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Tracing pre-service teachers' understanding of specialized content knowledge for sequencing and structuring mathematics learning in low-resource Malawi classrooms

Everton Jacinto, Arne Jakobsen, Raymond Bjuland
University of Stavanger

Abstract

Teaching quality is especially critical in under-resourced Malawian classrooms, yet little is understood about how pre-service teachers build the specific knowledge and skills required for effective mathematics teaching. This study explores how two first-year Malawian pre-service teachers approach sequencing and structuring mathematics learning—crucial aspects of Specialized Content Knowledge (SCK). Through thematic analysis of data from a teacher education college and primary schools, we found that pre-service teachers often rely on their training and low-cost instructional resources, which may limit responsiveness to students' needs and the ability to foster conceptual links between mathematical topics. However, as their training progresses, they demonstrate greater awareness of the complexities involved in effective mathematics instruction and show improved adaptability in sequencing and structuring lessons. These findings emphasize the importance of reflective, practice-based teacher education and highlight the need for further research on SCK development in low-resource contexts.

Keywords: Malawian primary education; mathematics teacher education; specialized content knowledge.

Introduction

Although substantial progress has been made globally in expanding access to primary education, ensuring that all children receive high-quality instruction remains a significant concern, particularly in contexts with limited resources. In the Republic of Malawi, where classrooms may host up to 80 pupils per qualified teacher, one of the highest ratios in sub-Saharan Africa (UNICEF, 2019), curricular reforms have been introduced to better prepare both in-service and pre-service teachers to meet the complex demands of their profession

(Kazima, 2014). These reforms aim to ensure that teacher candidates acquire the pedagogical knowledge and practical skills necessary to deliver the national curriculum effectively, even in overcrowded and under-resourced schools (Malawian Institute of Education, 2017). Such initiatives reflect a growing global recognition that expanding access to education must be matched by investment in teaching quality (UNESCO, 2016).

To yield meaningful results, it is essential to clearly define the knowledge required for effective teaching. Although this topic has been extensively researched in mathematics education (Ball et al., 2008; Ball, 2017; Rowland et al., 2005), literature examining how pre-service teachers acquire and develop an understanding of such knowledge, particularly in low-resource contexts, remains limited. A recent study by Jakobsen et al. (2018), conducted across all teacher education colleges in Malawi, indicated that while pre-service teachers' mathematical knowledge for teaching increases as they progress through their training, their overall growth remains limited and varies considerably between colleges. Consequently, further investigation is required to explore how this knowledge can be cultivated more effectively and enhance the quality of mathematics education (Mwadzaangati, 2019).

In response to this call, this study examines Malawian pre-service teachers' understanding of the knowledge—particularly SCK—required to effectively sequence and structure mathematics learning in primary schools. This focus reflects the priorities outlined in the Malawian teacher education curriculum, which positions sequencing and structuring as central components of mathematics instruction (Malawian Institute of Education, 2017). Moreover, this knowledge aligns with the broader aims of teacher education programmes, as our previous research has shown that it provides valuable insights into the knowledge that teachers draw upon to support mathematics learning (Jacinto & Jakobsen, 2020).

Mathematical Knowledge for Teaching

One of the current debates among researchers, educators, and educational stakeholders concerns the scope and nature of the knowledge required for teaching (Grossman, 1990; Shulman, 1986; Tardif et al., 2002). Although various frameworks have been proposed, *Mathematical Knowledge in Teaching* (Rowland et al., 2005), *Mathematics for Teaching* (Davis & Simmt, 2006), and *Mathematical Knowledge for Teaching* (Ball et al., 2008) are among the most widely accepted in mathematics education. In developing the Mathematical Knowledge for Teaching (MKT) framework, Ball et al. (2008) focused on the mathematical knowledge necessary to perform the core tasks of teaching. They proposed six domains: three extending Shulman's (1986) concept of pedagogical content knowledge—Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC)—and three related to subject matter knowledge—Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and Specialized Content Knowledge (SCK). These domains have guided teacher education programmes in preparing teachers to navigate the complex demands of classroom instruction (Ball, 2000). However, much remains to be done to clarify and support the development of these domains, particularly for teaching diverse learners in varied settings.

This study contributes to the literature on MKT by examining the role of SCK and engagement with teaching tasks in the development of pre-service teachers' instructional skills (Ball et al., 2008; Ball, 2017; Jakobsen et al., 2018; Jacinto & Jakobsen, 2020). SCK is defined as the knowledge required to make specific mathematical content visible and accessible to students (Ball et al., 2008) and is considered one of the most demanding components of subject matter knowledge. It involves more than content mastery; it requires the ability to unpack concepts in pedagogically appropriate ways (Ding, 2016). For example, when teaching division, teachers must understand not only procedural variations but also how to distinguish and apply conceptual models such as "take-away" and "comparison."

Despite its widespread recognition, Ball et al. (2008) offered limited guidance on how SCK can be developed. Acquiring SCK is especially challenging in teacher education, as pre-service teachers often lack sustained opportunities to work directly with students and tasks that elicit mathematical thinking (Jacinto & Jakobsen, 2020). Practical engagement with teaching tasks, such as linking representations, modifying textbook content, or adjusting task difficulty, is essential for developing this knowledge (Ball et al., 2008; Ding & Heffernan, 2018). As such, expanding the components of SCK within teacher education programmes is necessary (Hine, 2015; Lai & Clark, 2018), particularly to identify core teaching practices that foster mathematical understanding across various educational settings (Jacinto, 2020).

The knowledge needed for sequencing and structuring mathematics learning: a specific aspect of SCK

A key aspect of SCK is the teacher's ability to choose appropriate sequences of activities to support students in mastering mathematical content (Sazama, 2017). Researchers have examined the value of this ability in the context of pre-service teachers' learning to teach. For instance, Simon (2016) found that designing mathematical task sequences contributes to the development of new abstractions and supports reflection and conceptual learning in the classroom. In a related study, Meikle (2014) explored how pre-service teachers selected and sequenced solution strategies to enhance students' mathematical understanding. Although the participants used similar strategies, their underlying rationales varied. Likewise, Galant (2013) showed that selecting and sequencing mathematical activities helped pre-service teachers better understand students' difficulties and misconceptions and how students responded to the tasks.

However, organizing teaching tasks meaningfully is often challenging for pre-service teachers, who tend to see a mathematics lesson as a chronological but disconnected set of steps (Star & Strickland, 2008). Many also rely on their own teaching experiences to select and sequence classroom activities and often consider students' misconceptions when planning lessons (e.g., Charalambos, 2016). These findings have led researchers such as Abas (2016), Kang (2016), and Mallart et al. (2018) to conclude that pre-service teachers frequently struggle to connect core teaching tasks with educational standards and students' learning requirements.

This study provides insights into how pre-service teachers learn and use the knowledge required to sequence and structure mathematics learning in primary schools. The goal is not only to enhance our understanding of how SCK is interpreted and applied by pre-service teachers, but also to shed light on how this domain can inform teacher education practices, particularly in settings where curricula are still being developed, such as in Malawi.

Research Design and Participants

The study is part of a doctoral research project involving six first-year pre-service teachers enrolled in a teacher education programme in Malawi (see Jacinto, 2020). All the participants volunteered to participate in the study project. Data collection began at the start of their training, which was referred to as the initial moment (IM). During this phase, the participants completed a questionnaire that explored their preferences for teaching mathematics and the types of knowledge they believed were necessary for teaching mathematics in primary schools. Based on their responses, individual audio-recorded interviews were conducted, in which the pre-service teachers were asked to elaborate on their answers and provide concrete examples of how they would apply this knowledge in the classroom.

The second moment (SM) of data collection occurred during the participants' six-month teaching practicum in primary schools. This phase involved video-recorded classroom observations and post-lesson interviews. The observations focused on identifying instances where the pre-service teachers applied the knowledge they had previously described as important. Following each lesson, post-lesson interviews were conducted to encourage reflection and discussion about the specific teaching tasks and how their knowledge was used in practice.

This article presents data from two first-year pre-service teachers, referred to by the pseudonyms Denise and Martin, who participated in both the IM and SM. Both were enrolled in the same teacher education programme and completed their supervised teaching practice at two different primary schools located in remote areas of Malawi. These participants were purposefully selected for in-depth analysis based on their shared interest in teaching mathematics after graduation and the distinct ways in which they exhibited an understanding of the knowledge that mathematics teachers need to effectively sequence and structure students' learning. Their cases offer insights into how variations in the development of SCK can emerge even among novice teachers in similar training environments but working in different school contexts.

To explore the reasons for these differences, we conducted a qualitative thematic analysis across the cases. This method allowed us to "explore the understanding of an issue or the signification of an idea" (Attride-Stirling, 2001, p. 387). Following Braun and Clarke (2012), we used an inductive coding approach, allowing themes to emerge from the data rather than being guided by pre-existing categories. The coding and analysis processes were developed collaboratively, ensuring shared interpretations and deeper engagement with the data. Thematic analysis helped us categorize surface-level data to uncover meaning patterns and represent participants' experiences within their actual contexts. Through this process, we generated two central themes reflecting pre-service teachers' understanding of the knowledge needed to sequence and structure students' learning of mathematics: (1) Knowledge needed to sequence and structure students' mathematics learning; and (2) New forms of understanding upon reflection on the teaching tasks.

Findings and Discussion

This section is organised into two subsections, each addressing one of the two themes identified above. As will be illustrated, Denise and Martin initially described the knowledge required to sequence and structure students' learning of mathematics as a general concept derived primarily from school textbooks and teachers' guides. They emphasized the importance of beginning with simple and easy-to-understand activities and gradually progressing to more complex and dynamic tasks. However, through reflection on practical situations—where they were required to consider students' thinking and capabilities—this understanding began to evolve, becoming increasingly nuanced and context-sensitive.

The knowledge needed to sequence and structure students' learning

During their initial interviews, Denise and Martin provided complementary descriptions of the knowledge needed to sequence and structure students' mathematics learning, emphasizing the teacher's ability to organise and introduce new concepts gradually, from simple to complex.

The case of Denise

When asked about the knowledge and skills a teacher needs to effectively organise and conduct a mathematics lesson, Denise emphasized the importance of clear and accessible instruction. She stated, "The ability to make learners understand the content is one of the main characteristics of effective teaching in mathematics." She elaborated, "Sometimes mathematics can be very difficult for learners, so a good teacher needs to know mathematics very well to show learners that it is not [difficult]!"

Denise also highlighted the value of textbooks in lesson planning: "The textbook is very important for the teacher because it already shows you how the lesson should be conducted. […] In Malawi, the teacher usually starts revising what the students have learnt from a previous lesson and connects to it." She further explained, "I think this is a very good strategy for teaching mathematics. The teacher needs to teach from simple to complex, so everyone [students] can learn and use it [this knowledge]!" When asked to elaborate, Denise added, "Teachers can help them [the learners] by giving another view, so they can learn mathematics step-by-step."

In articulating these views, Denise demonstrated an emerging awareness of the importance of making connections across different aspects of the curriculum. In her view, teachers need a thorough understanding of the mathematics textbook for each grade level, as well as the ability to apply strategies that make mathematical content more accessible to students. She also emphasized that when planning and delivering lessons, teachers should consider how the content of previous lessons links to the goals and materials of the current instruction.

These findings align with other research, suggesting that at the beginning of teacher education, pre-service teachers' approaches to lesson organisation tend closely to follow curriculum guidelines (Charalambos, 2016; Sjöberg, 2018). Similarly, Çelik and Güzel (2017) found that pre-service teachers frequently associated knowledge of students' thinking with an awareness of what students had learned in prior lessons. For Denise, the progression outlined in school textbooks served as her primary reference for designing sequences of mathematics activities, a view also observed among participants in Simon's (2016) study.

The case of Martin

In response to a similar question to the one posed to Denise, Martin offered a complementary perspective, emphasizing the importance of a progressive sequence of activities aligned with the curriculum. He argued that "good teachers should know how to design a lesson logically, from simple to complex, as is done in the textbook." Martin expanded on this point by referencing both student learning and curriculum structure: "When you think about mathematics, teaching is about helping learners to progress from known to unknown, from simple to complex," he explained. He also referred to the guidance provided by official documents: "The [primary school] curriculum already identifies the simple concepts that they need to learn to understand the more complex ones, so the teacher does not need to do much about it."

Martin's ideas became clearer during practical reflection. When asked what teachers need to know in order to organise their mathematics instruction effectively, he responded as follows:

> If the teacher has recently graduated, he or she will probably use the textbook. But an experienced teacher will probably know the content very well and will have lots of strategies to teach it. Students appreciate when teachers use different methods and resources, like stones and sticks, to teach multiplication. The textbook suggests such approaches, but teachers can, of course, use other ways. (Martin, IM—Interview)

As shown in the excerpt above, these pre-service teachers' views on sequencing and structuring students' learning are linked to their knowledge of various teaching strategies. When discussing the organisation of teaching, both Martin and Denise argued that to carry out mathematics teaching tasks, teachers should know the mathematical curricular content, which progresses from simple to complex, and be able to connect concrete to abstract concepts. Previous studies have shown that understanding mathematical content and curriculum is a powerful resource for teachers (Hill et al., 2005) and that in the early stages of teacher education, pre-service teachers often rely on the curriculum and textbooks when planning lessons (König et al., 2020). This tendency was also evident in Martin's case study. However, Martin also acknowledged the value of using instructional strategies beyond those provided in standardised curricula and textbooks.

Both Martin and Denise viewed sequencing and structuring students' mathematical learning through the lens of curriculum progression. For them, this involved a simple-to-complex approach, allowing students to build their understanding progressively. However, while Denise relied solely on textbook content for lesson design, Martin appeared more flexible, seeing curricular guidelines as a foundation for exploring varied instructional strategies. In summary, at the beginning of their teacher education, Denise and Martin held complementary views on the knowledge and skills needed to teach mathematics in primary schools.

Viewed through Ball et al.'s (2008) MKT framework, both these pre-service teachers' responses reflect aspects of SCK, such as "recognising what is involved in using a particular representation," "connecting a topic being taught to topics from previous and future lessons," and "appraising and adapting the mathematical content of textbooks" (Ball et al., 2008, p. 400). However, their shared preference for a "simple-to-complex" teaching approach reflects a linear view of mathematics learning that may overlook the need for instructional adaptation based on students' specific needs and learning trajectories.

New forms of understanding the knowledge needed to sequence and structure students' learning upon reflections on teaching tasks

Lesson observation during the practicum provided insight into how Denise and Martin's views on sequencing and structuring students' mathematics learning—initially anchored in a "simple-to-complex" framework—were enacted and refined through lesson planning and teaching. To elaborate on our findings, we refer to one of Denise's lessons on multiplication for Standard 2 (Grade 2) students and one of Martin's lessons on the geometric representation of fractions for Standard 4 (Grade 4) students as illustrative cases. Selected teaching episodes from each lesson were discussed in post-lesson interviews to probe further into the types of knowledge that these pre-service teachers identified as essential for sequencing mathematical activities during initial teacher education (IM).

The episode selected for analysis in Denise's case occurred during her instruction on mathematical concepts and procedures, specifically, multiplication by three. In contrast, the episode from Martin's lesson took place as he responded to students' approaches to solving a task involving the geometric representation of fractions. These instructional moments—presented as excerpts—served as valuable analytical sources, as they prompted pre-service teachers to reflect on how to better support students' mathematical learning.

Data from both episodes were analysed in relation to the features of SCK outlined by Ball et al. (2008), particularly those relevant to explaining mathematical concepts and procedures and interpreting students' thinking and problem-solving strategies (Hill et al., 2004).

Connecting content elements to explain multiplication concepts and procedures. Denise began the lesson by organizing students into groups and asking each group to solve the expression "2 × 3 =", which she wrote on the blackboard. A student from each group then came to the front and wrote the answer "6" after the equal sign. To illustrate the concept visually, Denise drew two sets of three vertical lines (lll lll) on the board and explained that this representation corresponded to the expression 2 × 3. She then presented a similar task—4 × 3—and represented it with four sets of three vertical lines (lll lll lll lll), as shown in Figure 1. Through this approach, Denise aimed to connect symbolic notation with visual groupings to support students' understanding of multiplication.

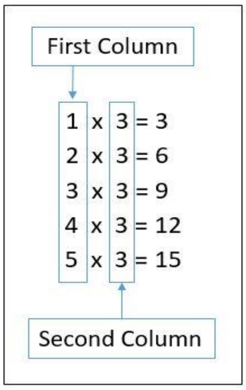
FIGURE 1: Representations used by Denise to explain multiplication by three.

Denise's approach to representing multiplication reflects an interpretation based on the order of operations. Her strategy suggests that the position of each number in the expression determines how the combination should be interpreted and represented. In the excerpt below, taken from the post-lesson interview, Denise explains how she conceptualised and implemented this model of teaching multiplication.

EXCERPT I: Order of operations in multiplication.

Line Interlocutor Speech

1	Researcher:	Okay, here you are teaching them how to multiply five by three, but it seems that you are also teaching multiplication of three by five. Is that right?
2	Denise:	Yes! But multiplication of three by five is at an advanced level! In Standard 2, we start with small numbers first.
3	Researcher:	Okay, but somehow you are also teaching multiplication of numbers by five, are not you?
4	Denise:	Yes! But there is a difference. Here, this one [first column] represents the number of groups and this one [second column] is the quantity of objects in each group.
5	Researcher:	So, the goal is …?
6	Denise:	They have to understand the meaning, you know… the numbers in that position mean what? First, they [the learners] have to find the number of groups, and then they have to find the number of objects in each group.
7	Researcher:	Hum … is that why you used the same structure throughout the lesson?
8	Denise:	Yes! Because if I change it, it will be different. The learners will be confused.
9	Researcher:	But they would understand, right? For example, they should be able to solve three multiplied by five.



10 Denise: Hum … maybe! But I think it will be very difficult for them. They are not used to such expressions!

Multiplication is one of the four fundamental arithmetic operations, reflecting an individual's capacity to perform repeated addition in a group (Steffe, 1994). In Denise's lesson, however, the teaching of multiplication extended beyond the mere concept of joining equal quantities; it encompassed a deeper conceptual understanding of the principles governing the relationship between the factors in each expression (as evidenced in lines 4 and 6 of Excerpt I). Notably, as indicated in Lines 2 and 4, Denise recognizes that although 5×3 and 3×5 yield the same product, the expressions may be interpreted differently. Her instructional strategy posits that the first factor represents the number of groups, whereas the second denotes the number of units within each group. Consequently, she contends that 3×5 is an unsuitable expression for Standard 2 learners in Malawi, as it requires working with larger numbers (as reflected in Utterances 2, 8, and 10).

Excerpt I further illustrates how the multiplicand and multiplier roles significantly inform Denise's approach to teaching multiplication by 3 in Standard 2. Denise acknowledges that teachers must understand the commutative property of multiplication to implement this approach effectively; however, she argues that this property need not be explicitly taught to students. This stance reveals a potential inconsistency concerning the purpose of teachers' mathematical knowledge, given that such knowledge is not translated into classroom practices. It is important to note that the Malawian primary school teachers' guide does not include the commutative property as a formal lesson topic. Instead, teachers and pre-service teachers are encouraged to introduce multiplication concepts progressively (Malawi Institute of Education, 2007). For instance, children in Standard 2 are expected to learn multiplication by 2 and 3, while Standards 3 and 4 focus on multiplication by 4, 5, and 6, and subsequently by 7, 8, 9, and 10. This sequential approach results in children learning to compute 7×3 in Standard 2 but encountering 3×7 only in Standard 4. The principle of ordering factors based on the positional meaning of each factor is consistently applied throughout all primary school standards in the country.

This pedagogical perspective offers insights into Denise's SCK, particularly her understanding of how to sequence multiplication activities according to students' academic readiness. Denise's awareness of students' capabilities serves as a justification for her choice of representations and explanations during the lesson. Her objective was not merely to teach a method for solving multiplication problems but to foster a conceptual understanding of multiplication. This instructional approach exemplifies her practical application of SCK in the classroom. Phillip et al. (2007) support this view, emphasizing that knowledge of content combined with knowledge of students enhances understanding of what constitutes SCK. Similarly, Thanheiser et al. (2010) assert that "connecting children's thinking and the elementary mathematics curriculum is also a constituent of teachers' specialized content knowledge" (p. 12).

The following excerpt illustrates how Denise acknowledges this instructional approach to multiplication and its implications for students' learning. The session began by asking her about the didactic resources she employed during the lesson.

EXCERPT II: Sequencing and structuring students' learning of multiplication.

Line	**Interlocutor**	**Speech**
1	Researcher:	So, you used sticks in this lesson, right?
2	Denise:	Yes! I used sticks in this lesson!
3	Researcher:	Why sticks?
4	Denise:	Well, this is what the textbook suggested! … But, for example, last week, I was teaching multiplication of numbers by two, so I was using shoes, because these objects come in twos. I was using shoes, ears, eyes, hands …. so that students can be familiar with that concept and can apply it in everyday life. I wanted to make them curious and motivate them to find objects that exist in pairs. Now, for three objects, it is more difficult, because practical examples are not locally available. So, today I used stones and sticks because they are more or less similar. I asked the learners to pack them in threes, and them use those packs to solve the problems.
5	Researcher:	What about in Standards 4 and 5?

6	Denise:	In Standard 4, we teach them to count legs of tables, desks, chairs … and in Standard 5, we use, for example, fingers, toes …
7	Researcher:	I see! So, if I am going to teach this topic, do I need to know that?
8	Denise:	Yes! You must know how to model multiplication to help learners understand the meaning, because it is valuable for them to know how to put them together and use it [this understanding] to multiply different groups of objects.
9	Researcher:	But, will they still multiply different groups of objects according to their levels? I mean, in Standard 2, multiplications only involve 2 and 3, in Standard 3, they learn multiplication by 4 and 5 …
10	Denise:	Yes, according to their levels … according to the lesson!

The procedure used to teach multiplication by two was the same as that adopted for teaching multiplication by three. In both cases, students learn to count in groups of two and three objects, respectively, a foundational skill that supports their ability to manage real-life situations requiring equal groupings of objects (Excerpt II, Line 4). Denise's approach to teaching multiplication by three reflected her belief that objects in nature rarely occur in groups of three (Excerpt II, Line 4). Consequently, she argued that teachers should be mindful of this limitation to better support students in making appropriate generalisations about related mathematical concepts (Excerpt II, Line 8).

According to Denise, teaching multiplication through grouping is advantageous because it enables students to understand the concepts and properties associated with multiplying by three. By aligning her instruction with the students' developmental level, she aimed to promote generalisations that emphasized the importance of factor order. In Standard 2, for example, learners are also introduced to multiplying two-digit numbers by three, such as 11 × 3 and 12 × 3, but only with products that do not exceed 99. Accordingly, the notion that students must understand how to organize objects into equal groups to perform multiplication (Excerpt II, Line 8) implies that they need to master counting by groups within the limits established by the curriculum. Despite these constraints, Denise emphasized that grouping remains a crucial instructional tool for fostering students' broader understanding of multiplication.

These insights suggest that Denise's instructional decisions reflect her view of multiplication pedagogy as a distinctive component of teaching practice, consistent with the teaching knowledge she shared during her initial interview (IM). Her approach also aligns with SCK, which involves the transformation of subject matter knowledge into pedagogical content knowledge (Worden, 2015). However, Denise contended that to make mathematical content accessible to learners, teachers must prioritize certain conceptual aspects over formal properties such as associativity or commutativity. In other words, her interpretation of the knowledge required for teaching aligns with the recommendations found in the national teachers' guide for Standard 2, while also reflecting a growing awareness of how abstract mathematical principles can enhance teaching and learning in the primary classroom.

Denise's case illustrates that the knowledge she draws upon when teaching multiplication is largely grounded in students' capacity to form basic generalisations. At the same time, this emphasis may limit attention to more abstract properties that could support the development of more complex understandings. From this pre-service teacher' perspective, SCK appears to represent only one dimension of the mathematical knowledge necessary to engage students in meaningful learning. For Denise, this involved selecting and adapting representations and methods that allowed students to work with unfamiliar problems while maintaining a strong focus on the conceptual features of the topic (Hill et al., 2008).

Considering students' capacities to create different instructional approaches.

The analyses presented in this section illustrate how Martin designed and implemented instructional tasks to support students' learning of fractions. While this practice can be understood as a component of the teacher's SCK, the primary objective is to examine how Martin developed his understanding of the role that SCK plays in the teaching of mathematics.

As previously mentioned, the episode under analysis took place during a lesson on fractions, an essential topic in the Standard 4 primary school curriculum. Martin began the lesson by writing several fractions on the blackboard. Next to each fraction, he drew a fraction bar (also known as a strip diagram), with the number of divisions corresponding to the denominator of the respective fraction. In the post-lesson interview, Martin explained: "The goal of the lesson was to make students explore ways to represent fractions in a geometrical format," a skill he believes helps students "develop an understanding of parts of a whole."

Figure 2 depicts five randomly selected students working at the blackboard to complete the fraction bars introduced by Martin.

FIGURE 2: Standard 4 students filling out strip diagrams according to their respective fractions.

As Martin anticipated, all students completed the task successfully, noting that “most of them were already familiar with this type of representation.” He then introduced two similar exercises, followed by a more complex task in which the fraction bars were arranged perpendicularly (Task 3 in Figure 3).

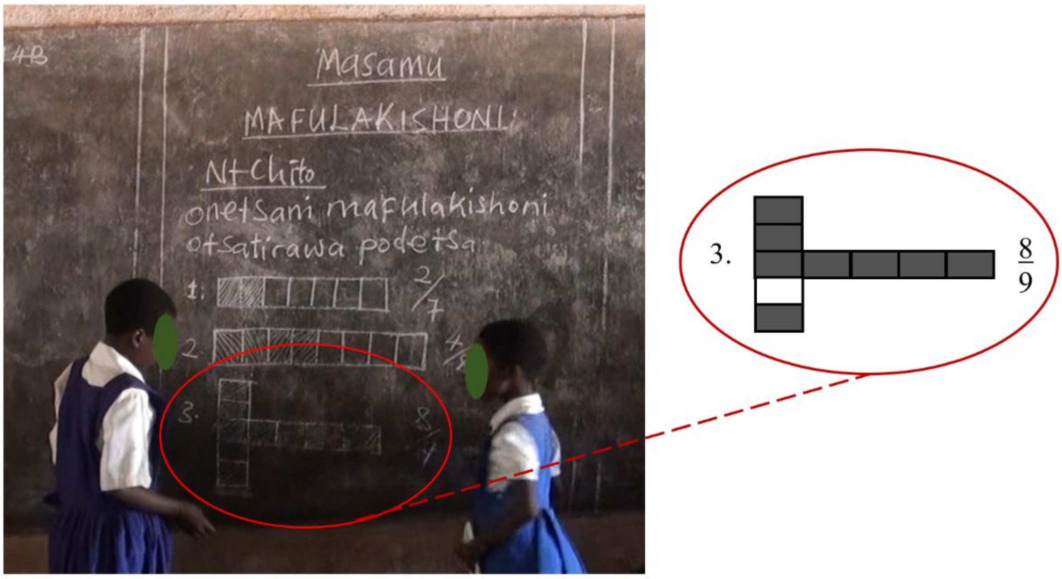


FIGURE 3: Students’ response to Task 3 requiring geometrical representation of the fraction 8/9.

As shown in Figure 3, the student on the left shaded one of the nine squares in Diagram 3.

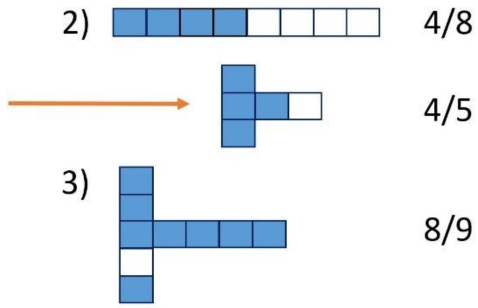
While previous strip diagrams had been presented horizontally, the use of both horizontal and vertical bars in Task 3 prompted students to engage with alternative forms of representation. This shift in format encouraged them to reconsider how fractions can be visually constructed. Martin elaborated on his interpretation of this response during the post-lesson interview.

EXCERPT III: Understanding the connections among activities to organise students' learning.

Line	Interlocutor	Speech
1	Researcher:	In this situation, why did that student choose to do that? [Use a different approach to solve the mathematical problem] Do you think this was a mistake?
2	Martin:	Yes! This was a mistake. She was supposed to put them all together not isolated!
3	Researcher:	But … look at the answer … is it not the same?
4	Martin:	Yes, the answer is the same, but I wanted her to put all coloured strips together.
5	Researcher:	Why?
6	Martin:	Because … you know … we need to follow the textbook, otherwise, the others [students] will be confused!
7	Researcher:	But … did you tell them you wanted all coloured blocks to be joined together?
8	Martin:	No, I did not! But I should have done that! It could avoid this mistake.

9	Researcher:	Ok … so, for this student, is there anything you could have done to help her?
10	Martin:	Yes, I could have added another example before this one, a simpler one. Like this …

Martin illustrated his idea by drawing the following figure:



11	Martin:	So, you can see that this example is much easier than the last one. So, it would help learners to understand and solve more complex problems like this one. [Martin points to the figure in Item 3 that represents the 8/9 fraction]
12	Researcher:	Hum …. Ok! But here … in this situation, would you not consider what she did as correct?
13	Martin:	I can consider it, but sometimes it is very hard to know what the learners will do …. That's why I think it is valuable, first, to focus on the textbook.
14	Researcher:	Interesting! So, if I come to Malawi as a teacher and decide to teach this content in Standard 4, what should I know? What should I do?
15	Martin:	If you are going to teach this topic in Malawi, you must consider what learners already know and use the textbook as the main reference.

As discussed above, Martin regarded the student's response as incorrect primarily because it did not conform to the procedures outlined in the textbook. He justified this stance by emphasizing the importance of following textbook conventions to ensure that all students develop a coherent sense of grouping and the capacity to construct complex geometric representations of fractional numbers (Excerpt III, Line 11). In the post-lesson interview, Martin elaborated: "It can be hard for them to count the bars if they don't put them together; as I said, they can get confused." He acknowledged that the student might have understood the task differently had he clarified beforehand that the fraction bars should be filled without leaving any gaps (Excerpt III, Line 8). Martin further noted that, because students require clear guidance, mathematics lessons must follow a logical structure, aligned with the instructional guidelines specified in the textbook for that grade level (Excerpt III, Lines 13 and 15).

The idea of providing simpler and more connected tasks for students emerged as a new pedagogical approach to sequencing and structuring mathematical learning. To support his students in understanding and representing fractions through different diagrammatic formats, Martin proposed an additional activity (Excerpt III, Line 10). This task resembled the intersecting-strip configuration from Diagram 3 (Figure 3) but used a reduced number of bars. According to Martin, this simplified version would help students conceptualize the strips as parts of a unified whole and reduce potential difficulties in solving comparable tasks. "They [learners] also have to know how to apply what they learn in the classroom to other diverse situations, not just familiar ones," he remarked.

Although the intermediate task proposed by Martin may not fully prevent confusion—since, as seen in the figure representing the fraction 4/5 (Excerpt III, Line 10), students could still leave gaps between shaded segments—it nevertheless highlights key considerations for sequencing and structuring tasks to support effective mathematics learning. Martin's reflections align with the notion of selecting and adapting classroom tasks (Rivas et al., 2012) to vary in complexity—both simplifying and extending them—which are essential features of mathematics teaching that rely on SCK (Ball et al., 2008).

Martin's approach to lesson design thus reflects a developing understanding of how visual representations can scaffold mathematical learning. He believed that once students mastered tasks involving horizontal bar models, they would be better prepared to tackle more complex problems, potentially extending beyond the textbook content (Excerpt III, Line 11). As he explained, knowledge of the tasks that support this kind of progression is fundamental for helping students build upon what they already know (Excerpt III, Line 15). Moreover, Martin's tendency to expand, reduce, or adapt lessons in response to students' abilities illustrates his emerging grasp of how SCK is applied in practice. His instructional decisions were guided not only by a desire to help students make sense of mathematical concepts but also by an ongoing effort to assess and leverage their understanding in designing meaningful learning experiences.

Final Considerations

The present work provides insights into pre-service teachers' understanding of a specific aspect of the SCK domain—specifically, the knowledge needed to sequence and structure students' learning of mathematics. Our findings illustrate how this understanding is shaped through the demands and realities of teaching practice. For novice pre-service teachers, engaging with this domain is particularly valuable, as it highlights the importance of experimenting with diverse instruments strategies that foster meaningful mathematical learning. These strategies also contribute to building classroom environments in which students can make coherent connections across concepts and progress toward clearly defined learning goals.

Although the two cases discussed are situated in the Malawian context, their relevance extends to other settings where teacher education remains underdeveloped. These cases offer valuable insights, as our analysis reveals that first-year Malawian pre-service teachers struggled to grasp the pedagogical knowledge necessary to organize instruction in alignment with curriculum guidelines and textbook structures. However, as they progressed in their training and gained classroom experience, they developed a more refined understanding of the competencies required for effective teaching in primary schools. The two pre-service teachers in our study not only began to better comprehend the nature of SCK, but also recognized how it could be applied more flexibly to support students' learning in mathematics.

Throughout the study, both participants often relied on their personal educational experiences when planning instruction. While they showed genuine interest in student learning, they found it challenging to respond to situations not directly addressed in textbooks or teacher guides. This gap limited their ability to create learning opportunities that emphasized connections between topics and representations in mathematics. Yet, we interpreted it as a necessary part of their learning process. Comparing their early reflections with those after their teaching practice reveals a more comprehensive understanding of SCK. Although they shared similar forms of understanding, they applied them in different ways—each aiming to align lesson content with strategies that support students´ learning of mathematics.

The insights gained from this research can inform the design of teacher education activities that help pre-service teachers connect practical teaching tasks with the theoretical knowledge needed for effective teaching. Understanding the diverse ways in which pre-service teachers develop their understanding and apply this knowledge in practice can help educators and researchers identify more effective approaches to support their professional growth as teachers.

It is important to acknowledge that this study is not exhaustive, and its findings should not be generalised beyond the specific context under investigation. Further empirical inquiry is required to examine additional dimensions of teaching knowledge and the instructional tasks embedded in the work of teaching, particularly within diverse educational settings. In the absence of such research, it remains uncertain whether existing teacher education programs adequately equip pre-service teachers to meet the complex and multifaceted demands of mathematics instruction. Ongoing investigation is therefore essential to advance a comprehensive understanding of effective mathematics teaching and to assess its feasibility in under-resourced classrooms.

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