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The Formation of the Teapot Effect under Different Forces

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Abstract: The phenomenon known as the "teapot effect", which refers to the movement of fluid along the outer wall of a teapot, has garnered significant attention in recent decades within the field of scientific research. Various methods, including the momentum balance method and the hydrodynamic equations, have been utilized to investigate this phenomenon. In this study, we employ a simplified model to examine the influence of different forces on the fluid as it moves along the solid surface. These forces include the adhesive forces between the solid surface and the fluid, the coupling force within the fluid, and gravity. Additionally, we explore the conditions under which the fluid can separate into distinct layers during the teapot effect. Some layers of the fluid may adhere to the outer wall of the teapot, while others may be expelled. The occurrence of this phenomenon is determined by the relative magnitudes of the different forces acting on the fluid. Furthermore, we analyze and calculate the effect's dependence on velocity. Through our calculations, we contribute to a deeper understanding of how these forces impact the teapot effect.

Keywords: Fluid, Teapot effect, Hydrodynamics, Mechanics, Classical physics.

1. Introduction

In the field of science and engineering, fluid flow is an extremely important and widely studied area. Understanding fluid flow plays a crucial role in the development of fields such as aerodynamics, ocean and atmospheric science, biology, chemical engineering, and aerospace technology. Simulation and understanding of fluid flow are essential in these fields. However, different shapes of containers and complex internal structures often give rise to a series of peculiar effects on fluid flow. One significant effect is known as the "Teapot Effect" [1–3]. The Teapot Effect refers to a phenomenon where the fluid flowing out of a curved and tilted container exhibits a peculiar behavior, seemingly flowing against the tilt. Although this phenomenon has been widely observed, the underlying mechanisms of fluid flow behind it have remained a challenging scientific problem.

To explain and predict the Teapot Effect, researchers have proposed various models and theories. Early theories attempted to describe fluid motion using complicated mathematical models. However, due to the involvement of fluid-container surface contact and interaction, these models were excessively complex and difficult to solve. With the rapid development of computer science and numerical simulation, researchers have begun using Computational Fluid Dynamics (CFD) methods to simulate the Teapot Effect.

In the current field, several models and simulation results regarding the Teapot Effect have been developed. Some of these models are based on fundamental principles such as the Navier-Stokes equations [4–6], turbulence models [7–9], and boundary conditions involving surface contact [10, 11]. Since the 1950s, the teapot effect has been extensively studied by different groups. It can be treated as a purely hydrodynamic process [12]. It can also be investigated via the principles of conservation of mass and momentum balances [13], taking into consideration the influence of inertia. This approach proved to be effective and provided a simplified yet semi-quantitative method for studying the phenomenon. More recently, the group led by E. Jambon-Puillet employed the momentum balance method to explore the formation of liquid helixes resulting from the teapot effect [14]. Additional research, such as that conducted by the Duez Group, has indicated that capillary effects and surface wettability play significant roles in the teapot effect [15].

These models focus on analyzing the mechanical characteristics of the flow and attempt to simulate the generation and evolution of the Teapot Effect using numerical methods. Other studies approach the topic from an experimental standpoint, using extensive observation and measurement data to validate the accuracy of the models. These models and experiments provide important references and foundations for our understanding of the Teapot Effect in fluid flow.

Nevertheless, despite the existing models and experimental research on the Teapot Effect, many unresolved issues remain. For example, the causes and mechanisms of the Teapot Effect have not been fully explained. Additionally, the current models and experiments have not sufficiently accounted for the microscale interaction between the fluid and the container surface. In this work, we try to analyze the teapot effect via another aspect by building a toy model focusing on the interactions between the layers of the fluid and the interactions between the fluid and the teapot surface. This will help to reveal the connections between the teapot effect and the interaction strength between the fluid and the teapot surface.

In this study, we explore various forces, including the adhesive forces between the fluid and the solid surface, the internal coupling strength within the fluid, the influence of gravity, and the impact of teapot thickness on the motion of liquid pouring out of a container. While our investigation does not encompass a comprehensive analysis of the teapot effect, we aim to approach the phenomenon using fundamental principles from college physics. We examine the effects of different force magnitudes on the fluid and discuss how the fluid behaves under various initial velocities. We also calculate the conditions under which the teapot effect can be observed. Additionally, we investigate the occurrence of layer separation within the fluid, where certain layers move along the outer wall of the container while others are ejected due to centrifugal force [16].

2. Ptheoretical Methods To Analyse the Teapot Effect

In recent years, extensive research has been conducted on the teapot effect, resulting in the development of numerous theoretical models. The underlying mechanics of the teapot effect are highly complex, involving intricate interactions between the fluid and the solid surface of the teapot, as well as interactions within the fluid itself [17, 18]. Typically, this phenomenon is investigated using the Navier-Stokes equations, complemented by appropriate boundary conditions. However, solving this problem completely remains a challenging task with significant difficulty for readers to understand.

In this study, we utilize Newton's laws to analyze fluid behavior, providing a reader-friendly approach suitable for college-level readers. The thickness of the fluid is denoted as e0, as shown in Figure 1. It flows over the solid surface of the teapot mouth with a thickness of r0, while the velocity of the fluid is represented by u.

One crucial factor contributing to the teapot effect is the interaction force between the fluid and the teapot. This force comprises both adhesive and capillary forces, whose magnitudes are closely associated with the properties of the teapot material and the liquid being used. Factors such as wetted area and surface wettability of the teapot influence the magnitude of these forces. However, determining the exact formula for the forces between the liquid and the teapot is a highly complex task that exceeds the scope of this study. Instead, we examine the conditions under which the teapot effect is observed by varying the values of these forces. For simplicity, we refer to all forces between the liquid and the solid surface as "adhesive forces". These forces act as part of the centripetal force, pulling the liquid toward the solid surface. Due to the small size of the teapot mouth, any change in fluid velocity as it moves along the solid surface can be neglected. Consequently, the adhesive forces only alter the direction of the fluid velocity vector ˙u, while their effect on ˙u itself can be safely disregarded.

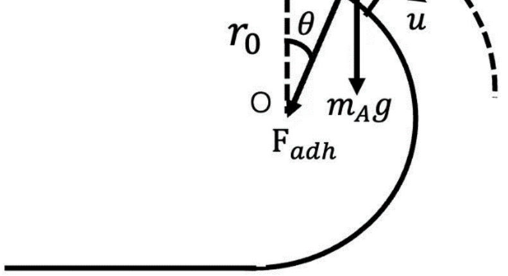

Fig. 1 Phase Schematic figure to show a period of liquid flowing over the solid surface of the teapot. The thickness of the teapot is characterized by the radius r0. The thickness of the fluid is e0. The velocity of the fluid is u. The fluid suffers adhesive force Fadh from the solid surface and also gravity mg.

In Figure 1, it can be observed that a portion of the liquid may either eject away from the teapot surface or move along it. In the subsequent calculations, we examine scenarios without and with the influence of gravity on the liquid, represented by Eq. (1) and Eq. (2), respectively. In these equations, the adhesive force serves as the centripetal force. In the scenario where the adhesive force is just sufficient to maintain the motion of the fluid cell along the teapot surface, the forces satisfy the following relation, both without and with considering gravity:

$$\mathrm{F_{adh}} = \mathrm{m}\frac{\mathrm{u}_{\mathrm{max1}}^2}{\mathrm{R}} = \mathrm{m}\frac{\mathrm{u}_{\mathrm{max1}}^2}{(\mathrm{r_0}+{}^{\mathrm{e_0}}/_2)} \tag{1}$$

$$\mathrm{F_{adh}} + mg\cos(\theta) = m\frac{u_{max1}^2}{R} = m\frac{u_{max2}^2}{(r_0+{}^{e_0}/_2)} \tag{2}$$

In Eq. (1), the centripetal force arises solely from the interaction force between the teapot surface and the liquid. However, in Eq. (2), the centripetal force is additionally influenced by gravity when the angle θ is smaller than π/2. The adhesive force F_{adh} acting on the liquid droplet is primarily determined by the teapot material and the properties of the liquid, and is generally independent of the fluid velocity u. Therefore, we can consider this force as a parameter. By fixing the value of F_{adh}, one can calculate the maximum velocity of the liquid beyond which it will eject from the solid surface. In the cases without and with the influence of gravity, these maximum velocities are denoted as umax1 and umax2, respectively. Due to the contribution of gravity, when the liquid ejects from the solid surface at an angle θ < π/2, it can be observed that umax2 > umax1. This signifies that the presence of gravity increases the maximum velocity of the liquid.

As discussed before, gravity only changes the direction of the fluid velocity, and its effect on the value of ˙u has been neglected. Solve the Eq.(1,2), we can get the results,

$$u_{max1} = \sqrt{\frac{F_{adh}(r_0+{}^{e_0}/_2}{m}} \tag{3}$$

$$u_{max2}\,(\theta) = \sqrt{\frac{(F_{adh}+mg\cos(\theta))(\mathrm{r_0}+\mathrm{e_0}/2)}{m}} \tag{4}$$

Two velocities correspond to the scenarios both without and with the presence of gravity. In this study, various values of the interaction forces and gravity will be considered in the calculations.

When the liquid is poured out, we gradually increase the velocity of the fluid until it eventually ejects away from the teapot surface due to inertia. As the angle increases, the term mg cos(θ) decreases. At a critical value of φ, the value on the right-hand side of Eq. (2) surpasses the value on the left-hand side. The critical angle can be expressed as:

$$\emptyset = \mathrm{acrccos}\left[\frac{\mathrm{m}\frac{\mathrm{u_c^2}(\emptyset)}{(\mathrm{r_0}+{}^{\mathrm{e_0}}/_2)}-\mathrm{F_{adh}}}{\mathrm{mg}}\right] \tag{5}$$

The liquid will eject away at $\theta \geq \emptyset$.

2.1. Layer separation in the teapot effect

There may be additional interesting phenomena occurring, such as some liquid close to the teapot surface flowing over the outer wall of the teapot surface, while other layers of the fluid eject away from the solid surface due to inertia. To analyze this layer separation phenomenon, we consider the fluid as a two-layer system, as shown in Fig. 2. The methodology and conclusions can be easily extended to a multi-layer fluid system. When the adhesive force between the fluid and the solid surface is relatively strong, while the force between layer-A and layer-B is weak, layer-A can be attracted to the solid surface while layer-B ejects away at a certain angle. In this section, we will investigate the conditions under which this phenomenon occurs. The ejection angle of layer-B is influenced by factors such as the geometry of the teapot spout, the fluid velocity, the relative magnitude of the adhesive force, and the coupling force within the liquid.

In Fig. 2, we denote the adhesive force between layer-A and the solid surface as FOA. There is also an attractive force FAB between layer-A and layer-B. The magnitude of FOA is typically determined by the wettability of the teapot material, while FAB is influenced by the coupling strength within the liquid. In the limit of an ideal gas, FAB approaches zero. When FOA is significantly greater than FAB, the layer separation phenomenon may occur in the teapot effect.

As the precise calculation of FOA and FAB is highly complex, we will utilize a simplified relation in our calculations, such as FOA = C × FAB, where C > 1 is a constant factor. The phenomenon of layer separation will be observed

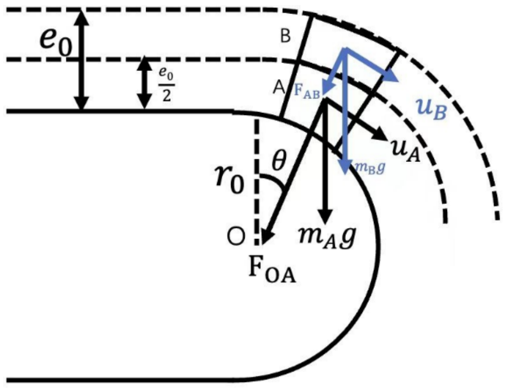

FIG. 2 Schematic figure to show a drop of fluid moving with a velocity ˙u. The liquid is treated as two-layer system to study the layer-sepration phenomenon in the teapot effect. The mass of the layer-A and the layer-B are the same, $m_A = m_B = m$.

When the following conditions are met:

$$F_{AB} + m_B g\cos(\theta) \le m_B \frac{u^2}{r_0+3e_0/4} \tag{6}$$

$$F_{OA} - F_{AB} + m_A g\cos(\theta) \ge m_A \frac{u^2}{r_0+{}^{e_0}/_4} \tag{7}$$

Assuming that the masses of the two layers are equal, $m_A = m_B = m$, we can calculate the angle at which the layer separation occurs for a given velocity u. The relationship for the ejection angle is given by:

$$\frac{1}{g}\frac{u^2}{r_0+{}^{e_0}/_4} - \frac{C}{C-1}\frac{F_{OA}}{mg} \le \cos(\theta) \le \frac{1}{g}\frac{u^2}{r_0+{}^{3e_0}/_4} - \frac{1}{C}\frac{F_{OA}}{mg} \tag{8}$$

$$\arccos\left[\frac{1}{g}\frac{u^2}{r_0+{}^{e_0}/_4} - \frac{C}{C-1}\frac{F_{OA}}{mg}\right] \ge \theta \ge \arccos\left[\frac{1}{g}\frac{u^2}{r_0+{}^{3e_0}/_4} - \frac{1}{C}\frac{F_{OA}}{mg}\right] \tag{9}$$

Where θ is located in 0 and 3.14. If there exists a specific value of θ that satisfies the aforementioned conditions, the layers will separate. The smallest value of θ that satisfies these relations will indicate the angle at which layer-B ejects while Layer-A remains attached to the solid surface.

To analyze the contribution of each force in the teapot effect, we will now consider a scenario where the teapot is made of a hydrophilic material. The interaction between the fluid and the teapot is much stronger compared with the coupling strength within the fluid. We will use the relation FOA = C × FAB with C = 10. The radius that characterizes the thickness of the teapot spout is assumed to be the same as the thickness of the fluid, denoted as e0 = r0 with r0 = 0.002 m. The meanings of e0 and r0 are explained in Fig. 1-2. In this case, the teapot spout is very thick and the fluid strongly adheres to the solid surface. When the fluid is poured out at different velocities u0, the lower and upper limits of the angle θ in Eq.(7) are plotted in Fig. 3. To account for the influence of gravity, we consider two cases: mg = 2FOA and mg = 10FOA.

In the left panel of Fig. 3, it can be observed that when the interactions between the teapot surface and the liquid are very strong, the relation in Eq. (9) is always satisfied at different fluid velocities during pouring. For instance, at a fluid velocity of u0 = 0.1m/s2. This demonstrates the occurrence of layer separation in the teapot effect. As the fluid velocity u decreases, the centrifugal forces acting on both layer-A and layer-B become smaller. Consequently, the layer-B is ”pulled” away from layer-A by gravity when θ > π/2. The ejection angle of layer-B becomes larger than π/2.

However, when gravity becomes more significant compared to forces such as FOA and FAB, the ejection angle of layer-B becomes very large. This means that at lower fluid velocities, for angles θ < π/2, gravity tends to pull both layer-A and layer-B towards the solid surface. For angles θ > π/2, gravity tends to pull layer-B away from layer-A, while layer-A remains adhered to the solid surface. As the fluid velocity u increases, both layers may be ejected together or remain stuck together without layer separation. Fig. 3 with different values of gravity also demonstrates

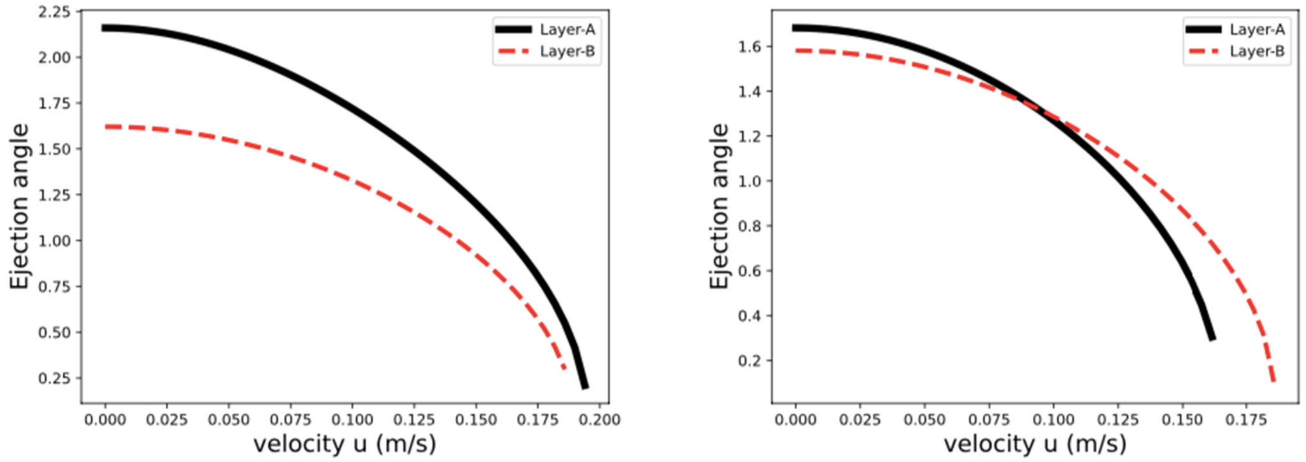

FIG. 3 The ejection angles of layer-A and layer-B in the fluid are plotted as a function of the fluid velocity u. In this analysis, the force between the fluid and the solid surface of the teapot, FOA, is assumed to be ten times greater than the forces within the fluid, FAB . This implies that the teapot is made of a hydrophilic material. The thickness of the teapot spout is assumed to be the same as the thickness of the fluid, i.e., $r0 = e0$. To account for the influence of gravity in the teapot effect, two different values of gravity are considered: $mg = 2FOA$ in the left panel and $mg = 10FOA$ in the right panel.

that gravity can influence layer separation in the teapot effect, which has not been extensively discussed in previous references.

When the thickness of the teapot spout is very small, as in the case of $r_0 = e0/5$, the centrifugal forces acting on layer-A and layer-B of the fluid become very large. Moreover, the centrifugal force on layer-A is even greater. This implies that layer-A tends to be ejected along with layer-B, which is not conducive to the phenomenon of layer separation. In Fig. 4, we have plotted the lower and upper limits of θ as given by Eq. (9). Compared with the previous Fig. 3, the range in which layer separation can be observed becomes smaller in Fig. 4. Layer separation can only be observed at a very large angle $\theta > \pi/2$ when the fluid velocity is very small. The comparison between the left and right panels in Fig. 4 indicates that gravity still plays a significant role in layer separation in the teapot effect.

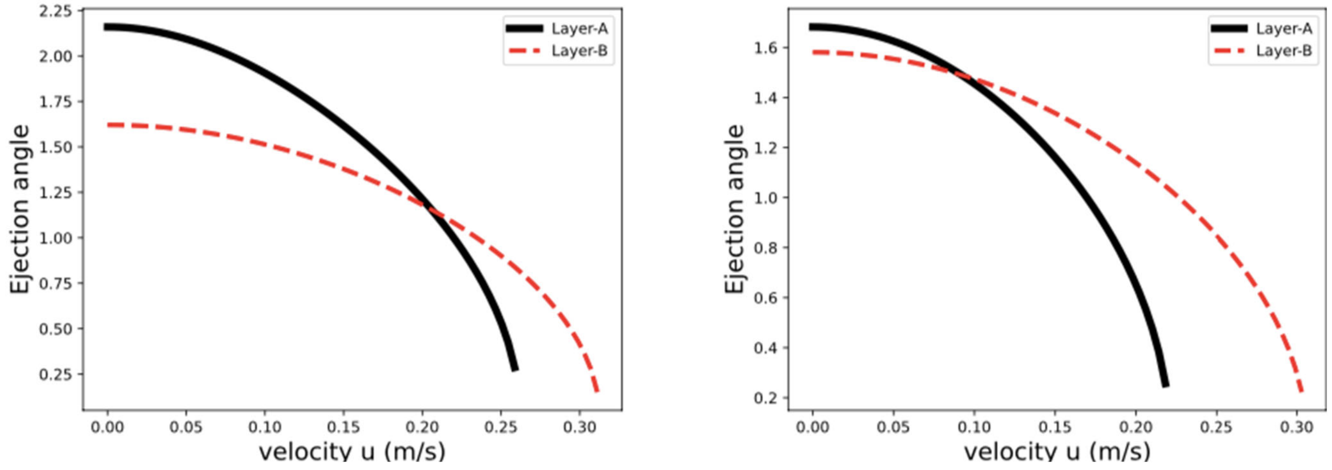

FIG. 4 The ejection angles of layer-A and layer-B in the fluid are plotted as a function of the fluid velocity u. In this analysis, the force between the fluid and the solid surface of the teapot, FOA, is assumed to be ten times greater than the forces within the fluid, FAB . This implies that the teapot is made of a hydrophilic material. The radius of the teapot spout, which characterizes its thickness, is taken as $r0 = e0/5$. To account for the influence of gravity in the teapot effect, two different values of gravity are considered: $mg = 2FOA$ in the left panel and $mg = 10FOA$ in the right panel.

In the final part, we will investigate the contribution of adhesive forces. When the teapot is made of a hydrophobic material, the adhesive force from the solid surface is small compared to the coupling strength of the fluid. As a result, layer-A and layer-B of the fluid tend to remain coupled together. The parameter $C = F_{OA}/F_{AB}$ ccharacterizes the relative magnitude of the liquid-teapot force and the liquid-liquid force (the force between layer-A and layer-B). By varying the value of C, we can observe how layer-B of the fluid ejects away from the teapot. In Fig.5, we consider gravity satisfying the relation $mg = (0.5, 0.7, 1)F_{OA}$. The corresponding ejection angle of layer-B at layer separation is plotted in Fig. 5. As the interaction force between the liquid and the teapot becomes stronger, the ejection angle of the liquid becomes smaller. One can observe the separation of layer-A and layer-B at $\theta < \pi/2$. Additionally, as gravity increases, the ejection angle becomes larger.

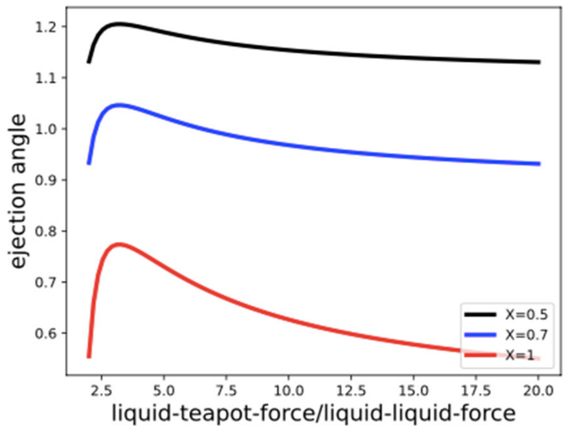

FIG. 5 The ejection angles of the layer-B at the layer separation. It changes with the ratio of the interaction force between the liquid and the teapot surface and the force between layer-A and layer-B. The radius of the teapot mouse characterizing the thickness of the teapot is taken as $r0 = e0/5$. Different values of the gravity is taken via $X = FOA/(mg) = 0.5, 0.7, 1$.

The above analysis can be readily extended to situations involving multi-layer fluids. In such cases, one can calculate the fraction of the fluid that ejects away from the surface of the teapot. For instance, it is possible for most of the liquid to be ejected away from the teapot, while only a very small fraction of the liquid flows along the solid surface of the teapot, as commonly observed in our daily lives. To determine the ejection angle in these scenarios, a set of equations similar to Eq. (6) to Eq. (7) can be established and solved. These equations allow for the calculation of the ejection angle.

3. Conclusion

In this study, we have developed a toy model based on Newton's laws to analyze the teapot effect, specifically focusing on the phenomenon of layer separation. Our approach captures the main characteristics of the teapot effect and is particularly suitable for college-level students to grasp the underlying principles. Several crucial factors contribute to the effect, including the adhesive forces between the liquid and the teapot surface, the strength of coupling within the liquid, the geometry and size of the teapot spout, and the fluid velocity. We have discussed how these factors influence the teapot effect. Furthermore, our model can be extended to more realistic scenarios where only a small fraction of the liquid flows along the outer wall of the teapot. This model provides valuable qualitative insights into understanding the teapot effect.

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