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Optimizing the Rear-Disconnected Steering Trapezoid for FSAE Racing Cars

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Abstract
The steering system of FSAE racing car plays a vital role in maintaining the direction and stability of the car. The design of the steering trapezoid is an important component of the steering system, and the post-disconnected steering trapezoid has been analyzed and optimized in this study. To improve the steering behavior of the car, a mathematical model of the post-disconnected steering trapezoid was developed. The objective function was established using the modified ideal Ackerman steering formula, and the geometric relationship between the actual inner and outer corners. Additionally, a constraint equation was established to ensure operational convenience of the steering mechanism. MATLAB was used to find the design variable parameters that satisfied the fatigue strength while ensuring that the actual internal and external angle relationship was maximally close to the ideal Ackerman steering. Furthermore, the dynamic change of the toe angle of the parallel two-wheeled beating was simulated using ADAMS, to ensure that the car met the stability requirement.
The results showed that the optimized post-disconnected steering trapezoid can significantly improve the steering behavior of the FSAE racing car. The mathematical model and optimization method developed in this study can provide a theoretical reference for the design of the steering trapezoid in future studies. Moreover, this study highlights the importance of considering the fatigue strength and dynamic simulation in the design process of the steering system for FSAE racing cars. Overall, the optimized post-disconnected steering trapezoid design can improve the stability and steering performance of FSAE racing cars, providing better handling during dynamic competition. The proposed method can be applied to the design and optimization of steering systems in various racing cars, making it a valuable contribution to the field of motorsport engineering.

Keywords: FSAE racing car, steering system, steering trapezoid, post-disconnected, ideal Ackerman steering, mathematical model, fatigue strength, dynamic simulation, stability requirement.

INTRODUCTION

Among the components of the FSAE, the steering system is directly related to the passing and steering stability of the car. The steering system of the FSAE racing car is the mechanism which is used to maintain or change the direction of the car. And it also can ensure a coordinated corner relationship between the steering wheels when the car is turning [1]. In the dynamic competition, the 8-word loop, high-speed obstacle avoidance and endurance race have higher requirements on the stability of the steering system, and the design of the steering trapezoid is a particularly important part of the steering system design [2]. So, a reasonable design of steering trapezoid not only improves the steering ability of the car, but also improves the overall performance of the car to a certain extent, which is of great significance for improving the performance of the car.
Presently, scholars at home and abroad have deeply analyzed and researched on the structural design of the FSAE racing car to the trapezoid from various angles.
For example, Song Xueqian and others established a mathematical model of the front-disconnected steering trapezoidal structure and collected data through an angular displacement sensor to correct the steering angle

[3]. ShaoFei and others realized the design of parallel Ackerman geometry steering trapezoid [4]. And ZhangKai and others proposed a steering trapezoidal joint optimization design method and improved the speed and corner stability of the FSAE racing car [5]. The above research provides different research methods for the steering trapezoid of the car, but the front layout is adopted in the layout of the steering trapezoid, and the research results are not fully applicable to the layout scheme adopted in this paper. So, in this paper we further analyzed and optimized the rear-disconnected steering trapezoid with the steering gear located behind the front axle.

Methodology

Steering trapezoid scheme selection

In the rules issued by the competition, it is clearly stated that the use of steer-bywire steering and electronically controlled steering is prohibited. Considering the light weight of the FSAE racing car, most of the teams use unassisted mechanical rack and pinion steering gears, which have simple structure, small size, easy to make, etc., can reliably meet the steering task [6].

Because of the compact layout of the FSAE racing car, in order to ensure good compatibility between the steering system and the independent suspension system, the disconnected steering trapezoid is often used. According to the different positions of the steering trapezoid and the rack and pinion steering gear with respect to the front axle, the arrangement can be divided into four types [1]. And the rear trapezoidal arrangement of the steering gear behind the front axle is simple. So, in this paper we use the rear-disconnected steering trapezoidal powerless mechanical rack and pinion steering gear with the steering gear behind the front axle. We analyze and design the FSAE racing car. And its structural diagram is shown in Figure 1.

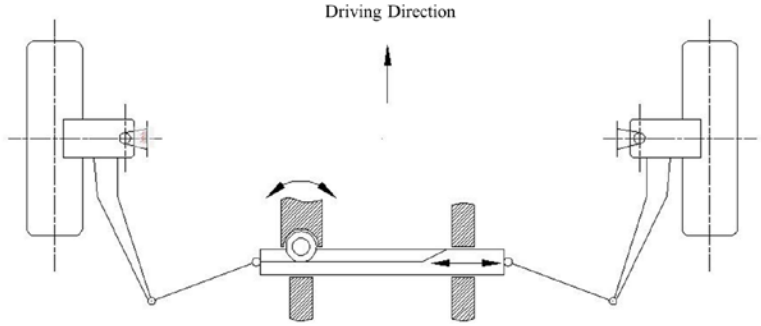

Figure. 1 *Rear-displacement steering trapezoidal structure intention of the steering gear behind the front axle*

Establishment and analysis of steering trapezoidal mechanism model

When the car is turning, in order to ensure that its four wheels are all rolling around the same instantaneous center point without sliding, that is, to meet the Ackerman steering [7], the angle of the inner and outer wheels of the front axle should satisfy the relationship as shown in Figure 2.

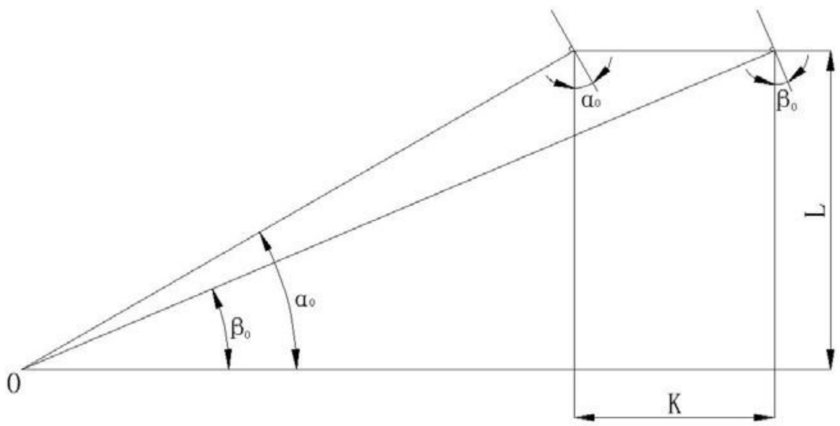

Figure. 2 *Ideal Ackerman Corner*

According to the geometric relationship, the concrete expression of the ideal inner and outer wheel angles when meeting Ackerman steering can be obtained:

$$\cot\beta_0 - \cot\alpha_0 = K/L \quad (1)$$

Where α_0 is the ideal inner wheel steering angle, β_0 is the ideal outer wheel steering angle, K is the main pin center distance, and L is the wheelbase. If the self-variation angle is assumed to be the ideal inner wheel steering angle α_0, then the angle of change is the ideal outer wheel steering angle β_0, the expectation of the variable angle β_0 can be expressed as:

$$\beta_0 = \mathrm{arccot}(\cot\alpha_0 - K/L) \quad (2)$$

However, in the actual car steering process, the tire will be subjected to the centrifugal force generated during the curve driving, that is, the cornering force. Under the action of the cornering force, the tire will have a certain deformation, so that the actual inner and outer wheel angles are smaller than the ideal inner and outer wheel angles. So, it is necessary to correct the ideal inner and outer wheel angles [5]. Thus, the Ackerman steering correction coefficient ε is introduced to correct the ideal outer wheel steering angle β_0. And the specific relationship between the corrected outer wheel steering angle β^* and the ideal outer wheel steering angle β_0 is :

$$\varepsilon = \frac{\beta^* - \beta_0}{\alpha_0 - \beta_0} \times 100\% \quad (3)$$

Then, putting formula (3) into formula (2), we can obtain the expected value of the corrected ideal outer wheel steering angle β^* as follow.

$$\beta^* = \varepsilon \times \left[\mathrm{arccot}\left(\cot\alpha_0 - \frac{K}{L}\right) - \alpha_0\right] + \alpha_0 \quad (4)$$

Place the selected steering gear on the rear trapezoidal broken rack and pinion steering trapezoid behind the front axle. And after simplification, the mathematical model can be obtained, as shown in Figure 3.

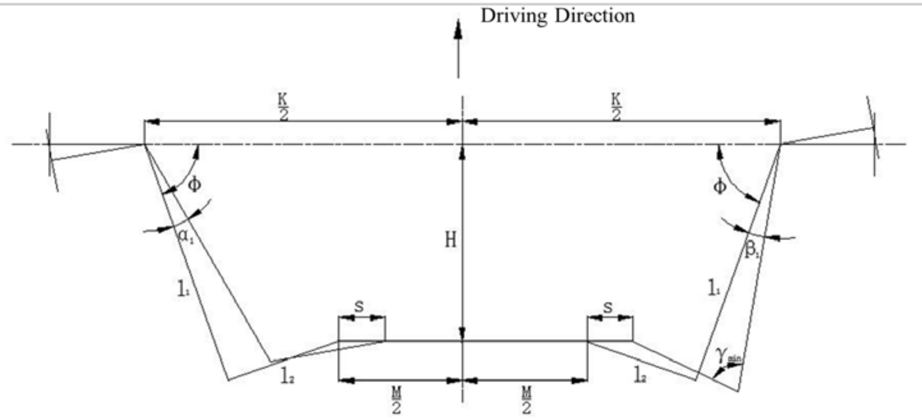

Figure. 3 *Rear Disconnected Steering Trapezoidal Model*

When steering, the steering gear drives the gear rack to move. And under the action of the tie rod, the inner and outer steering wheels will rotate to generate the steering angle [3]. During the steering process, the tie rod and the knuckle arm are hinged, and only the relative position change occurs between them, and the length of each rod is not change. Therefore, according to the geometric relationship between the tie rod and the knuckle arm, we can obtain the relative relationship between the various parameters of the steering ladder structure, which is as follows:

Before turning, the distance l_2 from the break point to the midpoint of the ball joint pin end of the knuckle arm is:

$$l_2 = \sqrt{\left(\frac{K}{2} - \frac{M}{2} - l_1 \times \cos\phi\right)^2 + (l_1 \times \sin\phi - H)^2} \quad (5)$$

Where l_1 is the arm length of the trapezoidal arm that turns to the trapezoidal structure, ϕ is the bottom angle of the steering trapezoid, H is the horizontal distance from the center line of the rack to the center point of the kingpin, and M is the length of the tie rod.

When turning, the movement stroke S of one side of the rack is:

$$S = -\sqrt{l_2^2 - \left[l_1 \times \sin(\phi\,\beta + {}_0) - H\right]^2} - l_1 \times \cos(\phi\,\beta + {}_0) + \frac{K-M}{2} \quad (6)$$

When the car is turning, the actual outer wheel steering angle β_1 is:

$$\beta_1 = \arctan\left(\frac{H}{\frac{K-M}{2} - S}\right) + \arccos\left(\frac{l_1^2 + H^2 + \left(\frac{K-M}{2} - S\right)^2 - l_2^2}{2 \times l_1 \times \sqrt{H^2 + \left(\frac{K-M}{2} - S\right)^2}}\right) - \phi \quad (7)$$

The actual outer wheel steering angle β_1 should be designed as close as possible to the expected value of the corrected outer wheel steering angle β^*. When the design value is closer to the expected value of the theoretical value, the car can be closer to the ideal Ackerman steering during the actual steering process. In order to objectively evaluate the closeness between the actual outer wheel steering angle β_1 and the expected value of the corrected ideal outer wheel steering angle β^*, we introduce an objective function $F(x)$ for evaluating the advantages and disadvantages of the steering trapezoid design. The specific expression is as follows:

$$F(x) = \mu \times \left| \frac{\arctan\left(\frac{H}{\frac{K-M}{2} - S}\right) + \arccos\left(\frac{l_1^2 + H^2 + \left(\frac{K-M}{2} - S\right)^2 - l_2^2}{2 \times l_1 \times \sqrt{H^2 + \left(\frac{K-M}{2} - S\right)^2}}\right) - \phi}{\varepsilon \times \left[\operatorname{arccot}\left(\cot \alpha_0 - \frac{K}{L}\right) - \alpha_0\right] + \alpha_0} - 1 \right| \times 100\% \quad (8)$$

Where μ is the correction factor, x is the design variable and $x = [M\ H\ \phi\ l_1]^T$.

Considering that FSAE racing cars are always driven under high-speed conditions, the wear pattern of the tire surface pattern is more serious. When evaluating the design of the steering trapezoid, it is necessary to take into account the influence of tire wear on it. Thus, the introduction of the correction coefficient μ makes the objective function $F(x)$ of the evaluation steering ladder design closer to the actual situation.

When setting constraints, we focus on the ease of operation of the steering mechanism. Because of the low quality of the FSAE racing car and the limitation of the installation space, the FSAE racing car generally does not use the steering assist device, which requires the entire steering mechanism to be maneuverable during handling. While in the steering trapezoidal mechanism, the direct influence of the handling of the portability are the length of the tie rod M and the bottom angle ϕ of the steering trapezoid. When the values of the design variables M and ϕ are selected too small, the steering load applied to the tie rods is too large, which is inconvenient for the racer to turn [1]. When the length M of the design variable tie rod is too large, the size of the entire steering trapezoid will increase, which will not only increase the difficulty of the FSAE racing steering mechanism, but also cause serious problems with the interference of the front suspension. Thus, selecting the length M of the appropriate tie rod and the bottom angle ϕ of the steering trapezoid have important significance for the manipulation and arrangement of the entire steering mechanism. The specific constraints are as follows:

$$\begin{cases} M - M_{min} \geq 0 \\ M_{max} - M \geq 0 \\ \phi - \phi_{min} \geq 0 \end{cases} \quad (9)$$

In the formula, M_{min} is the general design minimum of the tie rod, M_{max} is the general design maximum of the tie rod, and ϕ_{min} is the general design minimum of the base angle of the steering trapezoid.

According to the design experience, the general design minimum value of the tie rod is $M_{min} = 0.11K$, the general design maximum value of the tie rod is $M_{max} = 0.15K$, and the general design minimum value of the bottom angle of the steering trapezoid is $\phi_{min} = 70^\circ$.

At the same time, the entire steering mechanism should also have high transmission efficiency when maneuvering. The steering trapezoidal mechanism is essentially a multi-link mechanism. In the multi-link mechanism, the transmission angle can effectively reflect the transmission quality of the linkage mechanism. Thus, the steering angle is further constrained by the transmission angle. In the motion diagram of the steering mechanism designed above, the transmission angle is γ (As shown in Figure 3). And the larger the transmission angle γ, the better the work of turning the trapezoidal mechanism. During the steering process, the transmission angle x is constantly changing with the steering angle. In order to ensure the good transmission performance of the steering mechanism [8], the minimum transmission angle of the steering mechanism should be $\gamma_{min} \geq 40$□ . Considering that the FSAE racing car needs to meet the characteristics of high-speed continuous passage of multiple corners during the competition, the minimum value of the transmission angle of the steering mechanism is put forward here, that is, the minimum transmission angle of the steering mechanism should satisfy $\gamma_{min} \geq 50$□ .

During the steering of the car (Take the turn to the left as an example), when the car reaches the extreme position of the left steering, the relationship between the minimum turning radius D_{min} and the maximum value β_{max} of the steering angle of the front outer wheel can be obtained. The specific geometric relationship is as follows:

$\beta_{max} = \arcsin_{D} \min^{L}{}_{/\,2-a}$ (10)

In the formula, a is the main pin offset.

Through the study of the FSAE track, the minimum turning radius $D_{min} = 3m$ can be found, so the maximum value β_{max} of the front outer wheel steering angle can be determined. Through the geometric relationship, it can be found that the current steering angle of the steering wheel reaches the minimum value γ_{min} when the maximum value of the steering angle of the outer wheel is reached, so the constraint equation for the minimum value γ_{min} of the transmission angle can be obtained. The specific expression is as follows:

2

γmin= arccos*l* +*l* −□□□2*K*× ×−²*l*1*Ml*2−*S*□□□−*H* 2 ≥ 50□ (11)

According to the above analysis, the solution of the steering trapezoidal mechanism is essentially a constrained nonlinear programming problem. Under the constraints of the constraint equations (9) and (11), MATLAB is used to solve the minimum value of the objective function $F\ x(\ )$, which evaluates the advantages and disadvantages of the steering trapezoid design. Then we can get the best value of the design variable x .

After determining the optimal value of the design variables, the design parameters of the steering gear in the steering structure can be initially selected. In the rack and pinion steering gear, the movement stroke S on one side of the rack and the maximum angle W_{max} on one side of the steering wheel have the following relationship:

$\underline{360}^{\times W_{max}}$ (12)

$S = \pi \times \times m\ z$

In the formula, m is the modulus of the steering gear, and z is the number of teeth of the steering gear.

The modulus m of the steering gear should not be too small. If it is too small, the meshing point of the rack will be too close to the top of the tooth when meshing, causing the bending stress of the root to be too large, which is prone to chipping. Thus, the modulus m of the steering gear is generally 2-3mm [7].

In the selection of the number of teeth z of the steering gear, considering that the diameter of the indexing circle is small, if the number of teeth is too large, there will be a problem of difficulty in processing and low degree of coincidence of gear meshing transmission, and attention should be paid to avoiding the cutting phenomenon. Thus, the number of teeth z of the steering gear is generally selected from 8 to 19.

The maximum corner W_{max} of the steering wheel on one side generally requires no more than 10 □ from the middle to the left or to the right.

Thus, putting the optimal design variables into the formulas (6) and (12), and according to the above specific requirements for the rack and pinion mechanism parameters, the modulus m of the steering gear and the number of teeth z of the steering gear can be preliminary determined. Then, through checking the tooth surface contact fatigue strength of the gear, we can determine the gear parameters that meet the strength conditions. The specific process of the steering trapezoidal mechanism optimization design is shown in Figure 4.

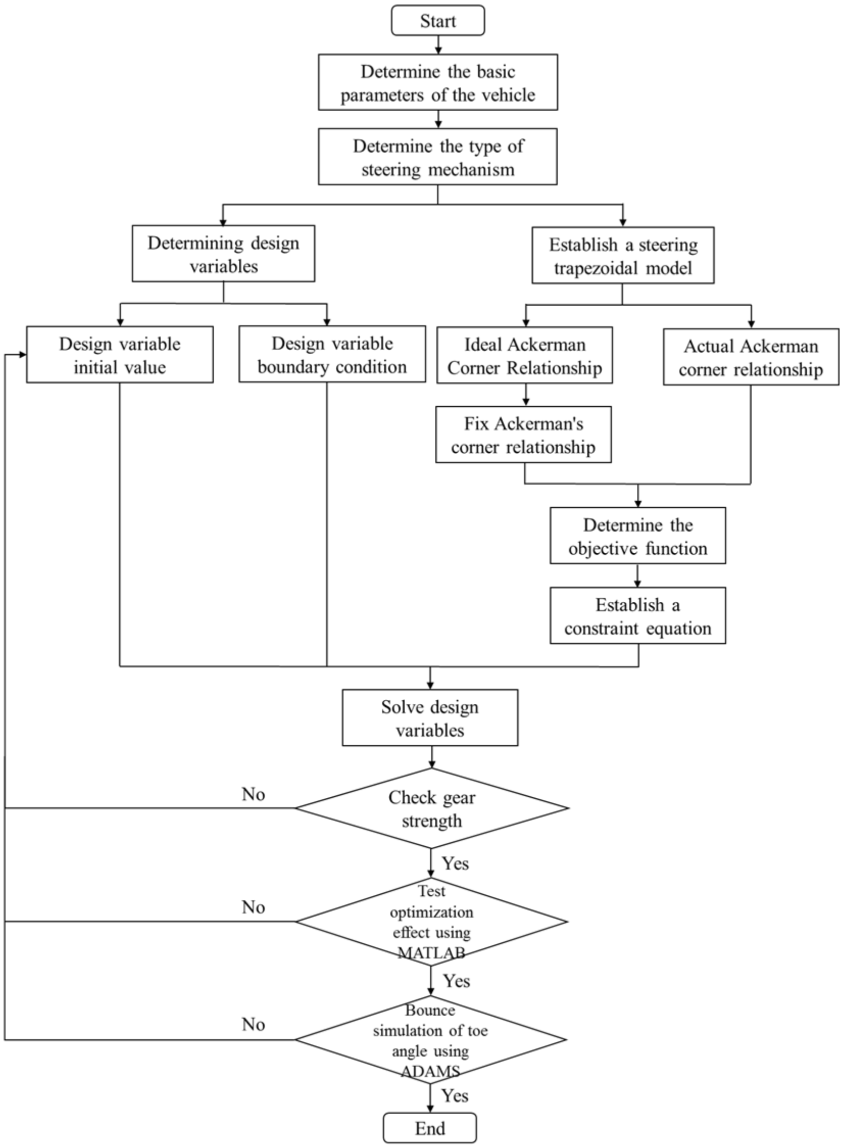

Figure. 4 *Steering Trapezoid Optimization Design Flow Chart*

Results and discussion

Steering trapezoid optimization design

According to the rules of the FSAE [6] and the design experience of the FSAE racing car, the initial values of the design variables and the upper and lower boundary conditions are determined as shown in Table 1.

Table 1 *Initial value of design variables and upper and lower boundary conditions*

Design variable	Initial value	Upper and lower boundary conditions
Length of tie rod *M* / mm	190mm	160mm-220mm
The horizontal distance from the centerline of the rack to the center point of the kingpin *H* /mm	35mm	30mm-40mm
corner of the Steering Trapezoid ϕ/□	75□	60 90□ –□
Arm length of trapezoidal arm	75mm	50mm-100mm

According to the above analysis of the steering trapezoidal mechanism model, using the fmincon function in the MTALAB optimization toolbox, formula (8) is used as the objective function of the solution, and equations (9) and (11) are used as the constraint equations, and Table 1 is used as the upper and lower boundary conditions of the design variables. And the optimized design results are shown in Table 2.

Table 2 *Steering mechanism technical parameter table*

Design variable	Design value
Length of tie rod M / mm	200mm
The horizontal distance from the centerline of the rack to the center point of the kingpin H /mm	39mm
corner of the Steering Trapezoid ϕ/□	79 □
Arm length of trapezoidal arm l_1 / mm	70mm
Rack travel on one side S /mm	44mm
Modulus of the steering gear m/ mm	2mm
Number of teeth of the steering gear z / teeth	13 teeth

In order to test the optimization effect of the steering trapezoid design, the relationship between the input angle of the outer steering wheel and the output angle of the inner steering wheel is drawn by MATLAB, as shown in Figure 5. When the angle curve of the actual inner and outer wheels is closer to the angle curve of the ideal Ackerman inner and outer wheels, it shows that the closer the FSAE car is to the ideal Ackerman steering when it is actually turning. It means that the smaller the difference between the two curves is, the better the actual steering effect is.

From Figure 5, it can be seen that after adopting the optimized design scheme of this paper, the actual corner curve is closer to the ideal corner curve than the initial corner curve. Thus, the design of the steering trapezoid in this paper can better match the ideal Ackerman steering relationship under the condition of meeting the design requirements and has better steering performance.

The parameter design of the steering mechanism not only affects the steering performance of the car, but also affects the steering stability of the car to some extent. During the steering of the FSAE, the relative motion of the steering mechanism and the suspension mechanism changes the toe angle of the wheel. While the size of the toe angle indirectly affects the wear level of the FSAE racing tires. Thus, in the design process of the steering mechanism, the degree of change of the toe angle during the running of the wheel should be minimized, so that the car maintains good steering stability during the steering process.

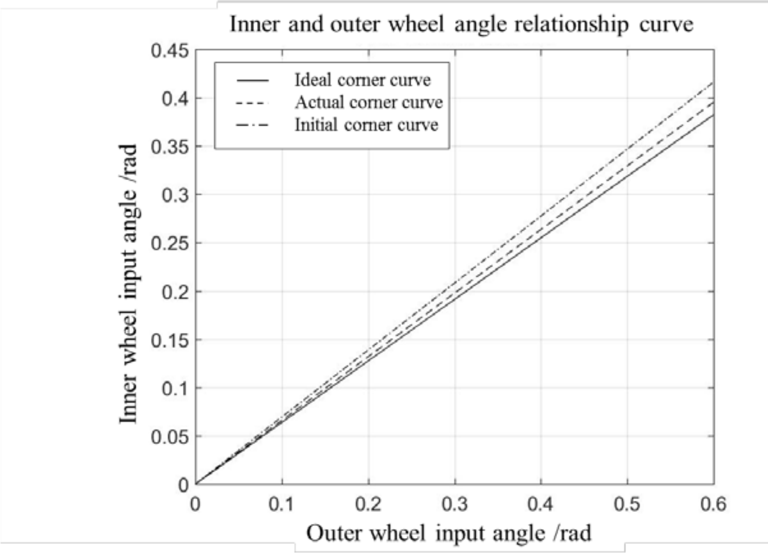

Figure.5 *Inner and outer wheel angle relationship curve*

The minimum value of the toe angle change during parallel double-wheel bounce is optimized using the Insight module in ADAMS. The optimization result is shown in Figure 6.

According to Figure 6, it can be seen that the toe angle of the wheel before and after optimization is reduced from 1.7957° to 1.4478°. After adopting the design of the steering mechanism of this paper, the degree of change of the toe angle when the wheel is beating can be effectively reduced. Thus, from the perspective of dynamics, the optimized design of this paper makes the FSAE racing car have higher steering stability when steering and achieves the expected optimization results.

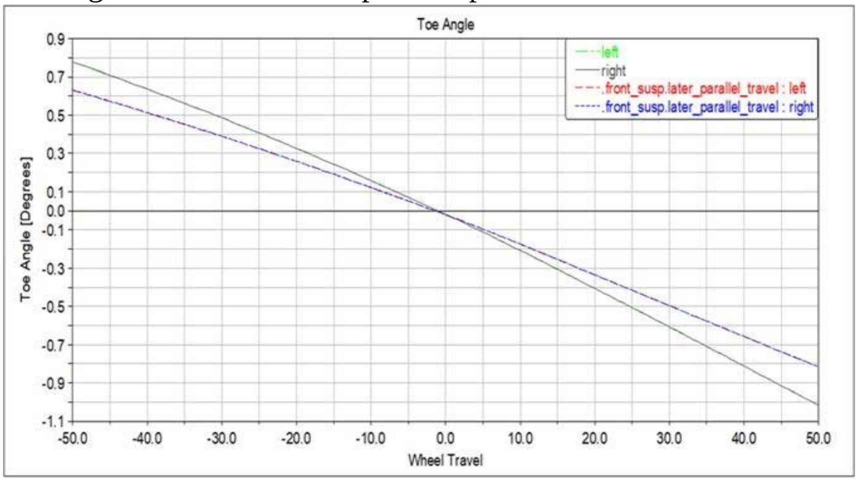

Figure.6 *Simulation Curve of Parallel Double Wheel Bounce Toe Angle Change*

Conclusion

In this paper, we analyze and model the rear-disconnected steering trapezoid of FSAE racing car and transform the steering trapezoidal multi-link mechanism into a mathematical model. And we modify the ideal Ackerman steering formula and consider the effect of the cornering force on the ideal Ackerman steering. The objective function is established by the geometric relationship between the modified ideal Ackerman steering formula and the actual inner and outer corners, and the operation convenience of the steering mechanism is constrained. After adopting the idea of nonlinear programming, and then determining the upper and lower boundary conditions of the design variables, the initial values of the design variables are continuously changed by MATLAB software, so that the final design structure can maximally close to the actual internal and external rotation angles under the condition of satisfying the fatigue strength. The ADAMS software is used to simulate the dynamic change of the toe angle of the parallel two-wheeled. It is verified that the steering trapezoidal design given in this paper not only has good steering performance, but also improves the steering stability of the car to a certain extent. And it also improves the dynamic performance of the FSAE racing car while driving in a corner. The results of the study provide a certain reference value for the design of the FSAE racing steering mechanism.

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