IDEALIZATION AND FORMALISM IN BOHR'S APPROACH TO QUANTUM THEORY

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PSA Nov 2002

Bohr held that quantum mechanical symbols find meaning only in the context of an experimental setting. Making a measurement requires establishing a correspondence between a property of the quantum object and a property of the measuring system via the introduction of a classical quantity with which the measuring instrument, classically understood, interacts. However, this correspondence is only approximate and involves the use of certain idealizations, and it is the commutation rules that tell us the limitations to this process. In this context I examine the caution by Daumer, et. al. (1996) against taking too seriously the idea of operators as observables. I conclude that Bohr would neither ascribe to such 'naive realism' about operators nor dismiss the formalism as unimportant to the understanding of quantum phenomena, although he would agree with the caution that the properties we can attribute to a system depend on the context of the experiment.

Introductory comments

My goal for this paper is to take some steps towards a new analysis of Bohr's views on quantum mechanics based on close look at the philosophical content of his work on atomic theory in the early 1920s. I argue that the main key to understanding Bohr's otherwise confusing (or complex and "subtle") explications of his philosophy is a re-consideration of the meaning and role of his correspondence principle in his approach to quantum theory. For Bohr, the correspondence principle was not merely a heuristic tool for the development of quantum theory; nor was it *primarily* concerned with the asymptotic agreement between classical and quantum theory. Rather, its main purpose was to connect theory-laden descriptions of phenomena with a developing theory of atomic structure—it was meant to bridge the epistemological gap between empirical phenomena and theory. And I will argue that Bohr maintained the need for such a tool in his interpretation of quantum mechanics. I am going to provide this picture of Bohr in the context of an issue raised by Goldstein (Daumer, Dürr et al. 1996) in the 1996 PSA conference concerning the meaning of quantum mechanical operators, where the authors of the presented paper argue against taking too seriously the standard quantum mechanical formalism and claim that Bohr himself pointed out some of the central theses of their argument. A comparison between Bohr and the Bohmian view presented in that paper will help clarify Bohr's argument that our ability to make meaningful statements about quantum systems depends on classical physics and a complete description of the experimental context, and in particular will help us understand the way in which Bohr relates these conditions to the meaning of the quantum formalism.

Naïve realism

In the paper presented by Goldstein at the 1996 PSA, Daumer et al. blamed what they call "naïve realism about operators" for the "contortions and perversions" of many interpretations of quantum mechanics. They argue, from a Bohmian perspective, that we ought not take too seriously the idea of "operator-as-observable" because this leads to our treating operators as representing actual properties of the system. It is only once one accepts the idea that quantum systems have properties corresponding to each operator that one has to explain what happens to those properties when they are not being measured. Daumer et al. argue that rather than treating operators as representing independent properties of a quantum system, we ought to understand them only as something like emergent properties of the entire experimental arrangement.[1]

Of course, from their Bohmian perspective, the only real property is position. Other so-called "properties" emerge from the wave equation representing the interaction of the particle with the apparatus. That is, these properties are "*merely* in the wave function," (1996, 389) although they

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[1] They take their inspiration from Bell, who argued partly for these reasons that the word "measurement" should be banished from our vocabulary regarding quantum mechanics. Bell himself refers to Bohr as having taught us that properties must be taken to apply to the whole system plus apparatus complex and as such only pertain to a particular experimental arrangement. Daumer et al. extend this critique of the meaning of the quantum mechanical formalism and argue that the idea of operator-as-observable (i.e., naïve realism about operators) is perhaps even more pernicious than the idea of "measurement".

caution even against this terminology, for they argue that we cannot *simply* make the move of attributing the property to the wave function instead of the particle, since the wave function is a specific "extremely complicated function" intimately tied to a particular experiment, and different experimental arrangements for what we may tend to think of as the same property may have "entirely different functions." (389) Thus, we ought not even think of this operator as a general property—rather, it is a function appropriate *only* for this particular experiment:

> The fact that the same operator plays a role in different experiments does not imply that these experiments have much else in common, and certainly not that they involve measurements of the same thing. It is thus with detailed experiments, and not with the associated operators, that random variables might reasonably be expected to be associated. (391)

Now, since we ought not treat operators as representing properties, then the non-commutativity between incompatible operators does not inform us at all about the nature of the properties of a quantum system. It can only tell us something about experimental arrangements, as Daumer et al. indicate in this remark about no-go theorems:

> The state of affairs described by the [no-go] theorems nonetheless logically implies the obvious conclusion, namely, that the incompatible joint values refer to different, and incompatible, experimental set-ups, just as Bohr told us all along. This mathematical incompatibility of "joint values" thus seems to attain genuine significance only to the extent that we are seduced by naive realism about operators. (391)

That is, we ought not take this formalism as an indication of the incompatibility of the actual or possible properties of a quantum system.

I would like to suggest that a truly Bohrian approach offers a middle ground that does not commit the alleged sins of naïve realism but also legitimates the attribution of properties corresponding to operators to a system. However, it does so by treating the classically-defined properties we attribute to a system as approximate idealized descriptions of the actual properties of that system in interaction with a particular experimental apparatus. As I suggested, the key to understanding Bohr's views of quantum mechanics is his correspondence principle.

The correspondence principle

Bohr's approach to the development of quantum theory was empirically driven. He attempted to use phenomena as a more secure starting point for the development of theory and searched for the most general features that any atomic theory was to have in the face of the novel introduction of quantum effects. Thus Bohr attempted to determine atomic structure from the properties of the radiation in the spectra, even in the absence of an understanding of the quantum mechanism for the emission of that radiation. Bohr described conclusions reached through this method as *"empirical deductions from the spectral evidence*." (1924b, 224: BCW 3, 579)

This approach assumed a sort of epistemological split between empirical phenomena and the atomic structure responsible for those phenomena and explicitly required a tool to allow one to bridge the gap between phenomena and theory. The correspondence principle was this tool, which together with Bohr's frequency condition ($\Delta E = h\nu$) acted as a bridge principle that let one make claims about atomic structure based on empirical phenomena and then let one deduce the empirical consequences of the model.[2]

The correspondence principle was based on the idea that the classical relationship between the wave properties of radiation and the oscillations of electrons in the atom must hold in its general terms. It was "a connection between the spectrum and the atomic model of hydrogen" (1920, 26: BCW 3, 249) which allowed one to reach conclusions about the oscillations of electrons in orbits. The relata of the correspondence principle were epistemologically distinct: on the one side were empirical generalizations of spectral phenomena and on the other were the properties of atomic structure responsible for those phenomena.

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[2] For example, one of the simplest and very first of these arguments was the way Bohr derived energy levels from the Balmer formula.

More specifically, the correspondence principle connected the frequencies of radiation in an atomic spectra with the Fourier components of the motion of an electron in an orbit. That is, while classically the motion of an electron in an orbit would simultaneously emit light in the frequencies of each of the Fourier components of its motion, the correspondence principle held that in a transition between stationary states the light emitted (or absorbed) would correspond in its properties to only one of these harmonics, which one depending on the difference between the quantum numbers of the states:

> On account of the general correspondence *between the spectrum of an atom and the decomposition of its motions into harmonic components*, we are led to compare the radiation emitted during the transition between two stationary states with the radiation which would be emitted by a harmonically oscillating electron on the basis of the classical electrodynamics. (1920, 51: BCW 3, 273) (emphasis added)

One has this access to the motion through the comparison the correspondence principle legitimates between the quantum emission of radiation through state transitions and the classical account of the emission process.[3] The correspondence principle let Bohr reach conclusions about the types of orbits that were allowed, what types of transitions between were allowed, and the effects on the orbits and transition possibilities from external forces. And while its quantitative application worked best only for states of high quantum number, Bohr maintained that at least qualitative properties could be connected for states of low quantum number, and held out hope that a more quantitative relationship could be found.

Matrix mechanics was, for Bohr, the solidification and clarification of this entire approach:

> In brief, the whole apparatus of the quantum mechanics can be regarded as a precise formulation of the tendencies embodied in the correspondence principle. (1925, 852: BCW 5, 280) [4]

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[3] This idea that the correspondence principle was primarily a connection between spectra and orbital motion, and not between classical and quantum theory in some more abstract sense, has not been entirely missed but has been generally overlooked. Two notable exceptions are Darrigol's (1992) historical treatment of Bohr's correspondence principle, and Tomonaga's (1962) textbook which takes seriously Bohr's ideas about correspondence.

[4] Born, Heisenberg, and Jordan made a similar claim in their 1926 paper, calling matrix mechanics "an exact formulation of Bohr's correspondence considerations" (Born, Heisenberg, and Jordan 1926; van der Waerden 1968, 322)

Matrix mechanics fulfilled the promise of the correspondence principle not merely in its retention of the forms of classical equations, but more fundamentally in the fact that it used *and solidified* the connection the correspondence principle had provided between empirical properties and the atom:

> [Matrix mechanics] operates with manifolds of quantities, which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. (1925, 852: BCW 5, 280)

The matrices are built out of elements which "symbolize" the oscillators that the correspondence principle connected to the spectral lines. The Fourier components of motion are retained but are no longer assumed to build up any classical orbital motion. The question now is how we are to understand this new formalism.

Idealizations

Bohr's interpretation of the new formalism explicitly concerned the use of classical theory as necessary idealizations. So let me take a moment to briefly address Bohr's attitude to idealizations before 1925. Bohr had always believed that there were limitations to use of the use of the correspondence principle and that these limitations were associated with the approximate applicability of idealizations. One of the most important postulates of his quantum theory had been the postulate of stationary states. Yet he explicitly admitted that this postulate was not universally applicable but rather, it was an idealization whose use was justified only in very specific situations.

And this applicability of the stationary state postulate was determined using classical physics. Bohr claimed that in order to apply quantum theory to the atom, we must be able to make

> the assumption that the motion within the stationary states can be described, to a close approximation, by the laws of ordinary electrodynamics, if only one neglects the reaction connected with the emission of radiation. (1924a, 2: BCW 3, 459)

Classical electrodynamics is clearly inconsistent with the concept of stationary states, because the reaction on the electron from the emission of light would lead to a decay of the orbit. However, we can still use it, but we can *only* use it, for those cases when according to classical physics the change in motion on the electron from the emission of radiation is small compared to the changes in motion from the forces within the atom—that is, its applicability is limited to those cases where we may in the classical account ignore to a first approximation the forces which correspond to the unknown quantum mechanism of radiation:

> In all processes to which quantum theory has been applied, the radiation which, according to the classical theory, is emitted during a period (a revolution of the electron) is but small. Hence, it is a very natural assumption that, in calculating the stationary states, we may neglect the influence of the radiation upon the motion …. We emphasize, however, that such a procedure represents only an approximation … (1922: BCW 4, 352)

The use of the correspondence principle was valid only under certain specifiable conditions, and even then provided for only a good approximation. And one determined whether the idealization was an appropriate approximation or not by examining the degree to which the classical physics used to ground the idealization was applicable.

The main point I want to make here is that our ability to attribute properties to the atom or to an electron in the atom depended on the degree to which we could approximately describe them with classical physics, and we in turn determined this degree on the basis of classical reasoning, although it was classical reasoning in the quantum context. Only when these conditions are met may we appropriately declare empirical phenomena to be evidence of the otherwise hidden properties of the atom; even then, the claims that we do make are only approximate, as our use of these concepts involves idealizations that do not strictly apply to the atom. This understanding of how Bohr approached quantum theory can help us understand what Bohr later meant with complementarity. Let me turn to this topic now.

Mutual exclusivity and the limitations of the correspondence connection

Bohr's introduction of complementarity involved the claim that results of measurements be given in classical terms which apply only to mutually exclusive experimental arrangements—i.e., that we can neither perform one experiment to measure two incompatible properties, nor perform two different experiments measuring incompatible properties in succession and treat it as one experiment which measured both. But it's not so clear what his argument for this mutual exclusivity was. Bohr claimed that our descriptions of experimental results involve a "neglect of quantum effects" and that the source of these "incontrollable" effects is the so-called "quantum of action" which we express in the quantum postulate. But Bohr also claimed that the only way the quantum postulate comes into the quantum mechanical formalism is through the commutation relations. In fact, Bohr clearly understood the uncertainty relations as "a direct consequence of the commutation rules." (1939, 18: BCW 7, 310)

I claim that despite the appearances of his predominately classical explications of complementarity, Bohr's argument for complementarity cannot be based merely on analyses from outside the quantum formalism. Rather, complementarity fundamentally relies on this formalism:

> In fact, the limited commutability of the symbols by which … variables are represented in the quantal formalism correspond to the mutual exclusion of the experimental arrangements required for their unambiguous definition. (1958, 312: BCW 7, 392)

Bohr thought that it is *only* through this formalism that we learn that the experiments are mutually exclusive.

Now what exactly is being limited in this mutual exclusivity? Bohr claimed that the quantum mechanical formalism was by itself merely abstract symbolism without connection to actual measurements. The way in which matrix mechanics is related to the correspondence principle indicates that the formalism receives its empirical content via a type of correspondence connection between empirical phenomena and quantum properties. However, this connection is limited, and outside an empirical context the quantum properties are now considered to be entirely abstract, as is the case for the components of the position matrix.

It is only through this process of attributing the corresponding classical properties to the quantum system that we can give the abstract symbols empirical meaning. The meaning of the symbols comes in the way in which we describe the interaction between the system and the measuring apparatus in classically-defined terms:

> We must recognize that a measurement can mean nothing else than the unambiguous comparison of some property of the object under investigation with a corresponding property of another system, serving as a measuring instrument, and for which this property is directly determinable according to its definition in everyday language or in the terminology of classical physics. (1939, 19: BCW 7, 311)

Making a measurement requires making a comparison between some property of the quantum object and some property of the measuring system. One makes this comparison by establishing a correspondence between the quantum mechanical symbol representing some property of the object and a classical quantity with which the measuring instrument, classically understood, interacts. This outside description of the classical behavior of the apparatus establishes what Bohr calls the "external conditions" of the phenomena—in the absence of these conditions the quantum symbols have no empirical content.

In a particular situation, a quantum symbol receives an essentially classical empirical meaning based on its role in the classical equations used for the description of the experiment, yet we cannot attribute the actual classical meaning to this symbol in general. Since the quantum properties of the object are not the same as their classical counterparts—at the very least they are not the same in their relations to one another—this process involves approximations or idealizations, and the commutation rules tell us the limitations to this process. Thus while we may be able to establish a correspondence with a classical idealization in a given empirical setting, this process inherently ignores quantum effects, and so requires that we ignore other aspects of the system not directly involved in the measurement. Given the quantum interaction between a system and the measuring instruments,[5] our ability to attribute properties to the system in the context of an experimental arrangement depends on our ability to neglect this interaction and independently describe the "external conditions" of the entire quantum phenomenon.

The correspondence principle had argued that certain elements of the asymptotic agreement between classical and quantum theory could hold in general and would thus allow us to bridge the epistemological gap between empirical phenomena and atomic properties. We knew that there were limitations to this process and that this correspondence agreement worked best only under certain specific circumstances. Now we find that some of these conclusions are still valid, but we have learned that the conditions for assuming that general correspondence do not hold simultaneously. For example, while we may still understand the quantum symbols in terms of representing an orbit or a definite stationary state, we learn from the commutation relations that the classical conditions determining the validity of these idealizations are mutually exclusive.

This argument is directly connected to the formalism, which Bohr understood as an outgrowth of the application of the correspondence principle:

> In particular is the essentially statistical nature of this account a direct consequence of the fact that the commutation rules prevent us to identify at any instant more than a half of the symbols representing the canonical variables with definite values of the corresponding classical quantities. (1939, 14-15: BCW 7, 306-307)

It is only in virtue of the commutation relations that we learn the limitations of our ability to connect the symbols representing quantum properties to the corresponding classical properties we take our instruments to measure. Complementarity is the *consequence* of this situation, and not something new added to the interpretation from the outside.

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[5] I believe that Don Howard's (1990; 1994) take on Bohr's understanding of this interaction in terms of entanglement is illuminating and perhaps the right way to understand Bohr.

Formalism, empirical content, and naïve realism

Bohr's view is thus consistent with the caution against naïve realism, as he does not take the specific idealized property that we attribute to a system in a particular experimental arrangement to be a universal or independent property of the system. Rather, it depends on the specifics of the apparatus, where the operator only gains content in an empirical setting is dependent on the entire experimental arrangement. We need to use classical reasoning to fix the external empirical conditions which then fix the appropriate equations governing the symbols. This is how we actually do determine quantum expressions, as we in general start with classical equations and then "quantize" them. The basis for this process begins outside the theory proper—the formalism of quantum mechanics does not on its own provide the recipe for this process, and so the application of the theory to any particular experimental arrangement depends on conceptual apparatus external to the theory—i.e., it depends on classical physics.

But Bohr would not accept the idea that the quantum operators ought not be considered to represent properties of the system under measurement. When we attribute a property such as position to the system as an idealized approximation, it is meant to serve exactly as that—an approximation of the unvisualizable properties in virtue of which the system is behaving as if it has a classically well-defined position. It is *not* classical position in its complete sense, because we know, for example, that the limitation of this idealized description indicates that the reaction of the system to a momentum measurement will not be the same as if it retained the classical position. Yet the point of this process is to capture as many as possible of the salient features of the quantum system in the experiment even though we cannot account for whatever it is about the quantum of action that limits our ability to maintain this coincidence outside the context of this experiment.

I would like to provide the following breakdown of the differences between the Bohmian caution against naïve realism and what I suggest would be Bohr's view on this topic:

Attitudes Regarding Naïve Realism

	Bohmian	Bohrian
operators	not to be understood as properties	empirical content depends on "external conditions"
properties	position only	idealizations & approximate
contextuality	at best, other "properties" apply only to entire experimental interaction	idealizations apply only in context of particular experiment
mutual exclusivity and incompatibility	apply only to experimental arrangements	express limitations to the applicability of idealized properties
role of classical physics	metaphysical in retention of position and determinism	empirical in the idealized description of quantum properties & the determination of the approximate applicability of idealizations
epistemology	precise position "hidden"	access to properties through approximate correspondence

While Bohr does argue that quantum properties are contextual and therefore probably would agree with the caution against taking too simplistically the idea of operator-as-observable, he would not go so far as to dismiss the formalism and declare that operators do not represent quantum properties at all. Rather, I argue, the Bohrian view is that the idealizations used for the process of describing measurements are appropriate for that context and do capture (albeit incompletely) important aspects of the properties of the system. Note, too, the different role classical physics plays in the two approaches: while in Bohmian mechanics it plays a sort of metaphysical role in maintaining determinism and in universally setting position as the principle property, for Bohr the role of classical physics is empirical. Classical physics is used in determining the empirical meaning of the quantum symbols by allowing us to establish a connection between quantum properties and the classically-defined properties we take our measuring instruments to measure; moreover, it allows us to determine the validity of the idealizations that are used in this process. Classical physics thus helps us determine which of the quantum properties are being "picked out" in any particular experiment.

That is, the Bohrian view is that we can attribute to the system an idealized, approximate classical description of the properties represented by the quantum symbolism. Quantum effects are evident only in the relations between incompatible idealizations, and it is only through these commutation relations that we know that these properties are incompatible. The formalism is thus fundamentally important to the Bohrian view in a way it would not be if properties were taken to be merely epiphenomenal or operational. In the latter cases, the formalism merely expresses facts about experimental arrangements and nothing interesting about quantum properties themselves. Yet because the Bohrian view allows that we can attribute properties into to the quantum system with appropriate reasoning, the commutation relations express a fact about both the success and limitations of the attribution of idealized properties to quantum systems. To interpret quantum mechanics, one needs *both* to understand the way in which non-commutativity and incompatibility is expressed in the formalism and to understand how the formalism is connected to empirical situations.

Complementarity thus reflects how this approximate correspondence between quantum and classical properties in one empirical situation relates to the approximate correspondence one can establish in another. The abstract formalism of quantum mechanics expresses these relationships and indeed demonstrates how different sets of classical quantities are incompatible, but the whole story about complementarity requires an account of the connection of this formalism to the various classical descriptions.

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