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# Introduction and Analysis of Typical Quantum Hall Effect

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**Abstract:** Through reading and organizing a large amount of literature, we have gained a certain understanding of the Hall Effect. Meanwhile, we have selected the Hall Effect at the macroscopic level, and the Quantum Hall Effect, Quantum Anomalous Hall Effect, and Quantum Spin Hall Effect at the quantum side as the research content of this paper. We first explain the principles of these four phenomena, then we summarize their discovery history and the latest research progress, introduce their applications, and finally, based on our comprehension and generalization of the literature, we give a summary and outlook. This paper is helpful for the in-depth study of phenomena such as Quantum Hall Effect and Quantum Anomalous Hall Effect from Hall Effect.

**Keywords:** Hall Effect, Quantum Hall Effect, Two-dimensional materials.

## 1. Introduction

The Hall Effect has an important scientific value and application in the field of electromagnetism, and some people take "Hall Effect" as a fixed object and summarize the Hall Effect family into the following eight types: 1. Hall Effect 2. Anomalous Hall Effect 3. Spin Hall Effect 4. Quantum Hall Effect 5. Quantum Anomalous Hall Effect 6. Quantum Spin Hall Effect 7. Anomalous Spin Hall Effect 8. Quantum Anomalous Spin Hall Effect. In this paper, from a quantum point of view, we introduce the Hall Effect - Quantum Hall Effect - Quantum Anomalous Spin Hall Effect - Quantum Spin Hall Effect.

## 2. Principle of Phenomena

### 2.1. Hall Effect

The physical meaning of the Hall Effect is that when a magnetic field perpendicular to the direction of the current is applied to a charged conductor or semiconductor, causing the charge carrier to be deflected, a transverse potential difference appears in the direction perpendicular to the current and the magnetic field according to the right-handed spiral rule. When the Lorentz charge force on the carrier is balanced by the electric field force, the charge carrier is no longer deflected and the potential difference is stable. This phenomenon is known as the Hall Effect.

### 2.2. Quantum Hall Effect

The Quantum Hall Effect is divided into the Integer Quantum Hall Effect and the Fractional Quantum Hall Effect. Under the experimental conditions of very low temperature and strong magnetic field, it can be observed that with the increase of the external magnetic field, the Hall coefficient no longer shows linear relationship with the magnetic field, but quantitative plateau, see Figure 1. At the same time, the corresponding longitudinal resistance falls to zero. The plateau is not related to the material properties of the sample and appears in

$$R_H = \frac{h}{ie^2} \quad \mathrm{i} = 1,2,3,\ldots \tag{1}$$

where e is the absolute value of the electron charge, h is Planck's constant, and i is a positive integer, determined by the electron density and magnetic flux density. This phenomenon is known as the Integer Quantum Hall Effect.

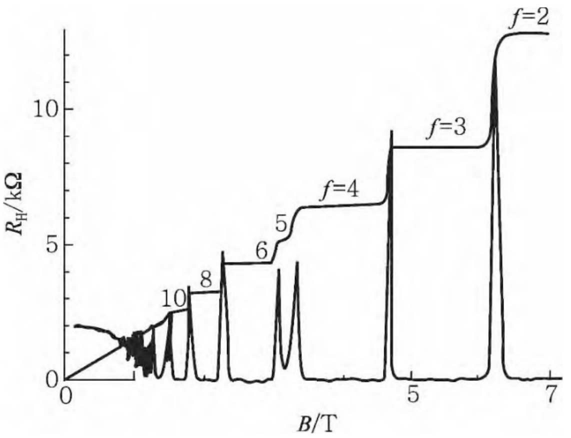

**Figure 1.** $R_H$-B diagram of the Integer Quantum Hall Effect[3]

In experiments on those samples of purer two-dimensional electron gas systems with high mobility at a lower temperature and stronger magnetic field than the Fractional Quantum Hall Effect, it is observed that the plateau of Hall resistance has a finer step structure, and the Hall resistance plateau appears at i = 1/3 , 2/3 , 4/3 , 5/3 ..., while the resistivity $\rho_{xx}$ at the current direction has a very small value, which is defined as the Fractional Quantum Hall effect, as following picture shows.

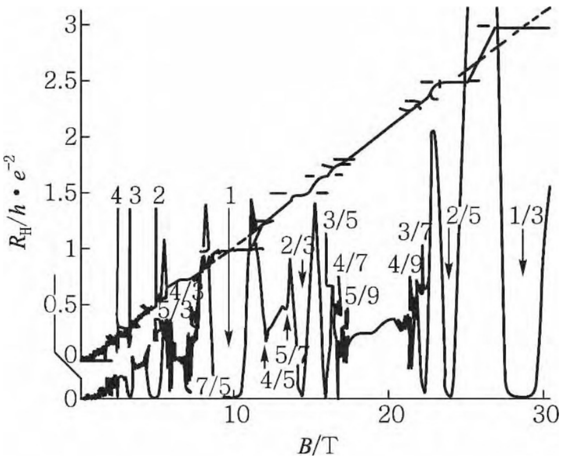

**Figure 2.** Fractional Quantum Hall Effect[3]

### 2.3. Quantum Anomalous Hall Effect

While the Anomalous Hall conductance is due to the spontaneous magnetization of the material itself, the Quantum Anomalous Hall Effect is the quantization of the Anomalous Hall Effect. The quantum effect is an overall quantum phenomenon exhibited by a macroscopic system consisting of a large number of particles under certain special conditions such as ultra-low temperatures. Under the usual conditions, the macroscopic object does not show quantum effects, but under special conditions such as lower temperature or higher particle density, the individual components of the macroscopic object will coherently combine and enter the lower energy quantum state through long-range correlation or recombination to form an organic whole, making the whole system exhibit peculiar quantum properties. The Quantum Anomalous Hall Effect differs from the Quantum Hall Effect in that it does not depend on a strong magnetic field but is generated by the spontaneous magnetization of the material itself, and the quantum Hall state can be realized in a zero magnetic field [1].

### 2.4. Quantum Spin Hall Effect

In a specific quantum well, under the condition of no external magnetic field, the surface of an insulator made of a specific material produces a special edge state that makes the edge of this insulator conductive and the direction of this edge state current is completely locked to the spin direction of the electron, this is the Quantum Spin Hall Effect [2] Like the Quantum Hall Effect, the Quantum Spin Hall Effect is also in the two-dimensional system, caused by edge states. However, the Quantum Spin Hall Effect is composed of two sets of edge states with opposite spin directions and running in opposite directions, and does not require an applied magnetic field. Since the electrons on the two sets of edge states make to move in opposite directions, the net charge current is zero and there is no Hall conductance. However, since the spins of these electrons are in opposite directions, a quantized spin Hall conductance is formed with a value of 2e/4π.

## 3. Discovery of Phenomena

### 3.1. Hall Effect

In 1879, a American physicist Hall, discovered that the charge carriers in the conductor are deflected by the Lorentz force perpendicular to the magnetic field and accumulate equal and opposite charges on both sides of the conductor perpendicular to the direction of the current and the magnetic field, resulting in a transverse potential difference U_H，while studying the force on a carrier conductor in a magnetic field. This phenomenon is named the Hall Effect.

### 3.2. Quantum Hall Effect

In 1974, Tsuheya Ando and Englert theoretically analyzed that when the Fermi energy level of a two-dimensional electron gas is between the Landau energy levels, the horizontal component of the resistance of the sample can fall to zero for a certain range of gate voltages, and the Hall resistance will appear as a step phenomenon. In 1980, Klaus von Klitzing discovered the Integer Quantum Hall Effect on the oxide surface of a metal-oxide-semiconductor field-effect transistor based on the idea of Tsuheya Ando that Hall Resistance can appear as a step phenomenon. In 1982, Qi Cui used GaAs and GaAlAs material to construct a quantum well at Bell Laboratories in New Jersey, in Restricting the electron to become a two-dimensional electron gas and giving a lower temperature and stronger magnetic field, it was found that the next order step of Hall resistance was three times the highest record of Klaus von Klitzing. Later, under fractional values of voltage，he has also observed the appearance of the Hall Effect plateau , which is the Fractional Quantum Hall Effect [3].

### 3.3. Quantum Anomalous Hall Effect

Since 2008, many scientific teams have predicted the Quantum Anomalous Hall Effect. In 2008, Chao-Xing Liu et al. predicted a new phenomenon in Hg1-yMnyTe materials, the Quantum Anomalous Hall Effect, based on the Quantum Hall Effect [4]. In 2010, Zhenhua Qiao et al. investigated the possibility of realizing the Quantum Anomalous Hall Effect in graphene and completed the calculation of the Berry curvature and a specific example of graphene[5]. After that, many research teams [6-9] also explored and studied the possibility of Anomalous Hall Effect in two-dimensional materials, until 2013, when the Institute of Physics, Chinese Academy of Sciences and the Department of Physics, Tsinghua University successfully observed Quantum Anomalous Hall Effect experimentally.

In 2010, the group completed the growth and transport measurements of 1 nm-6 nm thick thin films, thus making the growth measurements of quasi-two-dimensional ultra-thin films possible; in 2011, the group achieved the precise regulation of the energy band structure of topological insulators; in 2012, the group achieved the self-tuning of the energy band structure in quasi-two-dimensional insulators. In 2012, the group achieved spontaneous long-range ferromagnetism in a quasi-two-dimensional insulator, and used an applied gate voltage to precisely regulate the electronic structure in situ; in October 2012, the group observed that the anomalous Hall resistance of the material in a zero magnetic field reached the characteristic value of the Quantum Hall Effect under a certain applied voltage, thus announcing that the challenge of how to achieve the Quantum Anomalous Hall Effect in experiments was overcome. This result was published in Science magazine in March 2013, which officially represents the experimental discovery of the Quantum Anomalous Hall Effect.

### 3.4. Quantum Spin Hall Effect

In 2005, the Quantum Spin Hall Effect was proposed in a single-layer graphene sample by Kane and Mele et al. at the

University of Pennsylvania, USA [10]. However, in 2007, the work of Yugui Yao et al. at the Institute of Physics, Chinese Academy of Sciences showed that it is difficult to observe the Quantum Spin Hall Effect in graphene because the energy gap opened by spin-orbit coupling in graphene is only on the order of 1 μeV [11].

In 2006, Shousheng Zhang's group at Stanford University proposed that the Quantum Spin Hall Effect could be achieved in the HgTe/CdTe quantum well system [12], and this conclusion was experimentally confirmed by Laurens Molenkamp's group in Germany in 2007 [13].

# 4. Recent Research Progress

## 4.1. Quantum Hall Effect

In July 2022, Qingxin Li et al [14] fabricated a bilayer graphene/boron nitride heterojunction using a dry transfer technique. Under a strong magnetic field, they observed incompressible states accompanied by quantized Hall conductance with Landau energy level filling factors of -5/2, -1/2, and 3/2 . Experimentally, the quantum Hall state features strengthen and then weaken as the magnetic field is enhanced, corresponding to the polarization of the wave function of the Landau energy level. Such results imply that these observed even-denominator fraction quantum Hall states belong to the topological states described by the Pffafian wave function, providing a platform for future studies to perform interference experiments to test the non-abelian statistical properties.

## 4.2. Quantum Anomalous Hall Effect

In December 2021, Yanxin Li et al. reported the Quantum Anomalous Hall Effect observed in AB stacked $MoTe_2/WSe_2$ molar bilayers, paving the way for the discovery of Quantum Anomalous Hall Effect arising from the combined effect of strong correlation and topology in semiconductor Mohr materials [15]. In March 2022, Kang Zhang et al. discovered that the monolayer $NbTe_2$ transforms from a quantum spin Hall insulator to a Quantum Anomalous Hall insulator with increased electron correlation during the CDW phase by combining nontrivial topology and charge density waves (CDW). This finding established $NbTe_2$ as a promising candidate for exploring non-trivial topologies, electronic correlations, and exotic quantum states at the CDW intersection [16]. In the same month, Na Yang et al. predicted the existence of Quantum Anomalous Hall Effect in $Cr_2NF_2$ by first-nature principle. In addition, the biaxial strain in the a-b plane leads to an abundance of topological phases in the absence of SOC and an effective four-band tight-binding model is constructed to elucidate the origin of WF in the nontrivial band topology. The study also calculated the Berry curvature and anomalous Hall conductivity of $Cr_2NT_2$ and found them to be nonzero energy points around the Fermi energy level, indicating that monolayers of $Cr_2NT_2$ are promising candidates for spintronics applications [17]. The Anomalous Hall Effect was characterized. Also, by measuring the size-dependent breakdown currents, they confirmed that the chiral edge states are confined to the physical boundary with widths on the order of Fermi wavelengths [18].

## 4.3. Quantum Spin Hall Effect

In March 2022, Hairui Bao et al. found that Po has a three-layer structure with different valence states in the middle and two outer layers. Because this unique multivalent behavior of Po atoms favors the structural stability of the monolayer, it is considered as an intrinsic Quantum Spin Hall insulator with a large band gap [19]. In June 2022, Wang Xing et al. found a family of two-dimensional (2D) transition metal sulfidesMX5 can undergo Quantum Spin Hall Effect based on the First Principle[20].

# 5. Applications of Phenomena

## 5.1. Hall Effect

In the 150 years since the discovery of the Hall Effect, people have used the principle of the Hall Effect to develop a variety of Hall components for applications in automation control, communications, computing, and other fields. In medicine, the electromagnetic flowmeter made according to the Hall Effect can be used to measure the blood flow rate and blood flow during surgery. In daily life, the switch made according to the Hall Effect is used in household appliances such as rice cookers and refrigerators. In industry, the current strength can be measured according to the Hall Effect principle, and the current of DC, AC, or various waveforms can be directly obtained by amplifying the Hall voltage. In short, Hall Effect has a wide range of applications because its principle is simple and easy to implement.

## 5.2. Quantum Hall Effect

The application of the Quantum Hall Effect is mainly focused on providing resistance standards for electrical metrology. Conventional Quantum Hall Effect resistance standards use GaAs/Al GaAs heterojunctions as samples, which can provide uncertainty of 10-9 and high stability. Compared to the conventional quantum Hall Resistance standards, graphene can reproduce the Quantum Hall Effect at higher temperatures and lower magnetic fields, and has higher metrology accuracy. Considering the complex circuit situation of resistive elements in practical applications, where AC or high-frequency circuits may be encountered, China has also started to study resistive standard samples adapted to the AC Quantum Hall Effect [21]. In recent years, with the advancement of device preparation process, quantum resistance standards are developing toward miniaturization and low cost [22].

## 5.3. Quantum Anomalous Hall Effect

In the normal state, the electrons in electronic components do not have specific orbits and will collide due to irregular motion and thus lose energy. The Quantum Hall Effect is equivalent to setting a "rule" for the motion of electrons to improve the orderliness of electrons and reduce their collisions, thus reducing losses and improving the efficiency of electronic components. However, the usual Quantum Hall Effect requires the application of a very strong magnetic field, which is bulky and expensive. The Quantum Anomalous Hall Effect allows materials to achieve a quantum Hall state in a zero magnetic field, which is important for the reduction of losses in electronic devices needed in daily life. The Quantum Anomalous Hall Effect can help to break the bottleneck of Moore's law, and thus promote the progress of the information technology revolution.

## 5.4. Quantum Spin Hall Effect

Photonic Spin Hall Effect (SHEL) is an important one of Quantum Spin Hall Effect. SHEL is commonly found at the intersection of media and can be observed at the interface of metals, crystals, semiconductors and other materials, which is an important factor in designing optical experiments and optical devices [23]. The existence of Quantum Spin Hall Effect makes it possible to observe the internal Bing spin Hall phenomenon even in non-magnetic materials, and we can use this effect to generate spin flow, which will have a wide range of applications in the field of spintronics [24].

## 6. Summary and Outlook

Both the simple Hall Effect and the Hall Effect of quantum level are important physical phenomena. In-depth study of the Hall Effect at the quantum level in the future has irreplaceable significance in the condensed matter direction. For the quantum Hall Effect, we need to search for better reproducible materials for the phenomenon and its better application to resistive standards. Meanwhile, the development of quantum Hall resistor arrays is also an important development direction for quantum Hall resistors. For the Quantum Anomalous Hall Effect, how to break through the temperature bottleneck of the effect realization and how to find better material system to make the Quantum Anomalous Hall Effect more widely used in the future are what we are expected to think and research. For the Quantum Spin Hall Effect, the future of its more in-depth application can be of great significance in spintronics.

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