Modeling without Mathematics

Martin Thomson-Jones*†

Inquiries into the nature of scientific modeling have tended to focus their attention on mathematical models and, relatedly, to think of nonconcrete models as mathematical structures. The arguments of this article are arguments for rethinking both tendencies. Nonmathematical models play an important role in the sciences, and our account of scientific modeling must accommodate that fact. One key to making such accommodations, moreover, is to recognize that one kind of thing we use the term 'model' to refer to is a collection of propositions.

1. Introduction. My starting point for this article is the assumption that in order to understand how modeling works, we need to be thinking clearly about what sorts of things models are—or rather, what sorts of things they can be. And one key to getting that right, I will argue, is to pay attention to nonmathematical models.

Concrete, physical models aside, I will call a model a *mathematical model* when standard presentations of it in scientific contexts employ mathematical tools, and a *nonmathematical model* otherwise. Let me furthermore stipulate that the labels imply nothing else. In particular, then, to classify a model as mathematical in the current sense is not to say anything about what sort of object it is (nothing nontrivial, anyway). That a model is mathematical in this sense thus does not entail that it is a mathematical structure: things other than

*To contact the author, please write to: Philosophy Department, Oberlin College, Oberlin, OH 44074; e-mail: martin.thomson-jones@oberlin.edu.

†I am especially indebted to Arnon Levy and Edouard Machery for helpful correspondence on the chemoton model and on modeling in cognitive neuropsychology, respectively. Thanks also to Isabelle Peschard, Bas van Fraassen, and Michael Weisberg, my cosymposiasts; to Roman Frigg, Peter Godfrey-Smith, Mathias Frisch, and Phil Ehrlich for discussion at the PSA; and to Michael Strevens, Stephen Barker, Felipe De Brigard, Felipe Romero, and the other audience members at the Second Colombian Congress on Logic, Epistemology, and Philosophy of Science in Bogotá in 2012.

Philosophy of Science, 79 (December 2012) pp. 761–772. 0031-8248/2012/7905-0014$10.00

mathematical structures can be presented with mathematical tools.[1] If that is not already evident, I hope to make it so in what follows.

Another point of terminology: I will speak throughout of "nonconcrete" models. By this I mean simply to restrict attention to models that are not concrete, physical objects. One might, of course, call such nonconcrete models "abstract models," but that phrase is often used to convey something about the representational content of a model—roughly speaking, that it omits a great deal—whereas the issue at hand is one of ontological kind.[2]

When it comes to nonconcrete models, then, the primary focus of investigation has indisputably been mathematical models. But there are nonmathematical models, too, and important ones at that. The aim of this article is to explore some of the implications of giving this fact its proper weight.

First, the focus on mathematical models has surely lent plausibility to the thesis that nonconcrete models are, one and all, mathematical objects—set-theoretical structures, perhaps. I argue that the fact that many important models are (in my thin sense) nonmathematical gives us good reason to reject that thesis, and I offer an alternative account on which much talk of models—even of mathematical models—is about collections of propositions. I go on to provide several additional arguments for preferring this alternative view, including the fact that it solves a problem of individuation faced by its rival, and I respond to two objections.

Toward the end of the article, and far more briefly, I make two additional points: first, that my main thesis removes some of the pressure to see the targets of models as quasi-mathematical objects, to interpose data models between models and targets, and to construe model-target relations in mathematical terms and, second, that a desire to understand modeling as an activity and as a process provides us with another reason to take nonmathematical models seriously.

2. Nonmathematical Models. We should first consider an example of a nonmathematical model. Here I take my lead from Downes, who cites the textbook model of the eukaryotic cell: "in most texts a schematized [eukaryotic] cell is presented that contains a nucleus, a cell membrane, mitochondria, a Golgi body, [the] endoplasmic reticulum, and so on" (1992, 145).[3] This

1. This usage is thus a temporary divergence from some previous usage of my own (Thomson-Jones 1997, 2006) and from various ways other people have used the term 'mathematical model'.

2. For my own views about the best way to regiment talk of abstraction with regard to representational content, see Jones (2005).

3. It is important for Downes, too, that the model in question is nonmathematical, and my purposes overlap with his in some significant respects; my main thesis in the current article is not one Downes advances, however.

model is not a model of any particular cell, or even of any very specific type of cell. It is typically presented by means of a diagram and some surrounding text describing, for example, the functions of some of the organelles the cell contains. Crucially, no mathematics is employed.

It is not hard to find more examples. As Downes points out (1992, 145–46), biology textbooks typically go on to present equally nonmathematical models of the various organelles inside the cell, such as the mitochondria. And, importantly, the use of nonmathematical models is not restricted to pedagogical contexts. In evolutionary theory, for example, there is Maynard Smith and Szathmáry's (1995) discussion of Gánti's "chemoton" model, a model of a system that minimally meets certain requirements for life—separateness from its environment, metabolism, self-replication, and so on.[4] Again, the model is presented by means of a combination of descriptive prose and a diagram, and essentially without mathematics (20–23).[5] In the earth sciences, there are Wegener's models of continental drift (Giere 1988, chap. 8, esp. 273), and such areas of psychology as cognitive neuropsychology are replete with nonmathematical models.[6]

It seems to me, too, that there are nonmathematical models in even the relatively recent history of physics and chemistry. Two examples are the billiard ball model of gases and the nuclear model of the atom, both understood as models that transcended and outlived various detailed proposals about how to model the features of their targets (e.g., exactly how the molecules in a gas interact).[7] The idea that there are such models is suggested, I think, by the work of Kuhn (1970) and McMullin (1985), but I will not attempt to make a case for it here. At this point, I hope, the reader is happy to grant that there are nonmathematical models in the sciences and that such models can play important roles in scientific work. The question I want to ask now is, What does this tell us about the sorts of things models can be?

4. I am indebted to Arnon Levy for this example.

5. This case is, admittedly, a little tricky. Gánti, e.g., says that the chemoton model makes it possible to present an answer to the question, 'What is life?' with "exact mathematical methods" (2003, 1) and insists that even "the most elementary simplified description of chemotons" provides the basis for "an exact numerical investigation of [their] workings" (4–5). And Maynard Smith and Szathmáry refer to quantitative results due to Koch concerning the energetic possibility of cell division in certain circumstances (1995, 22). Nonetheless, Maynard Smith and Szathmáry's presentation of the chemoton model employs no mathematics (the use of the word 'two' and one mention of a ratio aside), and yet they put it to substantial theoretical use in developing their views about certain aspects of evolutionary theory.

6. Edouard Machery, personal communication.

7. See also the third point in sec. 7, below.

3. What Models Can Be. One possible view, the *mathematical structures view*, is that all talk of nonconcrete models is to be understood as talk about mathematical structures.[8] Perhaps no one holds exactly this view, simple and unqualified as it is. Nonetheless, I will take it as my foil: doing so will simplify my discussion, and a dominant tendency in current thought about modeling comes close enough to endorsing the mathematical structures view—by taking it that mathematical structures are at least always involved in nonconcrete modeling, for example, or by considering only cases of nonconcrete modeling that involve mathematical structures—that the arguments I present against it will bear quite directly on a range of views explicitly presented or implicitly promoted in the literature.[9]

The existence of nonmathematical models poses an immediate challenge to the mathematical structures view, for it is a strain to suppose that such nonmathematical models as the textbook model of the eukaryotic cell are mathematical structures. For one thing, it is not clear that the representational content of the model *could* be captured by a mathematical structure.[10] Even if it could, however, it seems quite implausible that when someone lays out the textbook model of the cell, he or she is actually presenting us with a mathematical structure, and so it is implausible that the model in question is *in fact* a mathematical structure. We thus have reason to reject the mathematical structures view.

Instead, I propose that the textbook model of the cell is a collection of propositions. The following propositions are among those that make up the model: that the eukaryotic cell has a membrane, that it has a nucleus, that the nucleus contains a nucleolus, that the nucleolus has such-and-such functions, and that the cell contains mitochondria. These are some, but not all, of the propositions contained in the collection of propositions that we can take the model in question to be.

I do not mean to restrict my proposal to nonmathematical models, however. Given that we can use mathematical language to express propositions, I propose that mathematical models can be collections of propositions, too.[11]

8. The mathematical structures in question might be state spaces with trajectories running through them or *n*-tuples of sets or other sorts of things. There are differences on this front; those differences will not matter in what follows.

9. For more on models as mathematical structures and further references to the literature, see Thomson-Jones (1997, esp. sec. 1.3; 2006). For recent examples of the tendency in question, see van Fraassen (2008) and Weisberg (forthcoming).

10. At least not given our actual practices surrounding the use of mathematical structures as representations (cf. Thomson-Jones 2011, 135–38).

11. Note that I say "can be," not "are." The reason for this will become clearer in the next section.

As an example, consider the Bohr model of the hydrogen atom. The following propositions are among those making up the Bohr model:

- the hydrogen atom is composed of a proton and an electron
- the electron is significantly lighter than the proton and moves around it in circular orbits
- the two particles carry opposite electrical charges, $\pm e$, and attract one another in accordance with Coulomb's law
- $F = ma$
- $(mv^2)/r = e^2/r^2$
- $mvr = n\hbar \qquad n = 1, 2, 3, \ldots$
- $r_n = n^2(\hbar^2/me^2) \qquad n = 1, 2, 3, \ldots$
- $v_n = (1/n)(e^2/\hbar) \qquad n = 1, 2, 3, \ldots$,

where m is the mass of the electron, a its acceleration, F the net force it experiences, v its velocity, r its orbital radius, and n indexes its permissible orbits.[12] Some of the propositions the model contains are typically expressed by means of equations and other expressions in mathematical language, and so the model is a mathematical one; nonetheless, it seems quite natural to regard it as a collection of propositions.

When a model is a collection of propositions forming a representation of a particular system, or a particular kind of system, I will call it a *propositional model*.[13] My proposal, then, is that when it comes to nonconcrete models, nonmathematical models are propositional models, and mathematical models can be.

I should immediately address one likely source of unease with this proposal, which is that it brings to mind the syntactic view of theories (also known as the Received View), a view almost universally regarded as misbegotten these days. The simple point, however, is that the view I am proposing here is not the syntactic view, nor does it have enough in common with that view to be deemed guilty by association. Propositional models are collections of *propositions*, not sentences, and that difference alone renders the view I am proposing invulnerable to some of the most well-known objections to the syntactic view. Moreover, the syntactic view was laden with tenets arising

12. The presentation of the Bohr model just given is partial, omitting (e.g.) any mention of energy. It is also historically misleading, but only in entirely standard textbook ways. Note, finally, that some of the propositions listed are consequences of combinations of others.

13. The clearest precursor here is Peter Achinstein's notion of a "theoretical model" (1968, 212–18); Michael Redhead took up Achinstein's analysis and explored it further in a later paper (1980). Achinstein's characterization of the theoretical model begins in much the same way as my characterization of the propositional model, but it goes further in a number of respects; for more on the differences, see Thomson-Jones (1997, 11–14).

from empiricist concerns about meaning and the like, none of which are part of my proposal.[14]

4. Two Senses of 'Model'. There is another issue likely to be troubling anyone familiar with the philosophical literature on scientific modeling: Am I saying that a mathematical structure cannot be a model, and thus that no model is a mathematical structure? The answer is no. Instead, I want to say that mathematical structures can indeed be models, in just the same sense that concrete, physical objects can be models. To spell this out, and make the overall view I am advancing more transparent, it will help to distinguish two senses of the term 'model'.[15]

One sense of 'model', and in some contexts the primary sense, is just that captured by the notion of a propositional model: in that sense, a model is a collection of propositions that together form a representation of a particular (kind of) system. In another sense of 'model', however, a model is an object used as a representation of a particular (kind of) system.[16] Given this, a mathematical structure clearly can count as a model, if it is used to represent a particular (kind of) system, for then it can be a model in the second sense.

The next thing to notice is that models of these two kinds can peacefully coexist; indeed, they can be intertwined. A propositional model can contain propositions about relations between its target and a certain mathematical structure. And that can be how we use the mathematical structure as a representation of the target—namely, by forming a propositional model concerning, in part, various relationships between the mathematical structure and the target. In that case, we will be employing both a propositional model of the target and something that is a model of it in the second sense; that latter something will be a mathematical structure.

We should also note the parallel we now have between mathematical structures as models and concrete objects as models: both are models in the second of the two senses I have just distinguished, and both can come to be models in that sense by being related to target systems by propositional models.

14. Another difference, of course, is that the syntactic view was about theories, whereas the current proposal is about models. For discussion of the talk of models that took place in the framework of the syntactic view of theories, see Thomson-Jones (1997, 14–16).

15. Or at least, two kinds of referent the word can have. I will sometimes talk of senses of 'model' in what follows, but my arguments could then be reformulated in terms of kinds of referent without loss.

16. Giere (1988, 80) and van Fraassen (2008, 23, chap. 11, and passim) provide prominent examples of this usage. There are other senses of 'model', too, of course, and other senses of the term that play or have played important roles in the literature on scientific modeling. For a considerably more extensive discussion, see Thomson-Jones (1997, esp. sec. 1).

To summarize, then, if the mathematical structures view is that

> All nonconcrete models are mathematical structures,

then one way of putting the core of the opposing view I want to argue for is this:

> Some nonconcrete models are mathematical structures, and some are collections of propositions.

This second formulation, however, might create the impression that it makes sense to expect a straightforward answer to all questions of the form "So which kind of thing is the (nonconcrete) such-and-such model?" when in fact I would take any such expectation to be mistaken. A better way of formulating the view I am advancing involves a little semantic ascent: when talking about nonconcrete models, we (philosophers and scientists both) sometimes use the term 'model' to refer to a mathematical structure and sometimes to a collection of propositions. In the case of most nonmathematical models (perhaps all), we do not use it to refer to a mathematical structure, as no mathematical structure is involved; instead, we use it to refer to a collection of propositions. In the case of many mathematical models (perhaps all), we use it both ways: mathematical modeling (often, and perhaps always) involves constructing a collection of propositions that are, in part, about relations between a certain mathematical structure and the target system, and we then use the term 'model' to refer both to the collection of propositions and to the mathematical structure.

Let us call the view I have just laid out the *propositional view*, not because it claims that all nonconcrete models are propositional models—it quite explicitly does not—but because it embraces propositional models and makes them fundamental. Although a mathematical structure can be a model, too, on this view, in many cases (and perhaps all) it comes to be a model because some propositional model claims that it stands in certain relations to a target system.

5. Arguments for the Propositional View. There are at least four reasons for preferring the propositional view to the mathematical structures view.[17] It is worth noting that, even though a recognition of the existence and importance of nonmathematical models provided the initial impetus for introducing the propositional view, three of the four reasons given below for adopting that view are quite independent of any such considerations.

5.1. Nonmathematical Modeling and Unification. As I have already argued, the propositional view accommodates nonmathematical modeling

17. Or five. Strictly speaking, sec. 5.1 identifies two distinct reasons.

quite comfortably, whereas it is hard to see how the mathematical structures view can do so. Relatedly, the propositional view offers a relatively unified account of mathematical and nonmathematical modeling, whereas it seems unlikely that any extension of the mathematical structures view that managed to accommodate nonmathematical modeling would yield a similarly unified picture.

5.2. Capturing Ordinary Talk. When talking about nonconcrete models in an unguarded way, both scientists and philosophers of science regularly employ such locutions as "One assumption of the model . . . ," "According to the model . . . ," and "It's true in the model that . . . ." These ways of talking make perfect sense if we take the speakers to be using the term 'model' to refer to a collection of propositions on such occasions. It is significantly more awkward to account for the use of such locutions if we take the speakers to be using the term 'model' to refer to some mathematical structure or other.

5.3. Individuating Models. The mathematical structures view has difficulty individuating models in the right way. This problem arises because some pairs of models employ the same mathematical structure—a certain model of a pendulum and a certain model of an electrical circuit, say. The mathematical structures view seems committed to identifying both the pendulum model and the model of the electrical circuit with the mathematical structure they have in common and, thus, to insisting that the pendulum model and the model of the circuit are one and the same model. Yet there is surely a sense of 'model' in which the pendulum model and the circuit model are distinct: one is a model of a pendulum, after all, and the other is not.

The propositional view avoids this difficulty. The sense of 'model' in which there are two distinct models in such a case is just the sense in which a model is a collection of propositions: clearly two collections of propositions can be distinct and yet concern, in part, the same mathematical structure. If, however, there is also a sense of 'model' in which it is correct to say that the model of the pendulum and the model of the circuit are the same model, the propositional view can accommodate that fact by claiming (as does the mathematical structures view) that the sense of model in question is the second of the two senses distinguished in section 4.

5.4. Essentiality of Content. In one sense of 'model', at least, it seems true to say each of the following things: (1) the Bohr model of hydrogen could not have failed to have aboutness—that is, it could not have failed to be a representation of something. (2) The Bohr model of hydrogen could not have been about anything other than hydrogen. (3) The Bohr model of hydrogen could not have said things about hydrogen other than the things it actually says. The propositional view can accommodate this fact by taking the phrase 'the Bohr model of hydrogen', as it appears in each of the three utterances in question, to refer to a certain collection of propositions. In that case, all three claims will be straightforwardly true—at least, if we assume certain standard views about propositions and take collections of propositions to be individuated by their members.[18] None of the three claims is true, however, if we take the phrase 'the Bohr model of hydrogen' to refer to a mathematical structure that represents the hydrogen atom as having such-and-such features in virtue of the ways we use it, for facts about the ways we use mathematical structures are surely contingent facts. Only the propositional view, then, can accommodate these intuitions about the essentiality of the content of the thing we refer to on at least some occasions when we use the phrase 'the Bohr model'.

6. Complaints. In turning to consider two objections one might have to the propositional view, I will again take the dialectical context to be a comparative one: as neither of the objections I will discuss provides a knockdown reason to reject the propositional view, the question I mean to address is whether either of them could give us good reason to prefer the mathematical structures view.

There are, of course, potential complaints I lack the space to discuss here. In particular, one might have worries about the presumed ontological commitment to propositions, and about the fact that no account has been provided of either the nature of propositions or how propositions themselves represent. I must limit myself to remarking that at least part of my response to such worries would be to argue that the propositional view fares no worse than the mathematical structures view in such respects. There is a more complex story to be told on another occasion, but for now I will focus instead on two quite different objections.

6.1. Sterility. It can be claimed that the mathematical structures view provides (and has provided) fertile ground for the development of accounts of explanation, prediction, confirmation, empirical adequacy, and the like, at least when those phenomena involve models: if models are mathematical structures, then we can hope to develop such accounts by bringing to bear the various mathematical and metamathematical tools we have at our disposal. One might then object that the propositional view seems sterile by comparison. What parallel work can be done by thinking about collections of propositions?

18. We might take propositional models to be *sets* of propositions in order to underwrite the latter assumption.

My response has two prongs: first, there are of course various bits of intellectual technology we can employ in thinking about collections of propositions. Logic is one example, as propositions stand in logical relations to one another, and the various theories of confirmation provide further examples, as we can take it that the entities that stand in relations of confirmation to one another (perhaps by functioning as the arguments of probabilities) are propositions. Second, remember that the propositional view encourages a peaceful coexistence between propositional models and mathematical-structure models (sec. 4). As a result, whenever it is appropriate to apply mathematical machinery to the task of understanding modeling—by grappling with the inner workings of the mathematical structures we sometimes use to model, or the relationships between those mathematical structures and data models, for example—such work can just as well be done under the auspices of the propositional view.

6.2. Nonpropositional Content. Suppose there are models that have representational content that cannot be captured by propositions (even inexpressible ones). Perhaps in some cases we are presenting models of that sort when we employ diagrams, for example. What should we say then?

I cannot examine the crucial supposition here, so I will simply point out that, at most, this would mean that we should expand our overall account of modeling to accommodate another kind of model. There is no reason here to deny that *one* kind of thing we pick out when we use the term 'model' is a collection of propositions, nor is there any reason to doubt the arguments given for that claim in section 5. And there is (thus) no reason here to prefer the mathematical structures view, even if, as seems unlikely, all nonconcrete models with nonpropositional content (still supposing there to be such) should turn out to be mathematical structures. That would only mean that some models are mathematical structures, a claim the propositional view already embraces.

7. Three Further Thoughts. I will close with three further thoughts, one about propositional models and two about nonmathematical modeling. I mention the first only as an idea for further exploration. According to the standard slogan, the semantic view of theory structure has it that a theory is a collection of models,[19] and the models in question are most often taken to be mathematical structures. Might there be advantages to thinking of theories (or some theories, at least) as collections of propositional models instead?

The second point is that exclusive attention to mathematical models has introduced a pressure to see the targets of our modeling practices—the

19. It is not easy to pin this claim on the central figures in the development of the semantic view, however (Thomson-Jones 1997, n. 53).

systems we are modeling—as protomathematical objects in one way or another; either that, or we find ourselves automatically interposing a mathematical counterpart of the target between the model and the target itself. And there is an associated pressure to see model-target relations as ubiquitously mathematical. Attention to nonmathematical models serves as a useful corrective to these tendencies. This is not to deny that there are mathematical data models or that they play an important role in scientific practice, but we should be careful not to insist on finding them wherever there is modeling. More generally, we should not be surprised if changing our understanding of what models can be makes a difference to the way we understand both model-target relations and the targets themselves (see also Downes 1992, 147–48, including n. 2).

Finally, we should bear in mind that if we want to understand the epistemology and methodology of modeling as an activity, then it will surely be important to understand all the stages of the modeling process, and even when the outcome of that process is a mathematical model, there will often be nonmathematical models lurking in the prehistory.[20] This point provides us with another reason to take nonmathematical models seriously and to insist on an account of the nature of models that can accommodate them.

8. Conclusion. There is a clear tendency for philosophers who are trying to understand nonconcrete scientific modeling to focus their attention on mathematical models, in some cases more or less exclusively. And although perhaps no one holds the mathematical structures view as stated, there is a related tendency to think only about mathematical structures when thinking about nonconcrete models. The arguments of this article are arguments for rethinking both tendencies. Nonmathematical models play an important role in the sciences, and our account of scientific modeling must accommodate that fact. One key is to recognize that an important notion of model in the sciences, and for the philosophy of science, is that of the propositional model.

REFERENCES

Achinstein, Peter. 1968. *Concepts of Science: A Philosophical Analysis*. Baltimore: Johns Hopkins Press.

Downes, Stephen M. 1992. “The Importance of Models in Theorizing: A Deflationary Semantic View.” In *PSA 1992: Proceedings of the 1992 Biennial Meeting of the Philosophy of Science Association*, vol. 1, ed. David Hull, Micky Forbes, and Kathleen Okruhlik, 142–53. East Lansing, MI: Philosophy of Science Association.

Gánti, Tibor. 2003. *The Principles of Life*. Commentary by James Griesemer and Eörs Szathmáry. Oxford: Oxford University Press.

20. This is perhaps one way to understand the role of the sorts of nonmathematical models I suggested we might find in physics and chemistry in sec. 2.

Giere, Ronald N. 1988. *Explaining Science: A Cognitive Approach*. Chicago: University of Chicago Press.

Jones, Martin R. 2005. "Idealization and Abstraction: A Framework." In *Idealization XII: Correcting the Model—Idealization and Abstraction in the Sciences*, ed. Martin R. Jones and Nancy Cartwright, 173–217. Amsterdam: Rodopi.

Kuhn, Thomas S. 1970. *The Structure of Scientific Revolutions*. 2nd ed. Chicago: University of Chicago Press.

Maynard Smith, John, and Eörs Szathmáry. 1995. *The Major Transitions in Evolution*. Oxford: W. H. Freeman.

McMullin, Ernan. 1985. "Galilean Idealization." *Studies in History and Philosophy of Science* 16:247–73.

Redhead, Michael. 1980. "Models in Physics." *British Journal for the Philosophy of Science* 31:145–63.

Thomson-Jones, Martin. 1997. "Models and the Semantic View." PhilSci Archive, University of Pittsburgh. http://philsci-archive.pitt.edu/8994/.

———. 2006. "Models and the Semantic View." *Philosophy of Science* 73 (Proceedings): 524–35.

———. 2011. "Structuralism about Scientific Representation." In *Scientific Structuralism*, ed. Alisa Bokulich and Peter Bokulich, 119–41. Boston Studies in the Philosophy of Science 281. Dordrecht: Springer.

van Fraassen, Bas C. 2008. *Scientific Representation: Paradoxes of Perspective*. Oxford: Clarendon.

Weisberg, Michael. Forthcoming. *Simulation and Similarity: Using Models to Understand the World*. Oxford: Oxford University Press.