Measurement Independence, *Parameter Independence* and Non-locality

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Abstract

In a recent paper in this Journal I formulated a conjecture relating *Measurement Independence* and *Parameter Independence*, in the context of common cause explanations of EPR correlations (San Pedro, 2012) . My conjecture suggested that a violation of *Measurement Independence* would entail a violation of *Parameter Independence* as well. Leszek Wroński (2014) has shown that conjecture to be false. In this note, I review Wroński's arguments and agree with him on the fate of the conjecture. I argue that what is interesting about the conjecture, however, is not whether it is true or false in itself, but the reasons for the actual verdict, and their implications regarding locality.

1 A conjecture on *Measurement Independence* and *Parameter Independence*

In one part of a recent paper in this Journal I presented and argued for a conjecture relating *Measurement Independence* and *Parameter Independence*, in the context of common cause explanations of EPR correlations (San Pedro, 2012).

Measurement Independence (MI) expresses the fact that measurement operations m_i in either wing of an EPR experiment are (statistically) independent of the common cause C postulated to explain the corresponding correlated outcomes, i.e. $p(m_i|C) = p(m_i)$.

Parameter Independence (PI), on the other hand, is usually imposed on postulated common causes of EPR correlations as a *locality* condition, expressing the requirement that measurement operations m_i in either wing must not influence whatever outcome O_j is to be obtained in the other wing of the experiment. In particular, for left wing outcomes PI demands that $p(O_L|m_L, m_R, C) = p(O_L|m_L, C)$ be satisfied (and symmetrically, $p(O_R|m_L, m_R, C) = p(O_R|m_R, C)$ for right wing outcomes).

My conjecture, then, was that whenever MI is violated PI does not hold either:

Conjecture 1 *If* Measurement Independence *is violated then* Parameter Independence *is also violated.*

This conjecture is false, as shown by Leszek Wroński (2014). Wroński's argument relies on what he calls the 'subset condition' (SC), which was implicitly assumed in my original discussion in defence of the conjecture (more about SC in a moment). In short, Wroński's claim is as follows: if SC applies then PI is trivially satisfied, so Conjecture 1 above is obviously false; and if SC is removed then infinitely many counterexamples to the conjecture can be constructed, so it is false in this case as well.

In this note I shall make two main points. The first is in relation to Wroński's arguments against the conjecture, and in particular with regards to his method of producing (infinitely many) counterexamples to it. Specifically, Wroński's algorithm to construct infinitely many counterexamples to the conjecture provides no more than a confirmation of something we already knew —but which did go unnoticed in my original paper: examples of common cause models of EPR correlations in which MI is violated but PI hold are not new. Huw Price (1994) retrocausal model for instance, which I briefly discussed in San Pedro (2012), is one such example.

My second point is more general and it has to do with what I think makes conjectures and other related claims —e.g. theorems propositions, etc.— interesting or particularly useful. The point is that these types of claims are interesting, beyond their formal particularities, mostly because of the conceptual interpretations they allow, and the implications that follow from them. This is indeed true for the case at hand. I shall thus argue that what is interesting about Conjecture 1 is not whether it is actually true or false but the implications of whichever verdict we might reach. I already discussed in San Pedro (2012) what the implications would be had Conjecture 1 turned out to be true. So now my original claims need to be reconsidered in the light of the conjecture being false, and new consequences identified. They are mostly related to locality issues.

Before proceeding to my comments on the above let me note that Conjecture 1 was formulated in the context of a common cause model I developed in the same paper (San Pedro, 2012, pp. 145-51). The conjecture was largely motivated by some of the model's features, and in particular by the fact that the common causes postulated in it displayed an explicit non-local character due to the violation of MI. There is no need to go through all the details of the model here but, since I will refer to it in the discussion below, it will be useful to point out some of its key features (see San Pedro (2012) for a detailed account):

(a) The model postulates a common cause C that is explicitly dependent on the EPR measurement operations m_i.[1] (Such explicit dependence may be interpreted causally under the right circumstances.)

[1] Note the change in the notation: while C denotes a 'generic' common cause event, C stands for a particular kind of common cause, as defined in the model outlined here. See San Pedro (2012) for full details.

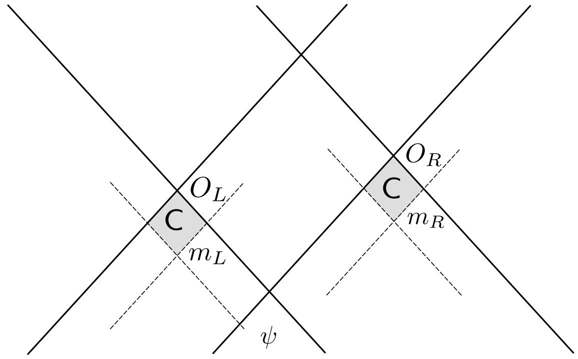

Figure 1: Space-time diagram for the model's postulated common cause events. Common causes are 'located' somewhere in the shaded areas, and display non-local causal influences that cause the corresponding EPR outcomes (see San Pedro (2012) for further details).

(b) The model's common causes C are postulated as taking place *after* the measurement operations.

(c) Because of (a) and (b) the (algebraic) structure of common cause events C is proposed to be a combination (in the sense of 'joint event') of the measurement events in both wings of the EPR experiment m_L and m_R, and a further hidden common causal factor Λ, possibly related to the source (i.e. the singlet state ψ), that is: $\mathsf{C} \subset m_L \wedge m_R \wedge \Lambda$.

(d) Together, (a), (b) and (c) entail that MI is violated in the model ...

(e) ... and also that the common causes defined in this way explicitly display non-local behaviour (see Figure 1 for the space-time structure of the model's common cause events).

Finally, I would like to stress that Conjecture 1 was proposed in the context of this model, but it is not a consequence of it. That is, the conjecture does not need to be true for the model to make sense. In other words, the model does not hinge or depend on the truth or falsity of the conjecture. In fact, as I shall argue in a moment, the model can even give us some clues as to how to understand the significance of the falsity of the conjecture.

2 The 'subset condition'

Because of the event structure described in (c) above, my common cause model in San Pedro (2012) satisfied what Wroński calls the 'subset condition' (SC), namely $\mathsf{C} \subset m_L \wedge m_R$ (Wroński, 2014, p. ????). I should note, however, that my discussion of Conjecture 1 in the last section of San Pedro (2012) assumes SC only implicitly. By making it explicit

Wroński (2014) has made clear its formal significance which, in fact, is at the core of his claims against the conjecture.

Wroński's argument distinguishes the case in which SC is required from the case in which SC is eliminated. I shall briefly discuss these two scenarios in turn.

(A). *Conjecture 1 without SC.* If SC is not required, counterexamples that render Conjecture 1 false can be shown to exist. Wroński not only provides specific counterexamples to the conjecture but a method to construct infinitely many of them (Wroński, 2014, p. ????).

As for the actual significance of this result —i.e. of an algorithm that produces infinitely many counterexamples to Conjecture 1— let me point out that we already have examples of common cause models that would satisfy PI but violate MI. In fact, I discussed one such model in San Pedro (2012), namely Huw Price's retrocausal common cause model (Price, 1994). Once more, I did not make any explicit reference to SC when discussing Price's model. But once SC is made explicit we can see that, as it happens, Price does not assume it in his model (not even implicitly). Thus, Price's common cause model constitutes a counterexample to Conjecture 1 of the same type as those provided by Wroński (2014).

In short, counterexamples to the conjecture if SC is not assumed are not new: and Wroński's algorithm to produce them (en masse) in a methodical manner is just a confirmation of this.

Let me stress, once more, that Conjecture 1 was originally formulated in the context of a common cause model which assumed, albeit implicitly, SC. Thus, strictly speaking, Conjecture 1 should be immune to arguments based on the existence of counterexamples such as Price's model, or those generated by Wroński's algorithm. This is a minor point nevertheless, for as Wroński (2014) has also shown the conjecture is false under the assumption of SC as well.

(B). *Conjecture 1 under the assumption of SC.* One may think that the case in which SC is required to hold is, in a sense, quite straightforward. For, as Wroński notes, it is rather trivial to see that requiring SC entails that PI always holds, no matter what: i.e. whether MI also holds or not (see Wroński (2014, p. ????) for the technical details). Thus, requiring SC makes Conjecture 1 trivially false, and of little interest.

Again, Wroński is completely right as far as the formal part of the question goes but, in my view, what is most interesting about Conjecture 1 is not whether it turns out to be true or false, but the consequences of the actual verdict. Indeed, this is basically why I take conjectures, propositions, theorems and the like to be particularly interesting in general. They are interesting not only because they may be proved, disproved, etc. —or because they turn out to be true or false in the case of a conjecture— but also and mostly because of the actual conceptual interpretations they lead us to and/or their implications.

In the case at hand, the consequences (of the falsity) of Conjecture 1 are mainly related to locality issues. In San Pedro (2012) I discussed what I took Conjecture 1 to mean if proved true. My main point there was that "did Conjecture 1 turn out to be true, it would seem to provide the grounds to claim that the model is non-local [...] since *parameter independence* is necessary for Bell's *factorizability* then, by Conjecture 1, a violation of *measurement independence* would also entail that *factorizability* is violated. This is generally taken to be a sign of non-local behaviour." (San Pedro, 2012, p. 153).

Now the falsity of Conjecture 1 forces a revision of such claims —as well as the intuitions behind them. Still, its consequences can be of use when addressing locality issues in the EPR experiment from a different perspective, i.e. from the perspective of the violation of MI.

3 *Measurement Independence* and locality revisited

Recall that I formulated Conjecture 1 in the context of a specific common cause model and what is more, it was motivated to a good extent by some of the model's features (see properties (a)–(e) above). In particular, Conjecture 1 seemed reasonable —and tenable as well— in the light of the explicit non-locality the model displayed as a consequence of the violation of MI (see Figure 1). The intuition behind the conjecture was in fact that non-locality, even if originating from a violation of MI, should in some way or other show up or have some consequence for what are usually taken as standard locality conditions in descriptions of EPR phenomena, such as PI. In other words, Conjecture 1, it seemed to me, nicely captured the fact that the model displayed non-local causal interactions as a consequence of a violation of MI. It being false therefore suggests that such intuitions are in need of revision. This is not the place for a full discussion of the issue and I shall only briefly go through the most immediate implications of the falsity of Conjecture 1.

We should first note that the aforementioned model satisfies PI —it does so in virtue of assuming SC. We must conclude, therefore, that, as long as we stick to the usual interpretation of PI as a locality assumption, the model must still retain *some* notion of locality. Recall on the other hand that the model's postulated common causes C violate MI, which gives rise to explicit non-local behaviour. In particular, the model contemplates the possibility of non-local causal influences both from measurement operations m_i on the postulated common causes C, as well as from common causes, C, on the corresponding EPR outcomes O_i (see Figure 1 and San Pedro (2012) for details). The model therefore turns out to be non-local, even if PI, i.e. a standard locality assumption, holds in it.

So Conjecture 1 being false, rather than associating non-locality with a failure of PI, suggests that common cause models such as the one I put forward in San Pedro (2012), which violate MI but satisfy SC, do indeed display some interesting kind of non-locality —which is in no way related to PI. This in turn suggests either that PI is not an adequate locality assumption, or that it does not capture the whole story as regards locality in EPR.

This is also related to a question I left open at the end of my original discussion of Conjecture 1: whether there is a fundamental difference between the non-locality that violations of MI seem to convey and that associated with a failure of PI. In the light of the conjecture being false, we can now say that the answer to this question is 'yes'.

The issue may be more subtle than it seems, however. For instance, Hofer-Szabó et al. (2013) have recently approached it in a more detailed and formally rigorous manner. Very roughly, they distinguish two different ideas of locality in an EPR scenario. The first, closely related to PI, involves the (unknown) postulated common cause C and is thus referred to as 'hidden locality', to emphasize its metaphysical origin. A second kind of locality, which the authors call 'surface locality', is represented by a statistical condition involving only the empirical facts of the EPR experiment, i.e. involving only measurement and outcome variables. The point is basically that these two locality conditions are not logically independent. In particular, Hofer-Szabó et al. show that 'hidden locality' in conjunction with a stronger version of MI —'strong no-conspiracy'— entails 'surface locality' (see Hofer-Szabó et al. (2013, p. 160) for details). This suggests, therefore, that PI (or 'hidden locality' for that matter) and MI ('strong no-conspiracy') both have an impact on an 'empirical notion' of locality in EPR, such as 'surface locality'. Put another way, the notion of locality encoded in PI falls conceptually short when it comes to empirically testable locality facts in EPR.

In sum, failure of the conjecture to hold tells us that the notion of non-locality that the model in San Pedro (2012) involves —as a consequence of a violation of MI— is fundamentally different from that resulting from a violation of PI. That is to say, the requirement of MI in common cause explanations of EPR correlations seems to encode some intuitions about locality as well. This observation opens the door for an alternative interpretation to the usual reading of MI in terms of conspiracy. Furthermore, it also suggests a different approach to locality issues in the EPR experiment —and by extension in quantum mechanics— other than those related to violations of PI.

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