 2,3\rangle$, and in two of these the first member is even, so prob$(Ax/Rxy) = {}^2/_3$. But prob$(Ax/(\exists y)Rxy) =$ prob$(x \text{ is even}/x = 1 \vee x = 2) = {}^1/_2$. So prob$(Ax/Rxy) \neq$ prob$(Ax \,/\, (\exists y)Rxy)$. The reason these probabilities are different is that there are more y's related to some x's than to others and this gives some x's more weight than others in computing prob(Ax/Rxy). If it were a necessary truth that each x had the same number of y's related to it then each x would have the same weight and then these probabilities would be the same. Thus taking $\#X$ to be the number of members in a finite set X, the following principle should be true:

$$\text{If for some natural number } n,\ \Box(\forall x)[(\exists y)Rxy \supset \#\{y \mid Rxy\} = n] \text{ then prob}(Ax/Rxy) = \text{prob}(Ax/(\exists y)Rxy).^{12} \tag{3.12}$$

[11]I have investigated this in "Probability and Proportions" (forthcoming), and the calculus of indefinite probabilities does turn out to be stronger in important ways than the standard probability calculus.

[12]This principle is a theorem of the general calculus of nomic probability constructed in "Probability and Proportions." Note also that it automatically holds for relative frequencies.

This is a kind of probabilistic principle of existential generalization. (3.12) implies a somewhat more general principle, and it is this more general principle that is required for an adequate treatment of the gambler's fallacy. Let us say that an n-tuple $(y_1, \ldots, y_n)$ is *distinct* if no object occurs in it twice. Let $\ulcorner Sx(y_1, \ldots, y_n)\urcorner$ be $\ulcorner y_1, \ldots, y_n$ distinct & Rxy_1 & . . . & $Rxy_n\urcorner$. If $\#X = n$ then there are $n!$ distinct n-tuples of members of X. Thus if the hypothesis of (3.12) holds:

$$\begin{aligned}
&\mathrm{prob}(Ax/y_1, \ldots, y_n \text{ distinct } \& \ Rxy_1 \ \& \ldots \& \ Rxy_n)\\
&= \mathrm{prob}(Ax/\sigma \text{ distinct } \& \ Sx\sigma)\\
&= \mathrm{prob}(Ax/(\exists\sigma)[\sigma \text{ distinct } \& \ Sx\sigma])\\
&= \mathrm{prob}(Ax/(\exists y_1) \ldots (\exists y_n)[y_1, \ldots, y_n \text{ distinct } \&\\
&\qquad\qquad Rxy_1 \ \& \ldots \& \ Rxy_n]).
\end{aligned}$$

Consequently, (3.12) implies the somewhat more general theorem:

$$\begin{aligned}
&\text{If for some natural number } n, \Box(\forall x)[(\exists y)Rxy \supset \#\{y|\,Rxy\}\\
&= n] \text{ then } \mathrm{prob}(Ax/y_1, \ldots, y_n \text{ distinct } \& \ Rxy_1 \ \& \ldots \& \ Rxy_n)\\
&= \mathrm{prob}[Ax/(\exists y_1) \ldots (\exists y_n)(y_1, \ldots, y_n \text{ distinct } \& \ Rxy_1\\
&\& \ldots \& \ Rxy_n)].
\end{aligned} \tag{3.13}$$

It is (3.13) that enables us to dispose of the gambler's fallacy. Letting $\ulcorner Rxy\urcorner$ be $\ulcorner Hy \ \& \ Tyx \ \& \ \#\{z|\, Tzx\} = 6\urcorner$, it follows immediately from (3.13) that the probability in (3.10) is the same as that in (3.11), and hence $s = {}^1/_2 < {}^{63}/_{64}$. Consequently, our intuitions are vindicated and (3.11) provides us with a defeater of type (D1) for the fallacious reasoning involved in the gambler's fallacy. It is somewhat surprising just how much powerful machinery is required to thus dispose of the gambler's fallacy.

4. Resolution of the Paradox of the Preface. We now have most of what is required to resolve the paradox of the preface. Consider a precise formulation of that paradox. We have a book b of the general description B, and we know that it is highly probable that a book of that general description makes at least one false claim. Letting T be the property of being true and Czy be the relation $\ulcorner z$ is a claim made in $y\urcorner$, we can express this probability as:

$$\mathrm{prob}((\exists z)(Czy \ \& \sim Tz)/By) = r. \tag{4.1}$$

Suppose N claims are made in the book: call them $p_1, \ldots, p_N$. Because r is high, (A1) gives us a prima facie reason for believing $\ulcorner(\exists z)(Czb \ \& \sim Tb)\urcorner$. The paradox arises when we observe that we are warranted in believing

$$(\forall z)[Czb \equiv (z = p_1 \vee \ldots \vee z = P_N)]. \tag{4.2}$$

The set consisting of $\ulcorner(\exists z)(Czb \;\&\; {\sim}Tz)\urcorner$, and $\ulcorner Tp_1\urcorner, \ldots, \ulcorner Tp_N\urcorner$, is a minimal set deductively inconsistent with (4.2), so if r is large enough it seems to follow that the members of this set are subject to collective defeat, and in particular that we are not warranted in believing any of the p_i's to be true. But this conclusion is obviously wrong. What I will now show is that this paradox can be resolved in a manner formally similar to the above treatment of the gambler's fallacy.

Let

$$s = \text{prob}[\![(\exists z)(Czy \;\&\; {\sim}Tz)/By \;\&\; (\exists x_1) \ldots (\exists x_N)\{x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots \&\; Tx_{N-1}\}]\!]. \tag{4.3}$$

This probability takes account of more information than does (4.1), so if it can be established that $s < r$ this provides part of what we need to obtain a defeater of type (D1) for the reasoning involved in the paradox. Let us turn to the evaluation of s. By (3.13):

$$\begin{aligned}&\text{prob}[\![(\exists z)(Czy \;\&\; {\sim}Tz)/By \;\&\; (\exists x_1) \ldots (\exists x_N)\{x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots \&\; Tx_{N-1}\}]\!] \\ &= \text{prob}\{(\exists z)(Czy \;\&\; {\sim}Tz)/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots Tx_{N-1}\}.\end{aligned} \tag{4.4}$$

By the classical probability calculus:

$$\begin{aligned}&\text{prob}\{(\exists z)(Czy \;\&\; {\sim}Tz)/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots \&\; Tx_{N-1}\} \\ &= \text{prob}\{{\sim}Tx_1 \vee \ldots \vee {\sim}Tx_N/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots \&\; Tx_{N-1}\} \\ &= \text{prob}\{{\sim}Tx_i/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots \&\; Tx_{N-1}\}.\end{aligned} \tag{4.5}$$

The gambler's fallacy is avoided by noting that we take the different tosses of a fair coin to be statistically independent of one another. In contrast to that, we are not apt to regard the disparate claims in a single book as being statistically independent of one another; but the important thing to realize is that insofar as they are statistically relevant to one another, they *support* one another. In other words, they are not negatively relevant to one another:

$$\begin{aligned}&\text{prob}\{Tx_i/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\& \ldots \&\; Tx_{N-1}\} \\ &\geq \text{prob}\{Tx_i/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)]\}.\end{aligned} \tag{4.6}$$

We may not know the value of the latter probability, but we can be confident that it is not low. In particular, it is much larger than the very small probability $1 - r$. By the probability calculus it then follows that:

$$\text{prob}\{\sim Tx_i/By \;\&\; x_1, \ldots, x_N \text{ are distinct } \& \; (\forall z)[Czy \equiv (z = x_1 \vee \ldots \vee z = x_N)] \;\&\; Tx_1 \;\&\; \ldots \;\&\; Tx_{N-1}\} < r. \tag{4.7}$$

Therefore, by (4.4) and (4.5) it follows that $s < r$.

The reasoning in the paradox of the preface proceeds by using (A1) in connection with (4.1). It follows that

$$s < r \;\&\; Tp_1 \;\&\; \ldots \;\&\; Tp_{i-1} \;\&\; Tp_{i+1} \;\&\; \ldots \;\&\; Tp_N \tag{4.8}$$

is a defeater for that reasoning. But this is not yet to say that the reasoning is defeated. In order for the defeater to defeat the reasoning in a straightforward way, we would have to be warranted in believing not only $\ulcorner s < r \urcorner$ but also $\ulcorner Tp_1 \;\&\; \ldots \;\&\; Tp_{i-1} \;\&\; Tp_{i+1} \;\&\; \ldots \;\&\; Tp_N \urcorner$. It is precisely the warrant of the $\ulcorner Tp_j \urcorner$'s that is called into question by the paradox of the preface. I will now argue that the reasoning is defeated in a more complicated way.

The principle of collective defeat requires that there are no defeaters for any of the prima facie reasons involved in the collective defeat other than those rebutting defeaters generated by the other members of the set. But it is unclear whether that assumption holds in the present situation. Specifically, the conjunction of (4.1) and (4.2) gives us prima facie justification for $\ulcorner \sim Tp_1 \vee \ldots \vee \sim Tp_N \urcorner$, but we have just seen that $\ulcorner Tp_1 \urcorner, \ldots, \ulcorner Tp_{N-1} \urcorner$ (conjoined with $\ulcorner s < r \urcorner$) gives us an undercutting defeater for that prima facie reason. However, as it is the warrant of the latter propositions that is at issue, it remains to be shown that this makes a difference and is sufficient to undermine the collective defeat. Let us consider a simplified version of this epistemic situation. Suppose A is a prima facie reason for B, C is a prima facie reason for D, B entails $\sim D$, D (together with background information) gives us a reason for E, and E is an undercutting defeater for A as a prima facie reason for B:

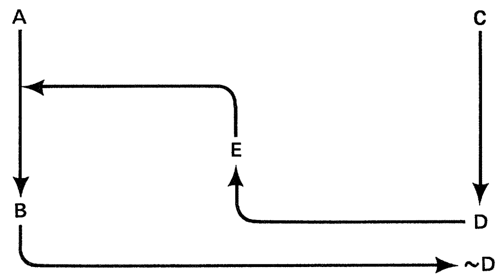

Figure 1.

B and *D* rebut one another so we have the potential for collective defeat, but *D* also gives us an undercutting defeater for our reason for believing *B*. My claim is that in a situation of this sort we are warranted in believing *D* and ~*B*. Basically, what this means is that undercutting defeaters take precedence over rebutting defeaters in the determination of what reasoning is defeated. This can be illustrated by considering the following rather poorly designed warning system. We have a room that is occasionally flooded with green poison gas. A large light is mounted on one wall and viewed through a window from across the room. The light is green in the absence of gas and white in the presence of gas. Unfortunately, as the gas is green, the white light looks green in the presence of gas. This fault in the system was recognized after it was built, and so a color TV camera was mounted directly before the light so that it can be viewed from close up and judged to be white without the interference of the gas. Now consider a case in which the light looks green as seen through the window but looks white on the TV monitor. Knowing what we do about the arrangement, we would judge that the light is white and the room full of poison gas. But consider the following epistemic map of the situation:

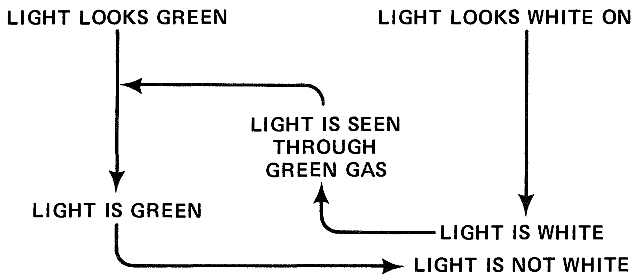

Figure 2.

If undercutting defeaters were not given precedence over rebutting defeaters we would have the inferences to 'The light is white' and to 'The light is green' collectively defeated. In fact, only the latter should be defeated. This illustrates my claim that undercutting defeaters take precedence in the determination of warrant. More precisely:

(UD) If we are warranted in believing *F*, *A* is a prima facie reason for *B*, *C* is a prima facie reason for *D*, *B* entails ~*D*, (*D* & *F*) is a reason for *E*, and *E* is an undercutting defeater for *A* as a prima facie reason for *B*, and we have no other relevant information, then we are warranted in believing *D* and ~*B*.

The next thing to notice is that the paradox of the preface fits this same epistemic map:

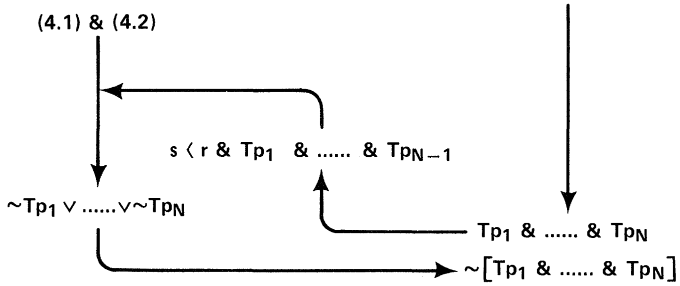

Figure 3.

Thus by (UD), we are warranted in believing $\ulcorner Tp_1 \ \& \ . \ . \ . \ \& \ Tp_N \urcorner$, that is, we are warranted in believing that all the claims in the book are true.

What I have argued is that the acceptance rule (A1), which was designed with the lottery paradox in mind, can also handle the paradox of the preface. The latter has turned out to be much more complicated than the lottery paradox, and its resolution involves an impressive amount of additional machinery. In particular, it involves the probabilistic existential generalization principle (3.12), and it involves principle (UD) concerning the relative roles of undercutting defeaters and rebutting defeaters in potential cases of collective defeat.

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