

The Persistence of Memory: Surreal Trajectories in Bohm's Theory

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Abstract

In this paper I describe the history of the surreal trajectories problem and argue that in fact it is not a problem for Bohm's theory. More specifically, I argue that one can take the particle trajectories predicted by Bohm's theory to be the actual trajectories that particles follow and that there is no reason to suppose that good particle detectors are somehow fooled in the context of the surreal trajectories experiments. Rather than showing that Bohm's theory predicts the wrong particle trajectories or that it somehow prevents one from making reliable measurements, such experiments ultimately reveal the special role played by position and the fundamental incompatibility between Bohm's theory and the relativity.¹

¹This paper is an extension of the discussion of surreal trajectories in Barrett (1999, 127–40). That section of the book was based on talk I gave at the Quantum Mechanics Workshop at the University of Pittsburgh in the Spring of 1997. And that talk owed much to conversations with Peter Lewis, David Albert, and Rob Clifton.

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Bohm's theory² has become increasingly popular as a nonrelativistic solution to the quantum measurement problem. It makes the same empirical predictions for the statistical distribution of particle configurations as the standard von Neumann-Dirac collapse formulation of quantum mechanics whenever the latter makes unambiguous predictions. Bohm's theory also treats measuring devices exactly the same way it treats other physical systems. The quantum-mechanical state of a system always evolves in the usual linear, deterministic way, so one does not encounter the problems that arise in collapse formulations of quantum mechanics when one tries to stipulate the conditions under which a collapse occurs. And Bohm's theory does not require one to postulate branching worlds or disembodied minds or any of the other extravagant assumptions that often accompany no-collapse formulations of quantum mechanics.

While Bohm's theory avoids many of the problems associated with other formulations of quantum mechanics, it does have its own problems. One problem, it has been argued, is that the particle trajectories it predicts are not the real particle trajectories. This is the surreal trajectories problem. If Bohm's theory does in fact make false predictions concerning particle trajectories, then this is presumably a serious problem. I will argue, however, that there is no reason to suppose that Bohm's theory makes false predictions concerning the trajectories of particles. Indeed, I will argue that a good position measuring device need never be mistaken concerning the actual position of a particle *at the moment that the particle's position is in fact recorded*.

While surreal trajectories are not a problem for Bohm's theory, the way that it accounts for the results of the surreal trajectories experiments reveals

²This is the theory described by David Bohm in 1952. Bohm's most complete description of his theory is found in Bohm and Hiley (1993).

the sense in which it is fundamentally incompatible with relativity, and this is a problem.

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On Bohm's theory the quantum-mechanical state ψ evolves in the usual linear, deterministic way, but one supposes that every particle always has a determinate position and follows a continuous, deterministic trajectory. The motion of a particular particle typically depends on the evolution of ψ and the positions of other (perhaps distant) particles. The particle motion is described by an auxiliary dynamics, a dynamics that supplements the usual linear quantum dynamics. In its simplest form, what one might call the *minimal version* (the version of the theory described by Bell 1987, 127), Bohm's theory is characterized by the following basic principles:

1. State Description: The complete physical state at a time is given by the wave function ψ and the determinate particle configuration Q .

2. Wave Dynamics: The time evolution of the wave function is given by the usual linear dynamics. In the simplest case, this is just Schrödinger's equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi \tag{1}$$

More generally, one uses the form of the linear dynamics appropriate to one's application (as in the spin examples discussed below).

3. Particle Dynamics: The particles move according to

$$\frac{dQ_k}{dt} = \frac{1}{m_k} \frac{\text{Im}(\psi^* \nabla_k \psi)}{\psi^* \psi} \text{ evaluated at } Q \tag{2}$$

where m_k is the mass of particle k and Q is the current particle configuration.

4. Distribution Postulate: There is a time t_0 when the epistemic probability density for the configuration Q is given by $\rho(Q, t_0) = |\psi(Q, t_0)|^2$.

If there are N particles, then ψ is a function in $3N$ -dimensional configuration space (three dimensions for the position of each particle), and the current particle configuration Q is represented by a single point in configuration space (in configuration space a single point gives the position of every particle). Again, each particle moves in a way that depends on its position, the evolution of the wave function, and the positions of the other particles.

Concerning how one should think of the role of the wave function in Bohm's theory, John Bell once said that "*no one can understand this theory until he is willing to think of ψ as a real objective field rather than just a 'probability amplitude.'* Even though it propagates not in 3-space but in $3N$ -space" (1987, 128). While the ontology suggested by Bell here is at best puzzling, the practical idea behind it is a good one: The best way to picture what the particle dynamics does is to picture the point representing the N -particle configuration being carried along by the probability currents generated by the linear evolution of the wave function ψ in configuration space. Once one has this picture firmly in mind one will understand how Bohm's theory accounts for quantum-mechanical correlations in the context of the surreal-trajectory experiments and the sense in which the theory is fundamentally incompatible with relativity.

Since the total particle configuration can be thought of as being pushed around by the probability current in configuration space, the probability of the particle configuration being found in a particular region of configuration space changes as the integral of $|\psi|^2$ over that region changes. More specifically, the continuity equation

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho v^\psi) = 0 \tag{3}$$

is satisfied by the probability density $\rho = |\psi|^2$. And this means that if the epistemic probability density for the particle configuration is ever $|\psi|^2$, then it will always be $|\psi|^2$, unless one makes an observation. That is, if

one starts with an epistemic probability density of $\rho(t_0) = |\psi(t_0)|^2$, then, given the dynamics, one should update this probability density at time t so that $\rho(t) = |\psi(t)|^2$. And if one makes an observation, then the epistemic probability density will be given by the system's *effective* wave function, the component (in the configuration space representation) of the total wave function that is in fact responsible for the post-measurement time evolution of the system's configuration. The upshot is that if the distribution postulate is ever satisfied, then the most that one can learn from a measurement is the wave packet that the current particle configuration is associated with and the epistemic probability distribution for the actual configuration over this packet.³ This is why Bohm's theory makes the same statistical predictions for particle configurations as the standard collapse formulation of quantum mechanics.

While it makes the same statistical predictions as the standard formulation of quantum mechanics, Bohm's theory is deterministic. More specifically, given the energy properties of a simple closed system, the complete physical state at any time (the wave function and the particle configuration) fully determines the physical state at all other times.⁴ It follows that, given a particular evolution of the wave function, possible trajectories for the configuration of a system can never cross at a time in configuration space. And this feature of Bohm's theory will prove important later.

Another feature of Bohm's theory that will prove important later is the

³See Dürr, D., S. Goldstein, and N. Zanghí (1993) for a discussion of the equivariance of the statistical distribution ρ and the notion of the effective wave function in Bohm's theory.

⁴We will say that a closed system is simple if the Hamiltonian is bounded and if the particle configuration always has positive wave function support.

special role played by position in accounting for our determinate measurement results. In order for Bohm's theory to explain why we get the determinate measurement records that we do (which is presumably a precondition for it counting as a solution to the measurement problem), one must suppose, as a basic interpretational principle, that, given the usual quantum mechanical state, making particle positions determinate provides determinate measurement records. Since particle positions are always determinate on Bohm's theory, this would guarantee determinate measurement records. And, at least on the minimal version of Bohm's theory, position is the only determinate, noncontextual property that could serve to provide determinate measurement records.⁵

The distinction between noncontextual and contextual properties deserves some explanation. Whether a system is found to have a particular contextual property or not typically depends on how one measures the property: one might get the result "Yes" if the contextual property is measured one way and "No" if it is measured another. Consequently, contextual properties are not intrinsic properties of the system to which they are typically ascribed. One might say that contextual properties serve to prop up our talk of those properties that we are used to talking about but which arguably should not count as properties at all in Bohm's theory. While the language of contextual properties provides a convenient (but often misleading!) way of comparing the predictions of Bohm's theory with the predictions of other physical theories, the predictions of Bohm's theory are always ultimately just predictions about the evolution of the wavefunction and the positions of the particles relative to the wavefunction.

⁵There is a sense in which one might say that some dynamical properties, like momentum, are also noncontextual in Bohm's theory, but, as we will see, the noncontextual momentum is not the *measured* momentum.

The upshot of all this is just that position relative to the wave function, or more precisely configuration relative to the wave function, is ultimately the only property that one can appeal to in the minimal version of Bohm’s theory to explain how it is that we end up with the determinate measurement records we do. And this means that *for an interaction to count as a measurement, it must produce a record in terms of the position of something*—it must correlate the position of something with some aspect of the quantum-mechanical state of the system being measured. So in order to explain our determinate measurement records on Bohm’s theory one must suppose that all measurement records are ultimately position records, records represented in the relationship between the particle configuration and the wave function. And since Bohm’s theory predicts the right quantum statistics for particle positions relative to the wave function, it predicts the right quantum statistics for our measurement records.⁶

3

In their 1992 paper Englert, Scully, Süssman, and Walther (ESSW) argued that the trajectories predicted by Bohm’s theory are not the real trajectories followed by particles, but rather are “surreal”. The worry is that the observed trajectories of particles are not the trajectories that the particles actually follow in Bohm’s theory. And if our observations are reliable and if Bohm’s theory predicts the wrong particle trajectories, then this is presumably a problem for the theory.

⁶The point here is that making the right statistical predictions concerning *particle configurations* is not necessarily a sufficient condition for making the right statistical predictions for our *measurement records*. One needs to make an extra assumption about the relationship between particle configurations and measurement records.

ESSW describe the surreal trajectories problem in the context of a two-path, delayed-choice interference experiment. John Bell (1980, reprinted in 1987) was perhaps the first to consider such an experiment in the context of Bohm's theory.

Consider an experiment where a spin-1/2 particle P starts at region S in a z -spin up eigenstate, has its wave packet split into an x -spin up component that travels from A to A' and an x -spin down component that travels from B to B' .⁷

[Figure 1: Crossing-Paths Experiment]

The wave function evolves as follows:

Initial state:

$$|\uparrow_z\rangle_P|S\rangle_P = 1/\sqrt{2}(|\uparrow_x\rangle_P + |\downarrow_x\rangle_P)|S\rangle_P \quad (4)$$

After the initial wave packet splits:

$$1/\sqrt{2}(|\uparrow_x\rangle_P|A\rangle_P + |\downarrow_x\rangle_P|B\rangle_P) \quad (5)$$

Final state:

$$1/\sqrt{2}(|\uparrow_x\rangle_P|A'\rangle_P + |\downarrow_x\rangle_P|B'\rangle_P) \quad (6)$$

Bell explained that if one measures the properties of P in region I , then one would observe interference phenomena (in this experiment, for example, one would observe z -spin up with probability one). The observation of interference phenomena is usually taken to entail that P could not have followed path A and could not have followed path B since, in either case, the probably

⁷Since the standard line is that position is the only observable physical quantity in Bohm's theory, this does not mean that P has a determinate z -spin; rather, it is just a description of the spin index associated with P 's effective wave function.

of observing z -spin up in I would presumably be $1/2$ (as predicted by the standard collapse formulation of quantum mechanics). In such a situation, one would say, on the standard view, that P followed a *superposition* of the two trajectories (which, on the standard interpretation of states is supposed to be neither one nor the other nor both trajectories). But according to Bohm's theory, P determinately follows one or the other of the two trajectories: that is, it either determinately follows A or it determinately follows B . On Bohm's theory, one might say that the interference effects that one observes at I are the result of the wave function following both paths.

If we do not observe the particle in region I , then P will arrive at one of the two detectors to the right of the interference region: either the one at A' or the one at B' . If it arrives at A' , then one might suppose that the particle traveled path A ; and if it arrives at B' , then one might suppose that it traveled path B . But such inferences do not work in the standard collapse formulation of quantum mechanics, where (according to the standard eigenvalue-eigenstate link) under these circumstances P would have traveled a superposition of the two paths. And such inferences do not work in Bohm's theory either, but for a very different reason. In Bohm's theory the particle really does travel one or the other of the two paths, it is just that its trajectory is not what one might at first expect.

In figuring out what trajectory Bohm's theory predicts, the first thing to note is that, by symmetry, the probability current across the line L is always zero.⁸ This means that if P starts in the top half of the initial wave packet, then it must move from S to A to I to B' ; and if it starts in the bottom half of the initial wave packet, then it must move from S to B to I to A' . That

⁸See Phillipidas, Dewdney, and Hiley (1979) for an explicit calculation of the trajectories in for a similar experiment. The explicit calculations, of course, show that possible particle trajectories never cross L . Bell cites this paper at the end of his 1980 paper on delayed-choice experiments in Bohm's theory.

is, whichever path P takes, Bohm's theory predicts that it will *bounce* when it gets to region I —in order to follow either trajectory, the particle P must accelerate in the field-free region I .

Concerning this odd bouncing behavior Bell says that “it is vital here to put away the classical prejudice that a particle moves in a straight path in ‘field free’ space” (1987, 113). But certainly, one might object, this is more than a prejudice. After all, this particle bouncing nonsense is a direct violation of the conservation of momentum, and we have very good empirical reasons for supposing that momentum is conserved. Isn't this alone reason enough to dismiss Bohm's theory? Put another way, whenever we observe which path the particle in fact travels, if we find it at A' , then we also observed it traveling path A and if we find it at B' , then we also observed it traveling path B . That is, whenever we make the appropriate observations, we never observe the crazy bouncing behavior (or any of the other violations of the conservation of momentum) predicted by Bohmian mechanics.

This puzzling situation is the basis for ESSW's surreal trajectories argument. The argument goes something like this:

Assumption 1 (explicit): Our experimental measurement records tell us that in a two-path interference experiment like that described above each particle either travels from A to A' or from B to B' ; that is, they never bounce.

Assumption 2 (implicit): Our measurement records reliably tell us where a particle is at the moment the record is made.

Assumption 3 (implicit): One can record which path a particular particle takes without breaking the symmetry in the probability currents that prevents the particle from crossing the line L .

Conclusion: The trajectory predicted by Bohm’s theory, where the particle bounces, cannot be the particle’s actual trajectory; that is, Bohm trajectories are not real, they are “surreal.” And if the trajectories predicted by Bohm’s theory are not the actual particle trajectories, then Bohm’s theory is false, and this constitutes very good grounds for rejecting it.

Dürr, Füsseder, Goldstein, and Zanghi (DFGZ) immediately responded to defend Bohm’s theory against the surreal trajectories argument:

In a recent paper [ESSW (1992)] it is argued that despite its many virtues—its clarity and simplicity, both conceptual and physical, and the fact that it resolves the notorious conceptual difficulties which plague orthodox quantum theory—BM [Bohmian mechanics] itself suffers from a fatal flaw: the trajectories that it defines are “surrealistic”. It must be admitted that this is an intriguing claim, though an open minded advocate of quantum orthodoxy would presumably have preferred the clearer and stronger claim that BM is *incompatible* with the predictions of quantum theory, so that, despite its virtues, it would not in fact provide an explanation of quantum phenomena. The authors are, however, aware that such a claim would be false. (1993, 1261)

And since Bohm’s theory makes the same predictions as the standard theory of quantum mechanics, DFGZ argue that ESSW cannot possibly provide, as ESSW describe it, “an experimentum crucis which, according to our quantum theoretic prediction, will clearly demonstrate that the reality attributed to Bohm trajectories is rather metaphysical than physical.” And with this DFGZ dismiss ESSW’s argument against Bohmian mechanics:

On the principle that the suggestions of scientists who propose

pointless experiments cannot be relied upon with absolute confidence, with this proposal the [ESSW] paper self-destructs: The authors readily agree that the “quantum theoretical predictions” are also the predictions of BM. Thus they should recognize that the [experimental] outcome on the basis of which they hope to discredit BM is precisely the outcome predicted by BM. Under the circumstances it would appear prudent for the funding agencies to save their money! (1261)

DFGZ conclude their defense of Bohm’s theory by making a point about the theory-ladenness of talk of particle trajectories and a point about the theory-ladenness of observation itself. But we will return to these two (important) points later, when we have the conceptual tools hand to make sense of them (Section 5).

In their reply to DFGZ’s comment, ESSW want to make it perfectly clear that they did not anywhere concede that Bohm’s theory had “many virtues” nor did they admit that the orthodox formulation of quantum mechanics was “plagued by notorious conceptual difficulties.” But, for their part, ESSW do seem to concede, as DFGZ insisted, that Bohmian mechanics makes the same empirical predictions as standard quantum mechanics: “Nowhere did we claim that BM makes predictions that differ from those of standard quantum mechanics” (1263).⁹ Rather than argue that Bohm’s theory made the

⁹But ESSW later make the following argument in favor of actually funding the surreal trajectories experiments that they describe: “Funding agencies were and are well advised to support experiments that have probed or would probe the “surprises” of quantum theory. Imagine the (farfetched) situation that the experimenter finds the photon always in the resonator through which the Bohm trajectory passes rather than the one predicted by quantum theory. Wouldn’t that please the advocates of BM?” (1263–4). This is, of course, very puzzling talk indeed once EWWS conceded that Bohmian mechanics makes the same empirical predictions as the standard theory—a proponent of Bohm’s theory

wrong empirical predictions, ESSW claim that the purpose of their original paper was “to show clearly that the interpretation of the Bohm trajectory—as the real retrodicted history of the [test particle that travels through the interferometer]—is implausible, because this trajectory can be macroscopically at variance with the detected, actual way through the interferometer” (1263). This last clause identifies the detected path with the actual path traveled by the test particle. This is their (implicit) assumption that particle detectors would be reliable (in a perfectly straightforward way) on the delayed-choice interference experiments that they discuss. ESSW conclude, “Irrespective of what can be said in addition, we think that we have done a useful job in demonstrating just how artificial the Bohm trajectories can be” (1264).

Again, ESSW’s claim is not that Bohm’s theory makes the wrong empirical predictions nor it is that the theory is somehow logically inconsistent; rather, they argue (on the implicit assumption that our particle detectors reliably tell us where particles are) that Bohm’s theory makes the wrong predictions for the actual motions of particles—that the predicted particle trajectories are “artificial,” “metaphysical,” and, at best, “implausible.”

While I agree with DFGZ that surreal trajectories are not something that a proponent of Bohm’s theory should worry about, the full story is a bit more involved than the sketch given in their comment on ESSW’s paper. In order to get everything straight, let’s return to Bell’s original analysis of the delayed-choice interference experiment in the context of Bohm’s theory.

would most certainly *not* be pleased if experiments showed that the standard quantum-mechanical predictions were false because this would mean that Bohm’s theory was itself false! When ESSW say things like this, it is easy to understand DFGZ’s frustration.

4

Bell’s analysis of the delayed-choice interference experiment provides a good first step in explaining why conservation-of-momentum-violating “surreal” trajectories do not pose a problem for Bohm’s theory. While Bohm’s theory does indeed predict that momentum (in the usual sense) is not conserved in experiments like that described above, Bell explained why one would never detect violations of the conservation of momentum. The short story is this: while the *actual* momentum (mass times particle velocity) is typically not conserved in Bohm’s theory, the *measured* momentum (as expressed by the results of what one would ordinarily take to be momentum measurements) is always conserved.

In order to detect a momentum-violating bounce in an experiment like that described above, one would have to perform *two* measurements: one to show which path the particle travels, (A or B) and another to show where the particle ends up (A' or B'). One might then try to show that a particle that travels path A , say, ends up at B' , and thus violates the conservation of momentum. But one will never observe such a bounce in Bohm’s theory *because measuring which path the particle follows will destroy the symmetry in the probability currents that generate the bounce*. That is, particles only exhibit their crazy bouncing behavior in Bohm’s theory when no one is looking!

Suppose (following Bell 1980) that one puts a detector on path B designed to correlate the position of a flag with the position of the test particle P (see figure 2). More specifically, consider a single flag particle F whose position (as represented by the quantum-mechanical state) gets correlated with the position of P as follows: (1) if P is in an eigenstate of traveling path A , then F remains in an eigenstate of pointing at “No” and (2) if P is in an eigenstate of traveling path B , then F ends up in an eigenstate of pointing at “Yes”. That is, the detector is designed so that the position of F will record

the path taken by P .

[Figure 2: Experiment where F 's position is correlated with the position of P]

While this experiment may look like the earlier one, introducing such a detector requires one to tell a very different story than the one told without the detector.

Given the nature of the interaction between P and F and the linearity of the dynamics, if P begins in the z -spin up state (a superposition of x -spin eigenstates), then the effective wave function of the composite system would evolve as follows:

Initial state:

$$|\uparrow_z\rangle_P |S\rangle_P |\text{“No”}\rangle_F = |S\rangle_P |\text{“No”}\rangle_F 1/\sqrt{2}(|\uparrow_x\rangle_P + |\downarrow_x\rangle_P) \quad (7)$$

P 's wave packet splits:

$$|\text{“No”}\rangle_F 1/\sqrt{2}(|\uparrow_x\rangle_P |A\rangle_P + |\downarrow_x\rangle_P |B\rangle_P) \quad (8)$$

M 's position is correlated with the position of P :

$$1/\sqrt{2}(|\uparrow_x\rangle_P |A\rangle_P |\text{“No”}\rangle_F + |\downarrow_x\rangle_P |B\rangle_P |\text{“Yes”}\rangle_F) \quad (9)$$

The two wave packets appear to pass through each other in region I (but they *miss* each other in configuration space!):

$$1/\sqrt{2}(|\uparrow_x\rangle_P |I\rangle_P |\text{“No”}\rangle_F + |\downarrow_x\rangle_P |I\rangle_P |\text{“Yes”}\rangle_F) \quad (10)$$

Final state:

$$1/\sqrt{2}(|\uparrow_x\rangle_P |A'\rangle_P |\text{“No”}\rangle_F + |\downarrow_x\rangle_P |B'\rangle_P |\text{“Yes”}\rangle_F) \quad (11)$$

Note that the position of F does in fact reliably record where P was when the position record was made. Because the wave function associated with the two possible positions for F do not overlap in configuration space, the position correlation between P and F destroys the symmetry that prevents P from crossing L . While the two wave packets both appear to pass through region I at the same time, they in fact miss each other in configuration space. In order to see how P and F move, consider the evolution of the wave function and the two-particle configuration in configuration space.

[Figure 3: The Last Experiment in Configuration Space]

If the two-particle configuration starts in the top half of the initial wave packet (as represented in Figure 3), then P would move from S to A to I to A' and F would stay at “No”. If the configuration starts in the bottom half of the initial wave packet, then P would move from S to B then F would move to “Yes” then P would move from B to I to B' . That is, regardless of where P starts, it will pass through the region I without bouncing. Moreover, F will record that P was on path A if and only if P ends up at A' and that P was on path B if and only if P ends up at B' . That is, if one makes a determinate record of P 's position before P gets to I , then P will follow a perfectly natural trajectory, and the record will be reliable. Again, recording the position of P destroys the symmetry that prevents P from crossing L .

This experiment illustrates why a measurement record is reliable in Bohm's theory whenever there is a strong correlation between the position of the system being observed and the position of the recording system. And since all measurements are ultimately position measurements on the minimal Bohm's theory, one might simply conclude that all determinate records produced by strong correlations are reliable in Bohm's theory and dismiss the surreal-trajectories problem as a problem that was solved by Bell before it was even

posed by ESSW. This is not such a bad conclusion, but the right thing to say about surreal trajectories is slightly more subtle.

Note that in order to tell a story like the one above, one must record the path taken by the test particle in terms of the position of something. Here the record is in terms of the position of the flag particle F . It is this position correlation that breaks the symmetry in the probability currents, which then allows the test particle P to follow a momentum-conserving trajectory. All it takes is a strong position correlation with even a single particle. And it is this that makes the final position record reliable.¹⁰

So what would happen if one tried to record the position of P in terms of some physical property other than position? This is something that is important to our making sense of the history of the surreal trajectories problem, but it is something that Bell did not consider.

¹⁰Bell explained that a good measurement record must make a macroscopic difference. He emphasized that a discharged detector is macroscopically different from an undischarged detector. This is also something emphasized by DFGZ (1993, 1262) in order to argue that one would not expect one of ESSW's detectors to generate a sensible record of which path the test particle followed "until an interaction with a suitable macroscopic device occurs." But note that all that really matters here is that the wave packets that correspond to different measurement outcomes (in terms of the position of F) be well-separated in configuration space in the F -position-record direction. This is not to say that Bell's (and DFGZ's) point concerning macroscopic differences irrelevant. If the flag is a macroscopic system that makes a macroscopic movement, then this will obviously help to provide the wave packet separation required for a reliable record. But while a macroscopic position correlation with a macroscopic system sufficient, it is not a necessary condition for generating a reliable record in Bohm's theory. See Aharonov, Y. and L. Vaidman (1996) for a discussion of partial measurements in Bohm's theory, measurements where the separation between the post-measurement wave packets in configuration space is incomplete.

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In order to avoid Bell's (preemptive) dissolution of the surreal trajectories problem ESSW must have had in mind a different sort of which-path detector than the one considered by Bell. Indeed, the experiments that ESSW describe in their 1992 paper employ detectors that record the path followed by the test particle in the creation of photons. A proponent of Bohm's theory might point out that since the theory is explicitly nonrelativistic and since the very statement of its auxiliary dynamics requires there to be a fixed number of particles, these experiments are simply outside the domain of the theory. But perhaps it is possible to capture at least the spirit of ESSW's experiments with experiments that are well within the domain of the minimal Bohm's theory.¹¹

Consider what happens when one *tries* to record P 's position in something other than position (one might naturally, and quite correctly, object that there is no other quantity in Bohm's theory that one *could* use to record P 's position, but with the aim of trying to revive the surreal trajectories problem, read on). Suppose, for example, that one tries to record P 's position in a particle M 's x -spin: that is, suppose that the interaction between P and M is such that if P 's initial effective wave function were an x -spin up eigenstate, then nothing would happen to M 's effective wave function; but if P 's initial effective wave function were an x -spin down eigenstate, then the spin index of M 's effective wave function would be flipped from x -spin up to x -spin down (since x -spin is a contextual property in the minimal Bohm's theory, the value of the x -spin record depends, as we will see, on how it is read, and one might

¹¹The experiment below shows what would happen if one tried to record the path taken by the particle in the x -spin of another particle. Dewdney, Hardy, and Squires (1993) tried to capture the spirit of ESSW's experiments by showing in graphic detail what would happen if one tried to record the path in terms of energy.

thus, quite correctly, argue that it is not a record of the position of P at all, but read on). In the standard von Neumann-Dirac collapse formulation of quantum mechanics (once a collapse had eliminated one term or the other of the correlated superposition!) one might naturally think of this interaction as recording P 's position in M 's x -spin. On this view, M might be thought of as a sort of which-path detector.

Continuing with the experimental set up, suppose further that M 's x -spin might then be converted to a position record by a detector with a flag particle F designed to point at “No” if M is in the x -spin up state and to point at “Yes” if M is in the x -spin down state. The conversion of the x -spin record (though it will turn out that there is no determinate M -record *until after this conversion is made* in the delay-choice interference experiments!) to a position record here consists in correlating the position of F with the x -spin of M (as represented by the quantum-mechanical state). The idea is that if P is in an eigenstate of traveling path B , say, then M will record this fact by the x -spin index on its wave function being flipped to x -spin down, which is something that might then be converted into a record in terms of the position of F through the interaction between M and F . In this case, F would move to record the measurement result “Yes”.

[Figure 4: One tries to record the position of P in the x -spin of M]

The effective wave function of the composite system then evolves as follows:

Initial state:

$$|\uparrow_z\rangle_P |S\rangle_P |\uparrow_x\rangle_M |\text{“No”}\rangle_F = |S\rangle_P |\uparrow_x\rangle_M |\text{“No”}\rangle_F \frac{1}{\sqrt{2}} (|\uparrow_x\rangle_P + |\downarrow_x\rangle_P) \quad (12)$$

P wave packet is split:

$$|\uparrow_x\rangle_M |\text{“No”}\rangle_F 1/\sqrt{2} (|\uparrow_x\rangle_P |A\rangle_P + |\downarrow_x\rangle_P |B\rangle_P) \quad (13)$$

The x -spin component of M 's wave packet is correlated to the position of P 's:

$$|\text{“No”}\rangle_F 1/\sqrt{2} (|\uparrow_x\rangle_P |A\rangle_P |\uparrow_x\rangle_M + |\downarrow_x\rangle_P |B\rangle_P |\downarrow_x\rangle_M) \quad (14)$$

The two wave packets pass through each other in configuration space:

$$|\text{“No”}\rangle_F 1/\sqrt{2} (|\uparrow_x\rangle_P |I\rangle_P |\uparrow_x\rangle_M + |\downarrow_x\rangle_P |I\rangle_P |\downarrow_x\rangle_M) \quad (15)$$

Then they separate:

$$|\text{“No”}\rangle_F 1/\sqrt{2} (|\uparrow_x\rangle_P |A'\rangle_P |\uparrow_x\rangle_M + |\downarrow_x\rangle_P |B'\rangle_P |\downarrow_x\rangle_M) \quad (16)$$

Then the position of the F is correlated to the x -spin component of M 's wave packet:

$$1/\sqrt{2} (|\uparrow_x\rangle_P |A'\rangle_P |\uparrow_x\rangle_M |\text{“No”}\rangle_F + |\downarrow_x\rangle_P |B'\rangle_P |\downarrow_x\rangle_M |\text{“Yes”}\rangle_F) \quad (17)$$

Note that here the symmetry in the probability current that prevents P from crossing L is preserved. That is, P bounces just as it did in the first experiment we considered.

[Figure 5: Last experiment in configuration space]

If the three-particle configuration begins in the top half of the initial wave packet (as represented in Figure 5), then P will move from S to A to I to B' then, when M and F interact, F will move to “Yes”. If the three-particle configuration begins in the bottom half of the initial wave packet, then P will move from S to B to I to A' and, when M and F interact, F will stay at

“No”. That is, the final position of the F will be at “No” if P traveled along the lower path and it will be at “Yes” if P traveled along the upper path. In other words, F ’s final position does not tell us which path P followed in the way that it was intended.

One might naturally conclude that the which-path detector is fooled by the late measurement, and defend Bohm’s theory against ESSW by denying their implicit assumption that the which-path detectors are reliable.¹² There is, however, another way of looking at a delayed-choice interference experiment where one tries to record the path in some property other than position. One might claim both that Bohm’s theory is true *and* that one’s detectors are perfectly reliable. This, it seems to me, is an option suggested by DFGZ’s discussion of the theory-ladenness of talk of trajectories and of observation itself near the end of their response to ESSW’s original paper.

Given their contention that the experiments described by ESSW could provide no empirical reason for rejecting Bohmian mechanics, DFGZ ask the question “So what on earth is going on here?”

The answer appears to be this: The authors [ESSW] distinguish between the Bohm trajectory for the atom and the *detected* path of the atom. In this regard it would be well to bear in mind that before one can speak coherently about the path of a particle, detected or otherwise, one must have in mind a theoretical framework in terms of which this notion has some meaning. BM provides one such framework, but it should be clear that within this framework the [test particle] can be detected passing only through the slit through which its trajectory in fact passes. More to the point, within a Bohmian framework it is the very existence of trajectories which allows us to assign some meaning to this talk

¹²In their 1993 paper, Dewdney, Hardy, and Squires argue precisely this.

about detection of paths. (1261–2)

It seems that there are two points here. The first point concerns the theory-ladenness of talk about trajectories. On the orthodox formulation of quantum mechanics, there is no matter of fact at all concerning which path the test particle traveled since it simply fails to have any determinate position whatsoever before it is detected. Indeed, insofar as ESSW's description of the surreal trajectories experiments presupposes that there are determinate particle trajectories, they are presupposing something that is *incompatible* with the very quantum orthodoxy they seek to defend! The point here is that any talk of determinate trajectories is talk within a theory. A precondition of such talk is that one have a theory where there are determinate trajectories, a theory like Bohmian mechanics.

DFGZ's second point, if I understand it correctly, concerns the theory-ladenness of observation, but this will first require some clarification. Their claim that in Bohmian mechanics a test particle "can be detected passing only through the slit through which its trajectory in fact passes" suggests that they were considering only experiments like those in the last section where the which-path detector indicates in a perfectly straightforward way the path that the test particle in fact followed. But DFGZ do in fact grant that there are situations where Bohm's theory predicts that a late observation of a which-path detector would find that the detector registers that the test particle traveled one path when it in fact traveled the other. They also grant that this is somewhat surprising. But they explain that "if we have learned anything by now about quantum theory, we should have learned to expect surprises!" And DFGZ maintain that even in such experiments the measurement performed by the which-path detector "can indeed be regarded as a measurement of which path the [test particle] has taken, but one that

conveys information which contradicts what naively would have been expected.” DFGZ then draw the moral that “BM, together with the authors [ESSW] of the paper on which we are commenting, does us the service of making it dramatically clear how very dependent upon theory is any talk of measurement or observation” (1262).

While it is not entirely clear what DFGZ have in mind, one way to read this is that, contrary to what is later argued by Dewdney, Hardy, and Squires (1993), DFGZ take the which-path detector to be perfectly reliable even in experiments where it “records” that the test particle traveled one path when it in fact (according to Bohm’s theory) traveled the other *once one understands what the detector is detecting*. On this reading, then, the point here is that since observation is itself a theory-laden notion, what one is detecting can only be determined in the context of a theory that explains what it is that one is detecting. But if this is what DFGZ had in mind, then what exactly does Bohm’s theory tell us that a which-path detector is detecting in the context of a late-measurement experiment? (And is it really possible to tell a plausible story where the detectors are perfectly reliable here?)

Perhaps the easiest answer would be to insist that when one tries to record the path taken by the test particle in a property other than position (in a delayed-choice interference experiment), one’s which-path detector simply works in exactly the opposite way that one would expect. The detector is perfectly reliable—it is just that when it records that the test particle traveled path *A*, the detector record (under such circumstances) really *means*, according to Bohm’s theory, that the test particle in fact traveled path *B*; and, similarly, on this view a *B* record *means*, according to Bohm’s theory, that the test particle traveled path *A*.

It seems, however, that this cannot be quite right. When one tries to

record the path that the test particle traveled in a property other than position (in the delayed-choice interference experiment), there is no determinate record whatsoever (on the minimal Bohm's theory) before the test particle passes through the interference region I because the which-path detector has not yet correlated the position of anything with the position of the test particle.

The right thing to say, it seems to me, is that while the which-path detector does not detect anything before one correlates the position of the flag F with M 's x -spin (on the minimal Bohm's theory), whenever one makes a determinate record in Bohm's theory using a device that induces a strong correlation between the measured position of the object system and the position that records the outcome, then that record will be perfectly reliable *at the moment the determinate record is made*. On this view, there is still a sense in which one can think of the detectors in the delayed-choice interference experiments as being perfectly reliable, but this will take some explaining.

As DFGZ suggest, we naturally rely on our best physical theories to tell us what it is that our measuring devices in fact measure, so what does Bohm's theory tell us about the *late-measurement* of the which-path detector in the delayed-choice interference experiment? Note, again, that there is no determinate record whatsoever before the late measurement. Also note that while the final position of F does not tell us where P was when P interacted with M , it does reliably tell us where P is at the moment that the x -spin correlation is converted into a determinate measurement record (when the position of F is correlated with the x -spin of M): if one gets the result "No" (x -spin up), then the theory tells us that P is currently associated with the x -spin up wave packet *wherever that wave packet may be*, and if one gets the result "Yes" (x -spin down), then it tells us that P is currently associated with the x -spin down wave packet *wherever that wave packet may be*.

So this is how it works. Since the only determinate noncontextual records in Bohm's theory are records in terms of the position of something, there is, strictly speaking, no determinate record of P 's position until we convert the correlation between the position of P and the x -spin of M into a correlation between the position of P and the position of F . And whenever this position correlation is made, we reliably, and *nonlocally*, generate a record of P 's position *at that moment*. If we wait until after P has passed through region I , then if F stays at "No", this means that P is associated with the x -spin up component which means that it is at position A' , and if F moves to "Yes", this means that P is associated with the x -spin down component which means that it is at position B' . The moral is that one cannot use a record in Bohm's theory to figure out which path P took unless one knows how and when the record was made.

But note that in this Bohm's theory is arguably better off than the standard von Neumann-Dirac collapse formulation of quantum mechanics. On the standard eigenvalue-eigenstate link (where a system determinately has a property if and only if it is in an eigenstate of having the property) one can say *nothing whatsoever* about which trajectory a particle followed since it would typically fail to have *any* determinate position until it was observed. If one does not worry about the unreliability of retrodiction in the context of the standard collapse theory (and ESSW do not seem to be worried about this!), then I can see no reason at all to worry about it in the context of Bohm's theory. Further, there is no reason to suppose that Bohmian particle trajectories are not the actual particle trajectories. Nor is there any reason to conclude that our good particle detectors are somehow unreliable. Rather than saying that a detector is fooled by a late measurement, one should, I suggest, say that the late measurement *reliably* detects the position of the test particle *nonlocally*.

On this view the surreal trajectories experiments simply serve to reveal the special role played by position and, ultimately, the nonlocal structure of Bohm's theory. As Bell explained, "The fact that the guiding wave, in the general case, propagates not in ordinary three space, but in a multi-dimensional configuration space in the origin of the notorious 'nonlocality' of quantum mechanics. It is a merit of the de Broglie-Bohm version to bring this out so explicitly that it cannot be ignored" (1987, 115). But one should note that it is not some subtle sort of nonlocality involved in the account of quantum-mechanical correlations here. The configuration space particle dynamics that accounts for the nonlocal correlations in the late-measurement experiments makes Bohm's theory incompatible with relativity.

But there is one more point that I would like to make before turning to a discussion of the relationship between how Bohm's theory accounts for surreal trajectories and its incompatibility with relativity. As suggested above (on the minimal Bohm's theory) whenever the position of one system is recorded in the position of another system via a strong correlation between the effective wave functions of the two systems (one that produces the appropriate separation of the wave function in the recording parameter in configuration space), then that record will reliably indicate where the measured particle is *at the moment the determinate record is made*. It is also the case (on the minimal Bohm theory) that all determinate records are ultimately position records. One can only take these facts to provide a solution to the surreal trajectories if one allows for Bohm's theory to tell one something about what one is observing when one observes (or, in somewhat different language, what constitutes a good measuring device). But it seems that this is precisely the sort of thing that one must be willing to do when entertaining a new theoretical option. One might dogmatically insist on holding to one's pre-theoretic intuitions concerning what one's detectors detect come what may, but this

would certainly be a methodological mistake.

6

Consider again the late-measurement experiment of the last section (see figures 4 and 5). If P begins in the top half of the wave function at S , it will travel path A to I in the x -spin up wave packet. That is, before P gets to I , the three-particle configuration will be associated with the x -spin up component of the wave function in configuration space. And this means that if one converts the spin record into a position record *before* the two wave packets interfere at I , one will get the result “No”. But if P continues to I , bounces, and the two-particle configuration is picked up by the x -spin down wave packet, then, since the two-particle configuration is now associated with the x -spin down wave packet, if one now converts the x -spin record into a position record, one will get the result “Yes” .

This means that one might instantaneously determine the value of the converted record at B (the record one gets by converting the M x -spin “record” into an F position record) by choosing whether or not to interfere the two wave packets at I . If the two wave packets pass through each other, then F will move to “Yes” when the spin record is converted; if not, then F will stay at “No” when the spin record is converted. So, if someone at I knew which path P was on (something, as explained earlier, that is prohibited in Bohm’s theory if the distribution postulate is satisfied), then he or she could use this information to send a superluminal signal to a friend on path B by deciding whether or not to interfere the wave packets at I . But regardless of whether one knows which path P is on, the theory predicts (insofar as one is comfortable with the relevant counterfactuals in the context of a deterministic theory) that one can instantaneously affect the result of

a measurement of M from region I , and one might take the possibility of superluminal effects here to illustrate the incompatibility of Bohm's theory and relativity.

This incompatibility is more clearly illustrated by considering the role that the temporal order of events plays in Bohm's theory. Consider the late-measurement experiment one more time. If one converts the spin record *before* the two wave packets interfere at I , then one will get the result "No"; and if one converts the spin record *after* the wave packets interfere, then one will get the result "Yes". But if the conversion of the spin record and the interference of the wave packets are space-like separated events, then the conversion event occurs before the interference event in some inertial frames and after the interference event in others. So in order to get any empirical predictions whatsoever out of Bohm's theory for this experiment whenever the conversion and interference events are space-like separated, one must choose a preferred inertial frame that imposes a preferred temporal order on the conversion and interference events. But having to choose a preferred inertial frame here is a direct violation of the basic principles of relativity. This is the sense in which the account that Bohm's theory provides of the late-measurement experiment is fundamentally incompatible with relativity.

If the distribution postulate is satisfied, then Bohm's theory makes the same empirical predictions as the standard von Neumann-Dirac formulation of quantum mechanics (whenever the latter makes unambiguous predictions) and the standard quantum statistics do not allow one to send superluminal messages (given the usual quantum statistics, one can prove a no-signaling theorem). So while Bohm's theory is not Lorentz-covariant, it explains why one would never notice this fact (just as it explains why one would never notice violations in the conservation of momentum).

A proponent of Bohm's theory might argue that nonlocally correlated

motions like the correlated motions in the conversion and interference events describe above is too weak of a relationship to be causal, and that Bohm's theory thus does not in fact allow for nonlocal causation. While such a conclusion would do nothing to make Bohm's theory compatible with relativity even if it were granted, I do not think that it should be granted. It seems to me that if any correlated motions should count as causally connected in Bohm's theory, then nonlocal correlated motions should as well. Nonlocal correlated motions, like local correlated motions (insofar as their are any truly *local* correlated motions!), are simply the result of the configuration space evolution of the physical state. The point here is that Bohm's theory handles nonlocal correlated motions precisely the same way that it handles events that one would presumably want to count as causal—like the correlated motion produced between a football and the foot that kicks it. Of course, one might resist the conclusion that nonlocal correlated motions are causally related by denying that there are any causal relationships whatsoever in Bohm's theory. But this would mean that even those explanations that one gives that look like causal explanations are not, and this seems to me to be putting things the wrong way around. Just as we look to our best theories to tell us how to build good detectors and to explain what it is that they detect, it seems that we should also look to our best theories to tell us something about the nature of causal relations. There is nothing inherently wrong with sitting down and deciding once and for all the necessary and sufficient conditions for events to be causally related. It is just that one risks adopting a notion of causation that is irrelevant to the sort of explanations provided by our best physical theories.¹³

¹³Michael Dickson (1996) has argued that it does not make any sense to ask whether a deterministic theory like Bohm's theory is local because of the difficulty supporting counterfactual conditionals in such a theory. He also suggests that the notion of causality may not make make sense in such a theory either (1996, 329). I agree that some intuitions

But regardless of what one thinks about causality, the particle trajectories predicted by Bohm's theory depend on one's choice of inertial frame, which means that the theory is incompatible with the basic principles of relativity. And this is the real problem.

7

It is the configuration space dynamics that makes Bohm's theory incompatible with relativity. But it is also the instantaneous correlated motion predicted by the configuration space dynamics that explains the quantum-mechanical correlations in Bohm's theory and makes the theory empirically adequate. And it is the configuration space dynamics that allows one to say that whenever the position of one system is recorded in the position of another system via a strong correlation between the effective wave functions of the two systems, then that record will reliably indicate where the measured system is *at the moment the determinate record is made*, which, it seems to me, is ultimately the best response to the supposed surreal trajectory problem.

But this leaves a proponent of Bohm's theory with a difficult choice. One might try to find some new way to account for quantum-mechanical correlations, one that does not require a preferred temporal order for space-like separated events where objects exhibit correlated properties. But it should be clear from the the configuration-space stories told above that such a theory would have to explain quantum-mechanical correlations in a way concerning what it would mean for a theory to be local or what it would mean for one event to cause another cannot be supported in a deterministic theory, but this does not mean that we can make no sense at all of what it would be for a deterministic theory to be local or for one event to cause another. Indeed, whether Bohm's theory is Lorentz covariant is a perfectly good sensible question concerning its locality—and it isn't.

that is fundamentally different from the configuration-space way in which they are explained by Bohm's theory. And, of course, actually finding such an alternative is much easier said than done.¹⁴ Or one might simply drop the requirement of Lorentz covariance as a feature of a satisfactory dynamics and settle for something weaker, perhaps something like *apparent* Lorentz covariance. But this would be an enormous theoretical sacrifice—presumably one that few physicists would seriously entertain.

¹⁴For other discussions of the incompatibility of Bohm's theory and relativity see Albert (1992) and Artzenius (1994). For a recent discussion concerning the difficulty in getting a Bohm-like auxiliary quantum dynamics that is compatible with relativity see Dickson and Clifton (1998).

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