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Some Considerations on Conditional Chances

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ABSTRACT

Four interpretations of single-case conditional propensities are described and it is shown that for each a version of what has been called 'Humphreys' Paradox' remains, despite the clarifying work of Gillies, McCurdy and Miller. This entails that propensities cannot be a satisfactory interpretation of standard probability theory.

1 Introduction

Donald Gillies' illuminating survey article on propensities (Gillies [2000]) discusses a number of responses to what has been called 'Humphreys' Paradox',[1] henceforth abbreviated to HP. The essence of HP is that single-case conditional propensities can lead to an inconsistency when principles such as Bayes' Theorem are used to invert those conditional propensities. There now exists a body of literature responding to HP, much of which has helped to refine our understanding of what the formal constraints on

[1] The name originates with Fetzer ([1981]).

propensities must be. My purpose here is to assess the major lines of response to HP in a way that I hope will illuminate the nature of conditional propensities. My conclusion will be that none of the existing responses undermines the principal consequence of HP that conditional single-case propensities cannot be standard probabilities. This indicates that the nature of propensities cannot be properly captured by standard probability theory.

It is not often remarked that at least three different versions of the paradox have been proposed. This variety of versions has arisen because the core idea behind the problem is easily conveyed informally, and discussions which present the problem in this informal way have fastened upon different aspects of the basic problem. As a result, not all of the replies to HP are replies to the same thing. The original version of the paradox (Humphreys [1985]) and the two other versions discussed here are formal paradoxes in the sense of giving rise to explicit inconsistencies. Other, more casual, formulations are paradoxes only in the sense of presenting a result which conflicts with ordinary expectations. It is the formal versions that require our attention because, for those, a satisfactory solution is required.

2 The basic issue

The essence of the issue can easily be conveyed. Suppose some conditional propensity exists, the propensity for D to occur conditional on C, $Pr(D \mid C)$.[2] For concreteness, consider the propensity of an individual to come down with influenza (event D), conditional upon his having been exposed to some other specific individual with this illness (event C). This is the kind of case to which propensities should be applicable, if single-case conditional propensities are countenanced at all, because a primary reason for introducing propensities was to deal with objective chances attached to specific situations. Standard theories of conditional probability require that when $P(D|C)$ exists, so does the inverse conditional probability $P(C|D)$. The value of $P(C|D)$ can easily be calculated within these standard theories by using the calculus of elementary probability, for example via a basic form of Bayes' Theorem:

$$P(C \mid D)=P(D \mid C)\ P(C)/P(D)$$

Yet the inverse propensity in the above example, $Pr(C \mid D)$, the propensity of the individual to have been exposed to the carrier, given that he gets influenza at some later time, is not related to $Pr(D \mid C)$ in any simple way, if indeed it is mathematically dependent at all. One might even doubt whether such an inverse propensity exists. As we shall see in a moment, it can easily be shown

[2] Propensities are indicated by the notation 'Pr'; probabilities by 'P'.

that the inverse probability calculations based on Bayes' Theorem and related results lead to an inconsistency when supplemented with a simple principle about conditional propensities.

3 The formal paradox

The original formal argument in Humphreys ([1985]) used an example as an illustration, although the example was intended to be representative of any system generating non-extremal conditional propensities. It involves photons being emitted from a source at time t and impinging upon a half-silvered mirror. Some of those photons are transmitted through the mirror, others are reflected from it, and what a particular photon[3] does is irreducibly indeterministic. We have two events $I_{t'}$ (a particular photon being incident upon the mirror) and $T_{t''}$ (that photon being transmitted through the mirror), where $T_{t''}$ occurs later than $I_{t'}$. There is also a set of background conditions B_t which includes all the features that affect the propensity value at the initial time t, which is earlier than the times of both $I_{t'}$ and $T_{t''}$. The fact that this single-case propensity value $Pr_t(.|.)$ is attributed at the time t and not at the time of the conditioning event is a matter of some importance, as we shall see.

The assumptions fall into two groups. The first contains assignments of conditional propensity values:

(i) $Pr_t(T_{t''} \mid I_{t'}B_t) = p > 0$

(ii) $1 > Pr_t(I_{t'} \mid B_t) = q > 0$

(iii) $Pr_t(T_{t''} \mid \neg I_{t'}B_t) = 0$

where the arguments of the propensity functions are names designating specific physical events. They do not pick out subsets of an outcome space as in the measure-theoretic approach.[4]

The second group consists of a single principle of conditional independence. It asserts that the propensity values of earlier events do not depend upon the occurrence or non-occurrence of later events:

(CI) $Pr_t(I_{t'} \mid T_{t''}B_t) = Pr_t(I_{t'} \mid \neg T_{t''}B_t = Pr_t(I_{t'} \mid B_t)$

(Note that here CI stands for conditional independence, not causal independence.)

[3] For the purposes of the example, I believe it is correct to speak of a particular photon. One may think of this in terms of the rate of emission of photons being so low that the probability of there being more than one emitted during a typical transit time is negligible.

[4] This is to keep the propensities oriented towards material, rather than formal, entities. By doing so, it requires significant metaphysical commitments to the existence of less than fully specific properties, such as 'is even', in order to maintain contact with common probability attributions, the discussion of which I shall not pursue here. Those who wish to avoid such commitments may simply think in terms of their favourite theory of events.

From these four assumptions alone, it is straightforward to show that the use of Bayes' Theorem results in an inconsistent attribution of propensity values, because $\Pr_t(I_{t'} \mid T_{t''}B_t) = 1$ when calculated from (i), (ii) and (iii) using Bayes' Theorem, but $\Pr_t(I_{t'} \mid T_{t''}B_t) < 1$ when calculated using CI and (ii). In addition, a familiar principle of probability theory, the multiplication principle, also results in inconsistent attributions from the four assumptions together with the use of one standard feature of probability theory (the theorem on total probabilities) which does not require the use of inverse probabilities for its proof.[5]

There are three issues of philosophical interest that emerge from the replies to this problem which have been formulated. The first is that they force us to consider how we attribute numerical values to conditional propensities. Are they values to be attributed on the basis of substantive theoretical considerations, on the basis of empirical data including experimental data, or, in certain cases, on the basis of broad a priori principles? There are interesting differences in the attributions of such values amongst the published responses to the paradox, and it is revealing to isolate the principles on the basis of which these attributions are made. The second issue is the need to be quite clear about what conditional propensities are—what bears them, what are their arguments, and whether they represent degrees of causal influence. The third issue concerns the dynamics of propensities and how their values change over time.

4 Values of conditional propensities

The three forms of the paradox discussed in the literature can be characterized in terms of the principles they use to attribute values to conditional propensities $\Pr_t(I_{t'} \mid T_{t''})$ when t'' is later than t'.[6] (I assume here, in addition, that t is earlier than t'.) The first principle is the *conditional independence principle* CI, which claims that any event that is in the future of $I_{t'}$ leaves the propensity of $I_{t'}$ unchanged; i.e. $\Pr_t(I_{t'} \mid T_{t''}) = \Pr_t(I_{t'})$. This principle reflects the idea that there exists a non-zero propensity at t for $I_{t'}$ to occur, and this propensity value is unaffected by anything that occurs later than $I_{t'}$. A second principle, which we may call the *zero influence principle*, holds that when t'' is later than t', $\Pr_t(I_{t'} \mid T_{t''}) = 0$. That is, any event $T_{t''}$ that is in the future of $I_{t'}$ is such that the propensity at t for $I_{t'}$, conditional upon $T_{t''}$, is zero. This second view is appealing to those who consider conditional propensities to represent the degree of causal influence between the conditioning and the conditioned events. A third principle, which we can call the *fixity principle*, first represented

[5] For details see Humphreys ([1985], p. 562).

[6] To avoid quibbles, we can set aside the case where t' and t'' are contemporaneous. Nothing of any great importance hinges on that case, except the truth status of $\Pr_t(I_{t'} \mid I_{t'}) = 1$. From here onwards, I shall drop explicit notation about the background conditions B_t unless otherwise indicated.

in Milne ([1986]), claims that when t'' is later than t', $Pr_t(I_{t'} \mid T_{t''}) = 0$ or $Pr_t(I_{t'} \mid T_{t''}) = 1$. This is because by the time the later event $T_{t''}$ has occurred, the occurrence or non-occurrence of the earlier event $I_{t'}$ is already fixed.[7] Each of these principles might be justified on the basis of an a priori argument, on the basis of a posteriori argument based on empirical evidence, or on a case-by-case basis using experimental manipulations. We have already seen how employment of principle CI leads to an inconsistency with the use of Bayes' Theorem. A parallel argument using the zero influence principle results in a similar inconsistency. In the case of the fixity principle, because this is an indeterministic system, it is not determined whether the event $T_{t''}$ will occur once the event $I_{t'}$ has occurred. So there will be cases in which $Pr_t(I_{t'} \mid T_{t''}) = 0$ on the fixity view, again producing an inconsistency with the results of Bayes' Theorem. Similar arguments give rise in each case to violations of the multiplication principle, as the reader can easily check.

5 Interpretations of propensities

A traditional taxonomy of propensities separates single-case accounts from long-run accounts. The former consider a propensity to be a disposition of a specific system to result in a specific outcome under specific test conditions. The latter takes a propensity to be a disposition of a specific system to produce specific values of frequencies under specific test conditions. A finer-grained account of single-case conditional propensities has emerged as a result of addressing HP. Four of these are the most important.

A *co-production interpretation* considers the conditional propensity to be located in structural conditions present at an initial time t, with $Pr_t(\cdot|\cdot)$ being a propensity at t to produce the events which serve as the two arguments of the conditional propensity. The positions of Miller ([1994], [2002]) and McCurdy ([1996]) fall into this category. Gillies ([2000]) has proposed a long-run version of the co-production interpretation. Miller, who first formulated the approach in his 1994 book *Critical Rationalism*, expressed the position in a later paper in this way:

> $P_t(A_{t'} \mid C_{t''})$ is the propensity of the world at time t to develop into a world in which A comes to pass at time t', given that it (the world at time t) develops into a world in which C comes to pass at time t''. (Miller [2002], p. 113)[8]

[7] There is a fourth position which asserts that inverse propensities are in general meaningless and hence that propensities are, at best, an incomplete interpretation of the probability calculus. I shall not discuss this position, because inverse propensities such as the propensity for an individual to have been exposed to a carrier given that he gets the 'flu' at some later time are clearly legitimate objects of discussion. Terms referring to them do not lack meaning, even if the associated propensities do not exist or, what is not the same, have value zero.

[8] Also in Miller ([1994]), p. 189.

In McCurdy ([1996], pp. 108–9), we find this description:

> The events described by the background conditions are responsible for the assignment of particular probability values to the members of the (previously established) event space, because these events are *responsible for the production of* the events in the event space [. . .] [T]he values assigned to conditional and inverse conditional propensities are intended to provide a measure of the strength of the propensity for the system to produce the two future events in the manner specified.

The key feature of co-production interpretations is that the representations of the conditions present at the initial time t are not included in the algebra or F-algebra of events within the probability space, in contrast to the events that occur as a result of those initial conditions, representations of which are a part of the formal probability space. The probability measure is defined only over those latter events, and the conditional probability measure is then defined in the usual way (for $P(B) > 0$) as $P(A \mid B) = P(A\&B)/P(B)$. In co-production interpretations, the conditional propensity is attributed to the system at the initial time t and it is a propensity for the system, characterized by the background conditions B_t, to produce events in the probability space that are related by the standard definition just given. The conditional propensity is not taken to be a property of the system at the time of the later conditioning event.

A co-production interpretation is in sharp contrast to a *temporal evolution interpretation*, which takes the propensity to have an initial value at t, with the propensity then evolving temporally, usually with its value changing under the influence of subsequent events. Within temporal evolution interpretations, the time order of events is crucial: when $t < t' < t''$, the propensity has an initial value at t, but this changes as later events occur between t and t' and the system evolves. When the conditioning event D occurs earlier than the conditioned event C (i.e. $t' < t''$), the conditional propensity $Pr_t(C_{t''} \mid D_{t'})$ is simply a temporal update of the original propensity $Pr_t(C_{t''})$ which was evaluated at the initial time t. When the time of the conditioning event D is later than that of the conditioned event C (i.e. $t' > t''$), the propensity evolves to the point where $C_{t''}$ either occurs or fails to occur at t'', and anything temporally subsequent is irrelevant. Principle CI is thus true for the temporal evolution interpretation.

Apparently similar to a temporal evolution interpretation, but in fact quite distinct, are *renormalization interpretations*. Here, instead of the temporal parameter t continuously evolving within real time, the conditioning event forces a jump from t to the time of the conditioning event, and the propensity value is determined at that new time. When $t < t' < t''$, this produces the same results as for the temporal evolution interpretation. But when t' is later than t'', the conditioning process 'jumps over' the events between t and t', including t'', and determines the propensity value at time t', in contrast to the temporal evolution interpretation, where the evolution of the propensity must pass through events in their temporal order. The fixity view is entailed by the renormalization interpretation. This is because for conditional propensities such as $Pr_t(C_{t''} \mid D_{t'})$, by the time the conditions $D_{t'}$ are in place, the event $C_{t''}$ either has already occurred or has failed to occur. One caveat: in standard probability theory, the renormalization procedure allows us to identify the conditional probability $P(C \mid D)$ with an unconditional probability $P_D(C)$ restricted to a new domain of subsets of D. Because propensities are features of the world and are not properties of sets, that approach is inappropriate for propensities.

The fourth position, the *causal interpretation*, takes the conditional propensity to represent the degree of causal influence between the conditioning event and the conditioned event. An advocate of a causal interpretation, although in the context of probabilistic conditionals rather than conditional probabilities, is Fetzer ([1981]). The causal interpretation entails the zero influence view, on the usual position that there is no temporally backwards causation.

Having laid out these positions, we can now consider the various responses to HP. I shall show that for each of the four interpretations of conditional propensities, at least one of the three principles CI—the fixity principle, or the zero influence principle, on the basis of which attributions of conditional propensity values are made—is true and hence that, for each of the four interpretations of propensities just described, a version of HP exists.

6 McCurdy's response

One direct response to the original version of HP is to argue that principle CI (i.e., $Pr_t(I_{t'} \mid T_{t''}B_t) = Pr_t(I_{t'} \mid \neg T_{t''}B_t) = Pr_t(I_{t'} \mid B_t) = q$) is false for the photon case and, by implication, that it fails for other situations having a similar structure. This position is taken in McCurdy ([1996]), where, as we have seen, he defends a co-production interpretation of conditional propensities.

I shall respond to McCurdy's arguments in two different ways. First, I shall argue that although a plausible case can be made for the co-production interpretation in my original photon example, a structurally similar example illustrates that this response will not work in that and other cases. Second, I shall argue that the co-production interpretation itself is seriously flawed as an interpretation of conditional propensities, at least in the sense that at best it preserves probability theory at the expense of losing the characteristic dispositional content of conditional propensities.

At the heart of McCurdy's argument against the original HP is his claim that he can establish, without any appeal to the probability calculus, that $Pr_t(I_{t'} \mid T_{t''}B_t) = 1$, a value that is inconsistent with the assignment $q < 1$ given to the conditional propensity by CI together with the original assignment (ii). Here is McCurdy's argument:

> Instead of utilizing the inversion theorems to determine the value of $\Pr_t(I_{t'} \mid T_{t''}B_t)$, the value can be arrived at as follows: the value of $\Pr_t(I_{t'} \mid T_{t''}B_t)$ must be one since the description of the system indicates that the system is arranged in such a manner that if the system produces a photon that is transmitted at t″ then the system must also produce a photon that impinges upon the mirror at t′. Indeed it is assignment iii) [i.e. that $\Pr_t(T_{t''} \mid \neg I_{t'}B_t) = 0$] that provides the information that the system is arranged in this manner, but it is the arrangement of the photon system itself—and not the value of $\Pr_t(T_{t''} \mid \neg I_{t'}B_t)$—that demands that $\Pr_t(I_{t'} \mid T_{t''}B_t) = 1$. (McCurdy [1996], pp. 110–1)

It should be clear from this quotation that McCurdy is not making the error of appealing to the fact that we can infer with certainty from the structure of the experimental arrangement that when a photon has been transmitted it must have been incident upon the mirror. Rather, it is the physical structure of the arrangement at t which he claims is the basis for the attribution of the value $\Pr_t(I_{t'} \mid T_{t''}B_t) = 1$, and an appeal to a co-production interpretation is clearly at work here.

The most direct evidence for this is his claim that:

> The fact remains that, although the events $I_{t'}$, $T_{t''}$, and $\neg T_{t''}$ lack common causal factors between the times t′ and t″, the events $I_{t'}$, $T_{t''}$, and $\neg T_{t''}$ share common causal factors that are effective between t and t′. Specifically, the photon transmission arrangement itself (described by B_t) provides a host of common causal factors. This fact is responsible for the failure of principle CI: if the system produces event $T_{t''}$, then it must have exhibited certain causal factors, some of which have an influence on the event $I_{t'}$. (McCurdy [1995], p. 116)

Because of some previous remarks in his paper alluding to factors responsible for the momentum of the emitted photons, it is possible that the photon example misleadingly suggests some quasi-deterministic aspects of the fundamentally indeterministic propensity at t, $\Pr_t(I_{t'}|B_t)$. So it may help to consider a somewhat different example, consisting of a radioactive source of alpha particles with a spherical radiation detector completely surrounding the source. The detector is shielded from all other sources of radiation but is of less than perfect reliability so that not all emitted particles are detected. The propensity for an alpha particle to be emitted in a specified time period is, I hope suncontroversially, taken to be fundamentally indeterministic. Let $\Pr_t(E_{t'} \mid D_{t''})$be the propensity at time t for an alpha particle to be emitted during the short time interval t′[9] conditional upon that alpha particle being detected at t″, where $t < t' < t''$. This example is then formally identical to the

[9] The use of a time interval rather than an instant introduces no essentially new considerations.

photon example, and it should be clear that because of the irreducibly indeterministic nature of radioactive decay, there are no common causal factors between t and the beginning of t′ (or the precise time of emission, if you prefer) on the basis of which one could truly assert that, if the system produces the event $D_{t''}$, then it must have exhibited certain causal factors between t and t′ some of which have an influence on $E_{t'}$. Principle CI is, then, true, and I believe evidently true, for this case—i.e. $Pr_t(E_{t'} \mid D_{t''}) = Pr_t(E_{t'} \mid \neg D_{t''}) = Pr_t(E_{t'})$. $Pr_t(E_{t'} \mid D_{t''})$—is thus not equal to unity as the co-production interpretation requires. Given that this is so, HP still stands.

My second reply to McCurdy's argument is more general and applies to any co-production interpretation of conditional propensities. A co-production interpretation does preserve the formal structure of probabilities on the event space, but at a price. The price is that under this interpretation, the structural basis of the propensity presents the relation between the conditioning and conditioned events as a relation between probability measures rather than as a material relation between concrete events. There is no propensity relationship between the conditioning and conditioned events in the conditional probability $P(A \mid B)$ on this view. It is thus not a single-case conditional propensity, strictly speaking. A major appeal of single-case propensities has always been their shift in emphasis from the outcomes of trials to the physical dispositions that produce those outcomes. To represent a conditional propensity as a function of two absolute propensities, as co-production interpretations do, is to deny that the disposition inherent in the propensity can be physically affected by a conditioning factor. This is, at root, to commit oneself to the position that there are conditional probabilities but only absolute propensities.

I now turn to the response given by David Miller in his 2002 paper to a different version of the paradox.

7 Miller's response

David Miller invented the co-production interpretation of propensities, versions of which may be found in his ([1994]) and ([2002]).[10] Miller formulates the paradox in this way:

> Intuitively we may read the term $P_t(A|C)$ as the propensity at time t for the occurrence A to be realized given that the occurrence C is realized. If C precedes A in time, this presents no extraordinary difficulty. But if C follows A, or is simultaneous (or even identical) with A, then it appears that there is no propensity for A to be realized given that C is realized; for

[10] In §9.6 of his ([1994]), Miller constructs a novel relative frequency account of probability but its merits can be considered separately from those of propensity theories.

> either A has been realized already, or it has not been realized and never will be. In a late work Popper wrote that 'a propensity zero means *no* propensity' (Popper [1990], p.13). If the converse holds too, we may conclude that $P_t(A \mid C)$ has the value zero unless C precedes A in time. But it is easy to construct examples in which none of $P_t(A|C)$, $P_t(A)$, and $P_t(C)$ is zero. The simplest version of Bayes's theorem, to the effect that $P_t(A \mid C) = P_t(C \mid A)P_t(A)/P_t(C)$, is thereby violated. (Miller [2002], p. 112)

We see from this quotation that the version of HP that Miller is addressing is based on the zero influence principle, reinforced by an appeal to fixity (although not to the fixity principle as a basis for attributing propensity values).

His response to the problem is contained in these two paragraphs:

> The objection involves a subtle misreading into the phrase 'given that' of an inappropriate temporal reference. Suppose that A is an occurrence that is realized, if it is realized at all, at time t′, and that C is an occurrence that is realized, if it is realized at all, at time t″. Talk of the propensity at time t for A to be realized (at time t′) given that C is realized (at time t″) does not mean that the realization of C at time t″ is supposed to be given at time t′. It means that the realization of C at time t″ is supposed to be given at time t. Of course, if t is earlier than t″, then this supposition is subjunctive. But provided that t is earlier than t″, there is no difficulty in principle in attributing a positive value to $P_t(A_{t'} \mid C_{t''})$. Note that if t″ too is earlier than t′, and C comes to pass at t″, then there is an innocuous sense in which the occurrence of C is given at t′—by the time t′ is reached, C has been realized. This is not the sense of the phrase 'given that' that is central to the theory of relative probability.
>
> Only if t is earlier than t″ can $P_t(A_{t'} \mid C_{t''})$ differ from $P_t(A_{t'})$, the absolute propensity at t for A to be realized at t′. We may set aside as uninteresting the case in which t is not earlier than t″. Now it should be obvious that to suppose at t that C comes to pass at t″ is not to suppose incoherently that every occurrence dated between t and t″ also comes to pass; it is not even to suppose at t that we are already at t″. Provided therefore that t is earlier than t″, to suppose at t that C comes to pass at t″ is not to suppose either that A comes to pass at t′ or that it does not come to pass at t′, even if t′ is earlier than t″. In consequence there is, if t′ is earlier than t″, nothing in principle that disallows $P_t(A_{t'} \mid C_{t''})$ from taking any value greater than zero. Of course, the value of $P_t(A_{t'} \mid C_{t''})$ will be either zero or unity unless t is earlier than t″. (Miller [2002], pp. 112–3)

Miller's response to HP as he has formulated the problem is indeed a powerful response to versions of HP that are based on the zero influence principle. It is also a response to versions that are based on the fixity principle. It is not, however, a response to the original HP, because subscribing to the principle CI does not entail attributing a zero value to the relevant conditional propensity. In Section 5 of Miller ([2002]), the co-production interpretation of conditional propensities is once again endorsed, and Miller asserts in Section 1

AQ: Please provide Table caption

Table 1

$t < t' < t''$	Principle CI $Pr_t(C_{t'} \mid D_{t''}) = Pr_t(C_{t'} \mid \sim D_{t''}) = Pr_t(C_{t'})$	Fixity $Pr_t(C_{t'} \mid D_{t''}) = 0$ *or* 1	Zero Influence $Pr_t(C_{t'} \mid D_{t''} B_t) = 0$
Co-production	True	False	False
Temporal evolution	True	False	False
Renormalization	False	True	False
Causal	False	False	True

that he is 'largely in agreement with' the arguments contained in McCurdy's paper. So it is perhaps taken to be implicit that the co-production interpretation provides an effective response to principle CI. If so, then one can bring to bear against this view the arguments presented in Section 6 above to the effect that one has retained the structure of conditional probabilities at the expense of removing from conditional propensities what has traditionally been considered essential to them.

8 Other possibilities

We can summarize the relation between the three principles and the co-production interpretation of conditional propensities as shown in the first line of Table 1.

Our discussion of the remaining elements of the table can be brief, but it requires a little elaboration on the three other interpretations of propensities—temporal evolution interpretations, renormalization interpretations, and causal interpretations. The reader may not find all of these interpretations congenial but they do capture, perhaps somewhat crudely, attitudes towards conditional propensities that I have encountered in published and unpublished discussions of HP.[11]

8.1 Temporal evolution

Recall that in this view, the value of the propensity is firmly rooted in present material conditions, and that value dynamically evolves through time. In that sense, we can view the time t as a parameter that is continuously updated. Because only current and not hypothetical future situations affect the value of the propensity in this interpretation, principle CI is true. For similar reasons, the fixity principle and the zero influence principle are false.

[11] An issue that I want to set aside here is the effect that HP has on our theory of rational degrees of belief through Lewis's Principal Principle or something akin to it. Propensities are objective features of the world and the view that they must be constrained by subjective probabilities is not one that I find attractive. But there is no doubt that ignoring the way the world is can be financially ruinous if one is inclined to gamble.

8.2 Renormalization

Here $Pr_t(C_{t'} \mid D_{t''})$ takes the occurrence of the event $D_{t''}$ as updating the initial probability assignment so that the conditions $D_{t''}$ at t'' are part of the basis of the propensity. With this view in mind, since t'' is later than t', the fixity principle is true and in consequence the zero influence principle will not be true in general. The principle CI will be false because $Pr_t(C_{t'})$ will not have extremal values in indeterministic contexts.

8.3 Causal influence

Within this interpretation, the conditional propensity captures the degree of causal influence of the conditioning event, D, on the main event of interest, C. Thus, assuming there is no temporally reversed causal influence, the zero influence principle is true for this interpretation.[12] Because the fixity principle also allows values of unity to be attributed to the conditional propensity, the fixity principle is false under this interpretation. Finally, since the degree of causal influence on $C_{t'}$ by $D_{t''}$ is zero, but in general $Pr_t(C_{t'}) \neq 0$, principle CI is also false for this interpretation.

This concludes our case-by-case evaluation of the twelve possibilities. We now see that for each of the four interpretations of conditional propensities, there exists exactly one principle governing the attribution of conditional propensities that is true under that interpretation. In consequence, for each of the four interpretations there is exactly one version of HP which shows that conditional propensities cannot be probabilities. The view that propensities are probabilities cannot be saved by switching interpretations.

9 Propensities to generate frequencies

Donald Gillies ([2000]) adopts what he terms a 'long run propensity' view. This is 'one in which propensities are associated with repeatable conditions, and are regarded as propensities to produce, in a long series of repetitions of these conditions, frequencies which are approximately equal to the probabilities' (p. 822). Although my own preference is for single-case propensities, which in certain stable conditions can ground long-run propensities through limit results, we need to consider Gillies' solution to HP, which he discusses using Milne's version and the frisbee-producing machine example of Earman and Salmon ([1992]). The latter example involves two frisbee-producing machines, one of which produces 800 a day with 1% defective frisbees, the

[12] It is this interpretation that is explicitly rejected in Miller ([2002], p. 115) and in Gillies ([2000], p. 831).

other of which produces 200 a day with 2% defective. The problematical propensity is the propensity for a defective frisbee to have been produced by the first machine. Gillies' account of this propensity, which has the value 2/3, is

> The statement Pr(M | D&S) = 2/3 means the following. Suppose we repeat **S** each day, but only note those days in which the frisbee selected is defective, then, relative to these conditions there is a propensity that if they are instantiated a large number of times, M will occur, i.e. the frisbee will have been produced by machine 1, with a frequency approximately equal to 2/3. (Gillies [2000], p. 829)

There is in this statement a reference to only noting occurrences within which a frisbee is defective, which tints the solution with an unnecessarily epistemic colouring, but that is easily eliminated by simply considering the set of outcomes involving defective frisbees. With that minor adjustment, Gillies' solution has the required objectivity,[13] but it reintroduces exactly the situation from which propensity accounts were intended to rescue us—the relativization of a relative frequency to a reference class and, within von Mises' and some other frequency theories, the need to provide an objective criterion of randomness. In so doing, it loses exactly the features of propensities which proved attractive to many of us. As such, despite its ingenuity, Gillies' solution cannot be a complete account of propensities.

10 Conclusion

The features of propensities explored here force us to confront an important question. If conditional propensities cannot be correctly represented by standard probability theory, what does that say about the status of probability theory? In Humphreys ([1985], Section IV), I tentatively suggested that probability theory should be viewed as a contingent theory. David Miller ([2002], p. 115) suggests something rather different: 'it is a factual matter whether propensities obey the calculus of probabilities,' and floats the idea, derived from Popper, that propensities are generalized forces. Whatever is the truth about these matters, HP is not a mere puzzle. At the very least it tells us that standard probability theory does not have the status of a universal theory of chance phenomena with which many have endowed it.

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[13] Objective in the sense of being a feature of the objects involved.

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