The Non-Equivalence of Einstein and Lorentz

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Abstract

In this paper, I give a counterexample to a claim made in Norton (2008) that empirically equivalent theories can often be regarded as theoretically equivalent by treating one as having surplus structure, thereby overcoming the problem of underdetermination of theory choice. The case I present is that of Lorentz's ether theory and Einstein's theory of special relativity. I argue that Norton's suggestion that surplus structure is present in Lorentz's theory in the form of the ether state of rest is based on a misunderstanding of the role that the ether plays in Lorentz's theory, and that in general, consideration of the conceptual framework in which a theory is embedded is vital to understanding the relationship between different theories.

1 Introduction

The existence of distinct but empirically equivalent theories is seen as a threat to the scientific realist. This is because empirically equivalent theories are often thought to be epistemically equivalent - that is, the evidence supports both theories equally - which has the consequence that the choice between them is underdetermined by the evidence. Many realists have responded to this challenge by arguing that empirical equivalence is not sufficient for epistemic equivalence. In particular, empirically equivalent theories may not be equally confirmed by the evidence. One notable advocate of such a position is Glymour (1977) who argues that spacetime theories which are empirically equivalent can receive different degrees of confirmation - if one spacetime theory has structure over and above the other, then that theory will be less supported by the evidence because it makes additional, unnecessary ontological claims. This avoids the problem facing the realist since if one theory is epistemically preferred to other empirically equivalent theories, then there is no underdetermination of theory choice.

There is another route for the realist. This is to argue that the cases the anti-realist gives as examples of underdetermined theories do not in fact correspond to distinct theories, but to one theory formulated in different ways, and therefore the underdetermination is dissolved. Norton (2008) follows this line, by arguing that instead of thinking, as per Glymour, that the theory with more structure is confirmed to a lesser degree, this extra structure should instead be interpreted as surplus structure. In this paper, I first present Norton's argument for why this allows us to treat empirically equivalent theories as theoretically equivalent. I then argue that despite being a case that Norton uses to support his argument, the case of Lorentz's ether theory and Einstein's theory of special relativity does not fit his framework because of the particular role that the ether plays in Lorentz's theory which means that it cannot be regarded as surplus, and therefore the issue of underdetermination remains. Finally, I look at some consequences of my argument for understanding the change from Lorentz's theory to Einstein's as well as the comparison of theories more generally.

2 Norton's Argument

I take Norton's argument to be the following:

P1. Theories that are empirically equivalent are often structurally similar.

P2. Consider two empirically equivalent theories, $T1$ and $T2$. There are two cases:

a) $T1$ and $T2$ are structurally equivalent.

b) $T1$ and $T2$ are structurally inequivalent, but one (say $T1$) has extra structure over the other ($T2$) which can be regarded as surplus.

P3. Theories are equivalent when they are structurally equivalent.[1]

C. Therefore, in either of the two cases above, the theories can be regarded as formulations of a single theory.

For case (a), the conclusion follows simply from $P3$. For case (b), the conclusion follows because if we regard the extra structure in $T1$ as surplus - that is, it plays no role of the theory of which it is part - and take in to account only the physically significant structure, then we have left structurally equivalent theories.

$P1$ is not particularly controversial since it makes a weak claim and it doesn't seem unreasonable to think that empirically equivalent theories will at least have some structure in common. The identification of theoretical equivalence with structural equivalence given in $P3$ is more controversial. First, in order to compare the structure of two theories, one first has to characterise the structure of a theory. Norton is assuming that there is a natural way to do this. Second,

[1] There is a wide literature on theoretical equivalence and Norton is not clear where he stands in this. In particular, Norton suggests that equivalence is both given by 'intertranslatability' and by structural equivalence. However, these are often not thought of as compatible views - the former is a syntactic relation between theories, using notions such as definitional equivalence, which Glymour (1970, 1977) endorses, while the latter is a relation between the mathematical structures of theories, which has been made more precise using the tools of category theory (see Halvorson (2016); Weatherall (2016a,c, 2017)). Since Norton wants to use comparison of mathematical structure to determine whether theories are equivalent, I take him to be endorsing a form of structural, rather than definitional, equivalence.

there are different ways of comparing the structure of theories and depending which criteria one uses, one might reach different conclusions regarding which theories are equivalent. Finally, one might think that structural equivalence is not a strong enough criterion for theoretical equivalence - one might also want to preserve something over and above structure, such as representational content.[2]

While the task of characterising and comparing the structure of theories in the general case is problematic, for the examples I look at in this paper I take there to be a relatively natural way of doing so. In particular, I will look at theories that are characterised in terms of their spacetime structure, which can be compared by looking at their automorphism groups. Instead, I will argue that the main problem with Norton's argument is that the two cases of empirically equivalent theories he considers, (a) and (b) above, do not cover all cases of empirically equivalent theories. I will present an example of two theories that are empirically equivalent, where there is a clear sense in which one has more structure than the other, but where it is not the case that one can think of the theory with more structure as having surplus structure. This is the case of Lorentz's ether theory and Einstein's theory of special relativity.

However, before I do so, I am going to present an example where Norton's argument does seem to apply neatly. This is the case of Newtonian gravitation - Newton's laws plus Newton's law of gravitation - set in Newtonian and Galilean spacetime respectively. Galilean spacetime consists of the structure $(M, \nabla, t_{ab}, h^{ab})$, a manifold isomorphic to $\mathbb{R}^4$ with a derivative operator, a temporal and spatial metric.[3] This corresponds to a foliation of spacetime into simultaneity hypersurfaces which each have the structure of Euclidean space. Newtonian spacetime consists of $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$, which adds to Galilean spacetime a privileged observer represented by a timelike vector with fixed 4-velocity ξ^a, which we can interpret as what Newton thought of as 'absolute space', giving a state of absolute rest, and therefore a notion of absolute velocity. From the way we have presented these spacetime structures, it is clear that Newtonian spacetime simply consists of Galilean spacetime + X, where X is some structure - namely, the structure corresponding to ξ^a. Therefore, if we regard ξ^a as surplus structure then we can regard the two theories as structurally, and thus according to Norton, theoretically equivalent.

A more rigorous way of seeing that Newtonian spacetime has more structure than Galilean spacetime is by comparing the automorphism groups of both structures. In this, we follow Barrett (2015), who suggests the following structural comparison:

(SYM*) A mathematical object X has more structure than a mathematical object Y if $\mathrm{Aut}(X) \subset \mathrm{Aut}(Y)$.

[2]See Nguyen (2017) and Coffey (2014) for views on theoretical equivalence that aim to go beyond formal accounts.

[3]The operator ∇ is a flat derivative operator on $\mathbb{R}^4$ compatible with t_{ab} and h^{ab}. Both t_{ab} and h^{ab} are smooth, symmetric fields on M, where t_{ab} has signature $(1,0,0,0)$ and h^{ab} has signature $(0,1,1,1)$, and they obey the orthogonality condition $h^{ab}t_{bc} = 0$. For details, see Malament (2012, 4.1).

The symbol $\subset$ is taken to be non-inclusive.

The automorphisms of a spacetime are diffeomorphisms which preserve the structure on the spacetime. So, for example, the automorphisms of Galilean spacetime, $(M, \nabla, t_{ab}, h^{ab})$, are diffeomorphisms which preserve t_{ab}, h^{ab}, and ∇. These turn out, not surprisingly, to be the Galilean transformations, given by spatial and time translations, rotations, reflections, and Galilean velocity boosts.

The automorphisms of Newtonian spacetime, $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$ are then diffeomorphisms which preserve the Galilean spacetime structure, but also preserve ξ^a. This means that the transformations which are automorphisms of Newtonian spacetime will be more restrictive than those of Galilean spacetime. In fact, they correspond to spatial and time translations, rotations and reflections[4], but not Galilean velocity boosts. So we see that it is true that:

Aut(Newt) $\subset$Aut(Gal)

Here, Newt and Gal correspond to Newtonian and Galilean spacetime respectively. Newtonian spacetime therefore has more structure than Galilean spacetime by (SYM*). And this 'more' structure clearly corresponds to that structure ξ^a, picking out a privileged frame of reference. Therefore, if we regard ξ^a as surplus structure, we are in effect equivocating between models in Newtonian spacetime related by a Galilean velocity boost, and return something which is struturally equivalent to Galilean spacetime.

I will now turn to the case of Lorentz and Einstein, and see how it differs to this case.

3 Background to Lorentz's and Einstein's Theory

In order to understand Lorentz's and Einstein's theory, I will consider how Maxwell's equations can be interpreted in different spacetime structures. First, Minkowski spacetime is given by (M, g_{ab}), where M is the manifold $\mathbb{R}^4$ and g_{ab} is a complete, flat and smooth metric with signature (1,3).[5] The metric g_{ab} gives Minkowski spacetime 'lightcone structure'. The automorphisms of Minkowski spacetime are the Lorentz transformations, which include the analagous translations, rotations and reflections to the Galilean case, as well as Lorentz boosts, which are analagous to Galilean boosts, except they do not preserve the spacelike vectors as Lorentz boosts do.[6] In Minkowski spacetime, Maxwell's equations admit the following, observer independent formulation, where the electromagnetic field is given by the smooth, antisymmetric tensor field F_{ab}, and the charge-current density is given by the vector field J^a:[7]

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[4]As long as these rotations and reflections are about ξ^a.

[5]For details, see Malament (2012, 2.1).

[6]Sometimes the term 'Lorentz transformations' is used to refer just to Lorentz boosts, but I will use the phrase to refer to all transformations that are automorphisms of Minkowski spacetime.

[7]The demonstration in this section follows that of Weatherall (2016b) and Malament (2012, 2.6).

$$\nabla_{[a}F_{bc]} = \mathbf{0} \tag{1}$$

$$\nabla_n F^{na} = J^a \tag{2}$$

To understand how the electric and magnetic fields are related to F_{ab} , we must consider the way in which they appear to different observers. In Minkowski spacetime, the electric field relative to a observer O at a point p with four-velocity v^a is given by:

$$E^a = F^a_{\ n} v^n \tag{3}$$

The magnetic field is given by:

$$B^a = \frac{1}{2}\epsilon^{abcd} v_b F_{cd} \tag{4}$$

$\epsilon_{a_1 \dots a_n}$is an orientation tensor, which gives a choice of 'handedness' or right-hand rule. It follows from (3) and (4) that the electric and magnetic fields are always orthogonal to v^a. We also need to define the electric charge density μ, and 3-current density j^a relative to O. These are given by:

$$\mu = J^a v_a \tag{5}$$

$$j_a = h^a_{\ b} J^b \tag{6}$$

h_{ab} is the spatial projection tensor at the point determined by v^a. We can also define F_{ab} in terms of the electric and magnetic fields, as:

$$F_{ab} = 2E_{[a}v_{b]} + \epsilon_{abcd} v^c B^d \tag{7}$$

Relative to a choice of constant timelike vector field v^a ($\nabla_a v^b = 0$), Maxwell's equations can then be given in the following form, where D is the derivative induced on hypersurfaces orthogonal to v^a (see Malament (2012, 1.10)):

$$\begin{gathered} D_b B^b = \mathbf{0} \\ \epsilon^{abc} D_b E_c = -v^b \nabla_b B^a \end{gathered} \tag{8}$$

$$\begin{gathered} D_b E^b = \mu \\ \epsilon^{abc} D_b B_c = v^b \nabla_b E^a + j^a \end{gathered} \tag{9}$$

Equations (8) and (9) correspond to Equations (1) and (2) respectively.

It is important to note that if one considers a different observer in Minkowski spacetime, represented by a different choice of timelike vector, Maxwell's equations in the form of (8) and (9) still hold. This is a consequence of the fact that the equations are invariant under the Lorentz transformations, which are automorphisms of Minkowski spacetime. Einstein's theory of special relativity takes spacetime to have the structure of Minkowski spacetime, and hence this is the interpretation of Maxwell's equations in Einstein's theory.

If we try to interpret these equations in Galilean spacetime $(M, \nabla, t_{ab}, h^{ab})$, problems arise because the spacetime symmetries are given by the Galilean transformations, where Galiean boosts preserve the spacelike vectors and not the timelike vectors, rather than the Lorentz transformations, where Lorentz boosts preserve neither spacelike nor timelike vectors. This means that if we consider a new observer, related to the first by a Galilean boost, Maxwell's equations in the form of (8) and (9) do not hold except for the case of constant electric and magnetic fields, with no sources.[8] Therefore, we cannot successfully interpret Maxwell's equations in Galilean spacetime.

In order to overcome this problem, we can adopt a privileged state of motion in the framework of Galilean spacetime and state that Maxwell's equations hold only in this frame of reference. But we can understand the adoption of a privileged state of motion as corresponding to moving to Newtonian spacetime, $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$. In this setting, ξ^a can be taken to represent the 'ether state of rest'.[9]

We can now understand Maxwell's equations by substituting ξ^a for v^a in equations (8) and (9), and define F_{ab} as:

$$F_{ab} = 2E_{[a}t_{b]} + \epsilon_{abcd}\xi^c B^d \tag{10}$$

Here, $E_a = h^{ab}E_b$. We then recover Maxwell's equations in the following form:

$$\nabla_{[a}F_{bc]} = \mathbf{0} \tag{11}$$

$$(\xi^a\xi^n - h^{an})(\xi^b\xi^c - h^{bc})\nabla_b F_{cn} = J^a \tag{12}$$

We can see explicitly that the ether state of rest, represented by ξ^a, is required to make sense of Maxwell's equations in Newtonian spacetime.

Using the resources of Newtonian spacetime, we can also define an inverse Minkowski metric:

$$g^{ab} = \xi^a\xi^b - h^{ab} \tag{13}$$

In Minkowski spacetime, F_{ab} and F^{ab} are related as follows:

$$F^{ab} = g^{ac}g^{bd}F_{cd} \tag{14}$$

and so equation (12) can be written in Minkowski spacetime as:

$$g^{an}g^{bc}\nabla_b F_{cn} = J^a \tag{15}$$

[8] For proof, see Weatherall (2016b).

[9] Although the ether state of rest plays an analogous role to absolute space in Newtonian Gravitation, it should not be identified with it. This is because the ether need not be said to be 'absolutely' at rest, it merely needs to be *one* state of motion in which Maxwell's equations hold.

This makes explicit the way in which the metric appears the same way in both formulations of Maxwell's equations. This gives the sense in which they 'say the same thing'.

I take Lorentz's theory to correspond to Maxwell's equations interpreted in Newtonian spacetime. Lorentz of course would not have thought of his theory in this way, since he did not have the resources to do so. However, it allows us to see clearly the connection between the introduction of the ether state of rest with the endorsement of the Newtonian picture of space and time.

This gives the basis of both theories. But we need to know more than Maxwell's equations to adjudicate between them. In particular, we need to consider the behaviour of *matter*. In Einstein's theory, the laws governing matter are Lorentz invariant. But Lorentz was born in the Newtonian tradition, where Galilean invariant Newton's Laws govern matter. So the reason we would want to endorse Lorentz's theory over Einstein's is if we characterize inertial frames of reference by the Galilean frames of reference - and this in turn would be if we believe that Newton's Laws hold in all frames.[10] If inertial frames of reference are Galilean one's then, by the argument above, we are forced to adopt Newtonian spacetime. But this theory on it's own - a combination of Newton's Laws and Maxwell's equations with moving observers charcterized by Galilean frames of reference - is inconsistent with experimental results, since it implies that we would be able to detect motion with respect to the ether, which several experiments failed to detect. What Lorentz did was construct a theory that maintained the Newtonian conception of space and time but was consistent with these experiments. He introduced a set of coordinate transformations, under which Maxwell's equations remained invariant. This is called his 'theorem of corresponding states':

> If there is a solution of the source free Maxwell equations in which the real fields E and B are certain functions of x_0 and t_0, the coordinates of S_0 and the real Newtonian time, then there is another solution of the source free Maxwell equations in which the fictitious fields E' and B' are those exact same functions of x' and t', the coordinates of S and the local time in S. - Janssen (1995, ch.3, p.48)

The frame S_0 is the rest frame of the ether, the frame S one moving through the ether. What this theorem says is just that Maxwell's equations are invariant under some coordinate transformations, given by:[11]

$$x' = \gamma(x_0 - vt_0) \qquad y' = y \qquad z' = z \qquad t' = \gamma[t_0 - (v/c^2)x_0]$$

The factor γ is defined as $\sqrt{1 - v^2/c^2}^{-1}$. This is just a mathematical theorem. In order to have any empirical consequences, Lorentz had to add a physical

[10] It might also be if we think that there is a notion of absolute simultaneity, but it seems the reason we would want to hold this is if it is implied by the laws.

[11] Originally, Lorentz included a factor on the right hand side of these equations to stand for a possible transverse deformation, but this was set equal to 1 in 1904 for dynamical reasons. These coordinate transformations are what we recognise now as a Lorentz boost.

assumption to the effect that the fields E' and B' are those fields that are produced in the frame S. This is obtained with what is called the 'generalised contraction hypothesis':

> If a material system, with a charge distribution that generates a particular electromagnetic field configuration in S_0, a frame at rest in the ether, is given the velocity v of a Galilean frame S in uniform motion through the ether, it will rearrange itself so as to produce the configuration of particles with a charge distribution that generates the electromagnetic field configuration in S that is the corresponding state of the original electromagnetic field configuration in S_0. - Janssen (1995, ch.3, p.47)

In other words, corresponding states - states related by the coordinate transformations given in the theorem of corresponding states - physically transform in to one another. It is important to emphasise that this is a physical (dynamical) transformation applied to *Galilean* frames, since Lorentz held on to the Galilean transformations as being the true coordinate transformations between observers. The coordinates x' and t' are auxiliary coordinates that represent the coordinates in which Maxwell's equations hold, not the true spacetime coordinates x and t of the moving frame. From these two posits, Lorentz was able to account for the negative result of any ether drift experiment.

It turns out that what is needed to ensure that the generalised contraction hypothesis holds is that all laws, including those governing matter, are Lorentz invariant. Of course, Lorentz needed to derive that this was true, or at least justify that it was the case in order to have a compelling theory - I refer to Janssen (1995) for such dynamical arguments that Lorentz gave. The important point is that for Lorentz, this fact about the Lorentz invariance of all laws is a coincidence - it is because of the behaviour of matter through the ether that it holds. To ensure full empirical equivalence with Einstein's theory, we also need that the rods and clocks of a moving observer measure the auxiliary coordinates and local time that is made reference to in the theorem of corresponding states. In other words, moving Galilean frames must appear 'as if' they are Lorentzian frames.

In what follows, when I speak of Lorentz's theory I will take it to be that which is empirically equivalent to Einstein's. Lorentz himself did not endorse this theory until after 1905, when Einstein published his theory.[12]And as mentioned before, this way of looking at Lorentz's theory in terms of Newtonian spacetime is an anachronistic way of doing so. However, I think that it captures the essential features of Lorentz's theory, and it is in this form that is particularly philosophically interesting, since we can compare Lorentz's and Einstein's theory directly in terms of their spacetime structure.

[12]Poincaré earlier suggested that observers will register the 'local time' of Lorentz's theory rather than the true Newtonian time, but Lorentz only accepted this later (see Janssen (1995, 3.5.4)).

4 Lorentz-Einstein as Counterexample

Barrett (2015) shows that under his standard of comparison (SYM*), Minkowski spacetime has strictly less structure than Newtonian spacetime. That is, the automorphism group of Newtonian spacetime is a strict subset of those of Minkowski spacetime. We therefore seem to have a case where Norton's argument should apply, and Newtonian spacetime can be interpreted as having surplus structure resulting in the equivalence of Lorentz and Einstein's theory. Norton himself introduces this example, suggesting that the surplus structure in Lorentz's theory is the ether state of rest. This is clear in the following quote:

> The mainstream of physics found Lorentz's view implausible and decided that his ether state of rest was physically superfluous structure, so that the two theories were really just variants of the same theory - Norton (2008, p.36)

However, by comparing the structure of both theories, it is clear that the ether state of rest is not the surplus structure that Norton is looking for. This is because the ether state of rest corresponds to that privileged reference frame ξ^a, which if removed gives back Galilean and not Minkowski spacetime, in which (as shown in Section 3) the laws of electromagnetism can not be formulated. In other words, simply removing the ether state of rest from Lorentz's theory neither results in a sensible theory, nor one that is structurally equivalent to Einstein's theory.

It is not clear what one could remove from Newtonian spacetime in order to recover Minkowski spacetime. What one is doing is removing the elements (t_{ab}, h^{ab}, ξ^a) and replacing them with g_{ab}, which, as we have seen, is a combination of the elements ξ^a and h^{ab}. We can understand it physically as removing the splitting in to separate notions of space and time given by the elements of Newtonian spacetime, and replacing them with some other notion where simultaneity becomes relative.

Another way of seeing this is that Galilean boosts map timelike vectors to other timelike ones while preserving the spacelike vectors, whereas Lorentz boosts map both timelike and spacelike vectors to different timelike and spacelike vectors. So Galilean boosts preserve the simultaneity structure of Newtonian spacetime, while Lorentz boosts do not. Consequently, while both Galilean and Minkowski spacetime are ways of removing the preferred state of rest from Newtonian spacetime and therefore the notion of absolute velocity, Galilean spacetime corresponds to doing just that, whereas Minkowski spacetime corresponds to something different - it removes the preferred state of rest, while also modifying the notion of simultaneity. This results in a completely different spacetime structure with a different interpretation.

So why might Norton have thought that the ether state of rest is surplus structure? One reason is that, like absolute space, it is in principle undetectable in Lorentz's theory. However, it is undetectable for a different reason than absolute space, and this difference corresponds to the reason why absolute space, and not the ether state of rest, can be regarded as surplus structure.

This can be seen by considering the symmetry argument that one can give for why we ought to regard absolute space as being surplus. Given a model of Newtonian Gravitation in which the laws hold, a transformation that adds a constant velocity to all particles gives another model in which the laws also hold and which is observationally equivalent to the original. But under such a velocity boost in Newtonian spacetime, the element which corresponds to absolute space - the distinguished vector ξ^a- is not invariant, and therefore solutions to Newton's laws related by a constant velocity boost correspond to distinct, but observationally indistinguishable, state of affairs. In particular, what is undetectable is precisely the state of rest represented by ξ^a. Therefore, the state of rest plays no role in the empirical consequences of the theory, and ought to be regarded as superfluous. And indeed, we can equivocate between models related by a Galilean velocity boost by removing the state of rest, which takes us to Galilean spacetime.

In Lorentz's theory, the situation is more subtle. In the rest frame of the ether, the laws governing matter are Newton's laws. Since Newton's laws are invariant under the Galilean transformations, this implies that inertial observers are related by Galilean boosts. But under a Galilean velocity boost, we do not get a solution to Maxwell's equations as a function of the Galilean coordinate x and t. So such a transformation does not map solutions to solutions, as it did in the previous example. But we do get an observationally equivalent state of affairs in the following sense - as a result of dynamical effects (the interaction of matter with the ether), Maxwell's equations (and all other laws) hold as a function of x' and t' which are the coordinates that moving observer measures, and so observers are unable to tell whether they are at rest or moving with respect to the ether.[13]

Therefore, although we can conclude that the ether is undetectable, this is not for the same reason as absolute space is in Newtonian gravitation, since Galilean velocity boosts in Lorentz's theory do not take solutions to solutions (as they do in Newtonian gravitation). Importantly, in Lorentz's theory, the ether is actually required in order to ensure that it is undetectable, since it is the interaction with the ether that provides the physical underpinning for the generalized contraction hypothesis, and thus the claim that moving observers can never detect the presence of the ether.[14]

This demonstrates that it is not undetectability on its own that makes something surplus. For some structure to be surplus, it must be that it can be removed to result in an empirically adequate theory. This depends not only on

[13]One might respond that if we consider a Lorentz velocity boost rather than a Galilean velocity boost, then we do get an analogous argument - such a boost maps solutions of Maxwell's equations to solutions and yet the ether state of rest differs. However, it is not clear that we can say that such models are observationally equivalent, since in Lorentz's theory such a transformation does not have physical meaning, other than when coupled to Galilean inertial observers.

[14]Lorentz tried to not make any specific assumptions about the makeup of the ether. One might argue therefore that the ether is not actually important to Lorentz's theory. However, the fact that he doesn't make any assumptions about how the ether is made up doesn't mean he doesn't think the ether is what justifies the dynamical effects.

which objects are undetectable, but what role the objects in a theory play, which depends on the conceptual framework of the theory as a whole. The ether is in principle undetectable in Lorentz's theory but it plays an ineliminable role in the theory, and therefore it cannot be thought of as surplus.

Another way we could have seen that the ether state of rest is not surplus in Lorentz's theory is by noticing that Maxwell's equations, written in Newtonian spacetime, make explicit reference to the structure corresponding to the ether state of rest (Equation 12). This is unlike Newton's laws written in Newtonian spacetime, where the laws only make explicit reference to structure found in Galilean spacetime. For example, Newton's second law in Newtonian spacetime can be expressed as:

$$F^a = mv^n \nabla_n v^a$$

We do not need a notion of spatial length for timelike vectors (which the structure ξ^a gives) to make sense of this equation. Of course, we have shown that we can reinterpret a quantity in Maxwell's equations written in Newtonian spacetime to give Minkowski spacetime, and in this sense Maxwell's equations don't require this bit of structure, but we don't need to reinterpret, or change the form of, anything in Newton's second law to recover Galilean spacetime. In the conceptual framework of Lorentz's theory, the ether state of rest is an essential piece of structure.

I have argued that Norton was wrong to think that the ether state of rest is surplus structure in Lorentz's theory. This does not rule out that one can still think of Lorentz's theory as having surplus structure. One might argue that if we think of Minkowski spacetime as a substructure of Newtonian spacetime, which we can do on the basis that Aut(Newt) $\subset$Aut(Mink), then we can think of Minkowski spacetime as resulting from equivocating between those models related by a Lorentzian velocity boost. The difference between the case of Galilean/Newtonian spacetime and the case of Minkowski/Newtonian spacetime - the fact that in the former, Newtonian spacetime is obtained by simply removing the state of rest, whereas in the latter, Newtonian spacetime is obtained by removing the state of rest but also modifying other structure - might just be due to the way we wrote down the spacetime structures.

The problem with this argument is that although one might be able to think of Newtonian spacetime as Minkowski spacetime + X (i.e. that Newtonian spacetime is structurally equivalent to something that corresponds to Minkowski spacetime + X), this does not say anything about how one interprets the spacetime structure. What Lorentz's theory is, under the characterization given above, is that theory of Maxwell's equations corresponding to a preferred state of rest in the framework of Galilean spacetime.

5 Consequences

5.1 Earman's Principle

Earman (1989, ch.3) argues that there is an important methodological principle that ought to determine what the structure of spacetime is:

> The symmetries of the dynamical laws should be the same as the symmetries of spacetime.

In order to understand this principle, we must know what it means to be a symmetry of the dynamical laws and a symmetry of spacetime. Symmetries of spacetime correspond to the automorphisms of the spacetime structure in question. The dynamical symmetries are not so easy to specify precisely - they correspond to transformations between solutions of the laws of motion. However, it can't be any such transformation, since this would imply that a map between arbitrary solutions counts as a symmetry.[15] It must be a transformation that preserves the physically significant structure, but the problem is: how do we know what the physically significant structure is, other than through the spacetime symmetries? In the context of spacetime theories, some people have argued that there is a relatively natural way of picking out the dynamical symmetries - the idea is to distinguish between the fields that represent spacetime structure from the fields which represent the matter content of spacetime, and then define a dynamical symmetry in terms of diffeomorphisms which act only on the matter fields and which maps solutions to solutions (See Earman (1989) and Pooley (2013)). Whether this will work for all spacetime theories is not clear, but it suffices for the simple cases at hand.[16]

The argument Earman (1989) gives for all dynamical symmetries being spacetime symmetries is that a dynamical symmetry without a spacetime symmetry would indicate some surplus structure in the structure of spacetime. This is because there would be two models of the theory which correspond to distinct spacetime possibilities (in other words, they are non-isomorphic) which nonetheless are observationally equivalent. For example, the theory of Newtonian gravitation in Newtonian spacetime has Galilean velocity boosts as a dynamical symmetry but not a spacetime symmetry. This indicates that Newtonian spacetime has surplus structure.

The argument for all spacetime symmetries being dynamical symmetries is that if the laws are generally covariant, then it follows from a transformation being a spacetime symmetry that it is also a dynamical symmetry. As we saw in Section 3, it is not possible to interpret the generally covariant version of Maxwell's equations in a spacetime structure - Galilean spacetime - which has a spacetime symmetry which is not a dynamical symmetry.

[15] See Belot (2013) for a more detailed argument against such a notion of symmetry.

[16] There have also been attempts to generalise this argument to cover all kinds of theories, for example by Dasgupta (2015) and North (2009). However, it becomes even more difficult to specify the dynamical symmetries in the general case.

Applying this to the case of Lorentz's and Einstein's theory, we find the following. In Einstein's theory, there is a perfect match between spacetime and dynamical symmetries - both are the Lorentz symmetry transformations.

In Lorentz's theory, the spacetime symmetries are the Newtonian ones while the dynamical symmetries are the Lorentz transformations, since the laws in Lorentz's theory are invariant under these transformations. Hence all spacetime symmetries are dynamical symmetries, but the group of dynamical symmetries is larger than the group of spacetime symmetries. If we follow Earman, this would imply that Lorentz's theory has surplus structure. The problem with Earman's argument in this case is that it doesn't take in to account how the dynamical symmetries are interpreted in Lorentz's theory. This is similar to the reasoning that we gave in the last section for why the ether is undetectable but not surplus in Lorentz's theory: Lorentz took moving bodies to be characterised by the Galilean transformations, since Newton's Laws, which were held to govern matter, are Galilean invariant. But since Maxwell's equations do not hold in these frames, a dynamical explanation was needed to explain their observations. The interaction between moving bodies and the ether was meant to provide this explanation. The resulting laws that did hold in these frames of reference were forced (in accordance with negative results from ether drift experiments) to be Lorentz invariant ones.

So while it is true that the laws are invariant under the Lorentz transformations, this is a result of dynamical effects in Galilean frames of reference, and not because Lorentzian frames correspond to the frames of reference for moving observers. Consequently, Lorentz's theory does fail the test of symmetry matching in the sense that the laws are invariant under a wider range of transformations than the spacetime structure. However, this is not due to Lorentz's theory having surplus structure, it is due to the peculiarity of Lorentz's theory that all laws transform in the same way in Galilean frames of reference. In other words, the Lorentz transformations only have meaning in Lorentz's theory when combined with Galilean frames of reference and the dynamical effects of the ether, and therefore one needs to maintain the structure of Newtonian spacetime in order to make sense of this dynamical symmetry in the context of Lorentz's theory.

This example shows how Earman's principle, by making reference only to formalism and not to the interpretation of theories, misses subtleties in comparing theories. While Lorentz's theory does fail the test of symmetry matching, this has to do with the whole conceptual framework on which the theory rests, and therefore one can't say that the problem with Lorentz's theory is simply that the spacetime and dynamical symmetries don't match, and that this means that the theory has surplus structure. If we want to maintain that Einstein's theory is superior because it satisfies Earman's principle, we have to say what it is about the matching of symmetries in this theory that makes it superior. We will look at some arguments in the following section.

5.2 Why did Einstein Supersede Lorentz?

If we want to maintain that Einstein's theory is a distinct, superior theory to Lorentz's, then we have to provide reasons for why Einstein's theory is to be preferred epistemically. The arguments given in this paper imply that it cannot be because Einstein's theory doesn't have surplus structure.[17]

One reason one might give is that Einstein's theory has *less* structure, since Minkowski spacetime has strictly less structure than Newtonian spacetime. Therefore, by Occam's razor, we ought to prefer Einstein's theory. However, there are reasons to be unsatisfied with this argument. Firstly, it doesn't explain *historically* why Einstein's theory was preferred - the comparison of spacetime structure has only been made relatively recently. Additionally, simplicity of spacetime structure is not all that we might care about in a theory - there are other theoretical virtues such as unification and explanation. It is also only one source of simplicity in a theory.

It might be that the simpler structure of Minkowski spacetime means that other theoretical virtues are met, which could be recognised historically as important. Michel Janssen argues that one virtue of Einstein's theory over Lorentz's is explanatory virtue[18] - two observationally equivalent states of affairs which receive very different explanations in Lorentz's theory receive a unifying explanation in Einstein's theory. For example, consider the case of a moving magnet inducing a current in a stationary wire. This receives a different explanation in Lorentz's theory from the case of the same wire moving through the magnetic field of the magnet when stationary, even though the current produced is the same. This is because in one case the magnet is stationary with respect to the ether and in the other the wire is, so they correspond to distinct situations. On the other hand, in Einstein's theory, the two cases are understood as the same situation just looked at from different perspectives.[19] Additionally, in Lorentz's theory the Lorentz invariance of all dynamical laws is in need of explanation on a case by case basis - it is just a coincidence that all matter behaves in the same way when they move through the ether. In Einstein's theory, on the other hand, the Lorentz invariance of the laws is explained by the Minkowski metric - Lorentz invariance is a symmetry of the spacetime, so the fact that the observations are the same in Lorentzian frames of reference can just be thought of as reflecting 'normal spatio-temporal behaviour' (Janssen (1995, ch.4, p.7)). In other words, inertial reference frames naturally become the Lorentzian one's. This argument suggests that the matching of symmetries in Einstein's theory means that the dynamics can now be explained by the spacetime structure,

[17]The question of why Einstein superseded Lorentz was raised in Zahar (1973a,b). He argues that while Special Relativity seems to have greater heuristic power than Lorentz's theory, the crucial reason is that Special Relativity continued into General Relativity, which had novel empirical success. While I do not want to deny that this is a reason for why Einstein's theory was eventually accepted, I am concerned here with the question of why the heuristic of Einstein's theory is superior to Lorentz's.

[18]See Janssen (2002a,b, 2009).

[19]This example is taken from Janssen (2002b), but it was also the example Einstein used to demonstrate the unsatisfactory consequences of Lorentz's theory.

which it couldn't be in Lorentz's theory.

The problem with this argument is that it takes the arrow of explanation to be from spacetime structure to dynamical laws. Some people have argued (see, for example, Brown and Pooley (2006)) that this gets things the wrong way round - dynamics should explain spacetime structure. On this view, it is not the case that Einstein's view is better because it allows the dynamics to be explained in terms of spacetime structure. However, this view also takes Einstein's theory having better explanatory power, since in Lorentz's theory the mismatch between dynamical and spacetime symmetries means that the spacetime structure is not explained by the dynamics.[20] Therefore, we can maintain the explanatory virtue of Einstein's theory over Lorentz's while remaining agnostic as to the arrow of explanation between dynamics and spacetime structure.

There is also an epistemological virtue to Einstein's theory - in Lorentz's theory, we are unable to observe the ether state of rest, whereas in Einstein's theory, we have no analogous undetectable structure. This is not sufficient on it's own since, as argued before, the ether is unobservable but not surplus. However, both the epistemological and explanatory benefits of Einstein's theory give reason to think that Einstein's theory is not only theoretically simpler (because it has less structure), but also *conceptually* simpler.

This is contrary to what Acuna (2014) argues - that these pragmatic virtues are tenuous, and that Earman's defence of symmetry principles in terms of physical and ontological simplicity is a better way of arguing for the superiority of Einstein's theory. However, he takes the violation of Earman's principle to reflect that the ether is superfluous. For example, he says:

> The violation of the symmetry principles in Lorentz's theory results in that the ether - in connection with the privileged ether rest frame and the Newtonian structure of space-time - becomes suspicious of representing nothing physical. The fact that special relativity does not postulate entities or structures that are dubious in this sense makes it a physically and ontologically simpler theory than Lorentz's. That is, the comparative simplicity of Einstein's theory is not a merely pragmatic virtue, but it reflects more solid ontological foundations. - Acuna (2014, p.294)

I have argued that this view is mistaken, and therefore that we need to incorporate pragmatic reasons which look not only to the formal structure of theories, but also how they are interpreted, in order to say why Einstein's theory is preferred.

[20] In fact, on this view a theory such a Lorentz's is not even coherent, since it maintains that the dynamical symmetries underwrite the spacetime symmetries and so there cannot be a mismatch between them. I think that this goes too far and that Lorentz's theory should be treated as a plausible theory, but I will not discuss this further here.

6 Conclusion

In this paper, I have given an example of two empirically equivalent theories - Lorentz's ether theory and Einstein's theory of special relativity - where the former has strictly more structure than the latter, but where one cannot regard this theory as having surplus structure in the sense that it is inessential to that theory and can be excised to give a theory structurally equivalent to the other. The reason for this is that removing the candidate for surplus structure - that corresponding to the ether state of rest - does not give something structurally equivalent to Minkowski spacetime, where Einstein's theory takes place. Therefore, contra Norton, this is not an example of one theory formulated in two different ways, where one formulation simply has surplus structure that the other doesn't have.

This highlighted the following. First, one cannot infer from something being unobservable, or absent in an empirically equivalent theory, to being surplus - one has to recognise the role that object/structure plays in the theory of which it is part. Second, one cannot simply look at the formal structure of theories to tell us what is problematic or which theory is to be preferred - one has to consider how the theories at hand can be interpreted and the conceptual framework in which they are embedded. In the case of Lorentz's and Einstein's theory, ignoring these points can lead to mistakenly thinking that the source of problems in Lorentz's theory is the presence of surplus structure.

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