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# Study on Multi-beam Line Measurement Problem Based on Geometric Mathematical Model

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**Abstract:** Measuring water depth has always been an important part of human exploration of the ocean. With the development of science and technology, multi-beam sounding technology has become a common means to measure the depth of the ocean. In view of the problem of multi-beam line measurement in Marine environment, this paper first makes the geometric diagram of different multi-beam depth measurement lines, and establishes a mathematical model of the coverage width and overlap rate according to the geometric relationship, so as to calculate the index values of different distances from the central point. On this basis, the slope and line direction angle are introduced to correct the cover width and establish the mathematical model of the cover width of multiple beam depth. Finally, based on the previous results and the corresponding references, a set of north-south survey lines are designed that can fully cover the whole sea area to be tested, with the overlap rate of 10%~20% and the spacing of 36.37m, and the corresponding conclusions about the multi-beam survey line and the coverage width are obtained.

**Keywords:** Multi-beam sounding geometry model, Covering the width, Overlap rate measurement line spacing.

## 1. Introduction

With the rapid development of Marine industry, Marine science and technology has begun to pay much attention to it. Human research on the ocean has never stopped, and various exploration technologies are also making continuous progress, which is beneficial for mankind to have a deep understanding of the ocean, explore and utilize Marine resources, and promote human development. When exploring the ocean, sounding technology is usually used to measure the depth of the sea floor. Single beam sounding is a commonly used water depth measurement technology. Its working principle is based on the nature of uniform propagation of sound waves in a uniform medium, emitting sound waves to the sea floor and measuring its propagation time and speed, and then calculating the depth of the sea water. However, the single beam sounding technology has its own limitations, only firing a single beam, and it needs to be encrypted in this process, resulting in poor measurement effect [1,2]. In order to improve the measurement efficiency of seabed topography, multi-beam sounding technology came into being. Compared with single beam sounding technology, multi-beam sounding can emit multiple beams at one time, and then receive the transducer to receive the sound waves returning from the seabed. One detection can get hundreds or even more water depth values in the plane perpendicular to the measuring line, with the advantages of high efficiency, wide coverage, high accuracy and automatic mapping [3,4]. According to the current development status of sounding technology, a multi-beam line measuring scheme with high efficiency, high precision and low cost, which is of great significance for the research of Marine science. Based on this, this paper will study the problem of measuring the ocean depth in a mathematical model.[5]

## 2. Data Source

The data presented in this paper are derived from http://www.mcm.edu.cn/.

## 3. Model Building and Solution

### 3.1. Establishment and solution of the model of 2 D multi-band coverage width and overlap rate

#### 3.1.1. Model establishment and analysis

Assuming that the plane perpendicular to the direction of the survey line is a diagonal line with the Angle between the horizontal plane, it is represented as a two-dimensional diagram as follows:

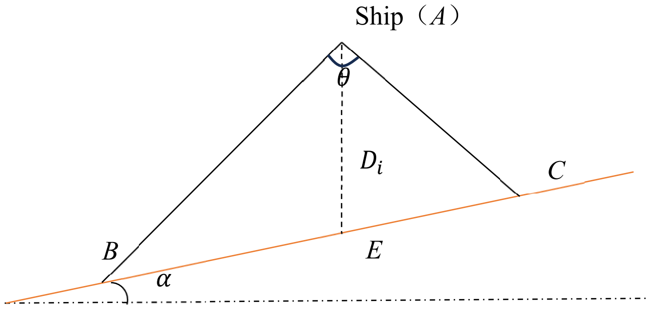

**Figure 1.** 2 D diagram of multiple beam sounding

In Figure 1, to simplify the study, this paper does not consider the influence of the actual transverse Angle[6] distance, the distance from the center distance is, make the diagram of different lines (figure 2), to establish a mathematical model of multiple beam depth coverage width and overlap rate between adjacent strips, and then calculate the index values of different positions.

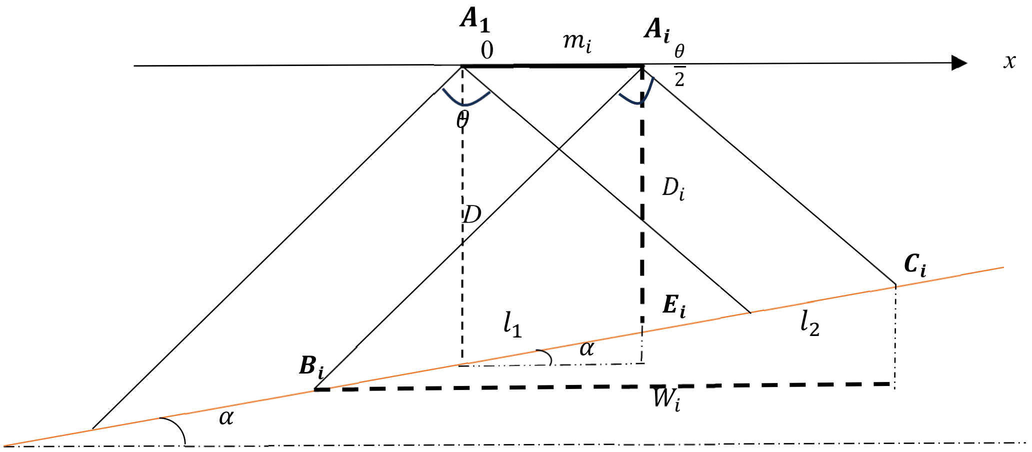

**Figure 2.** Geometric diagram of different lines of multi-beam sounding

Step1 Calculate seawater depth $D_i$

The seawater depth is that shown in Figure 2 $A_iE_i$ , There is a geometric relationship is that

$$D_i = D - m_i \cdot \tan\alpha \tag{1}$$

$D$ :Sea water depth at the central point of the sea area

$m_i$ :The distance of the line from the central point, $\alpha$ :slope,

Step2 Calculate the coverage width of the strip $W_i$

Let $B_iE_i = l_1$, $E_iC_i = l_2$ , $\Delta A_iB_iC_i$ , by the sinusoidal theorem:

$$\frac{l_1}{\sin\frac{\theta}{2}} = \frac{D_i}{\sin(90 - \frac{\theta}{2} - \alpha)} \tag{2}$$

$\Delta A_iE_iC_i$ , by the sinusoidal theorem:

$$\frac{l_2}{\sin\frac{\theta}{2}} = \frac{D_i}{\sin(90 - \frac{\theta}{2} + \alpha)} \tag{3}$$

Then, according to the geometric relationship, there are:

$$W_i = (l_1 + l_2) \cdot \cos\alpha \tag{4}$$

Integrate the above formula (2), (3) and (4) to obtain:

$$W_i = \left( \frac{D_i \sin\frac{\theta}{2}}{\sin(90 - \frac{\theta}{2} - \alpha)} + \frac{D_i \sin\frac{\theta}{2}}{\sin(90 - \frac{\theta}{2} + \alpha)} \right) \cdot \cos\alpha \tag{5}$$

$D_i$ :Water depth, $\theta$:Transducer Angle, $\alpha$ :slope;

Step3 The overlap rate of the first line and the previous line was calculated $\eta_i$

Due to the small slope given in this paper, the visual seabed topography is flat to simplify the calculation. By reviewing reference [7,8],if the ship is not considered as a particle and considering the influence of draft depth and acoustic zone, the following diagram is shown:

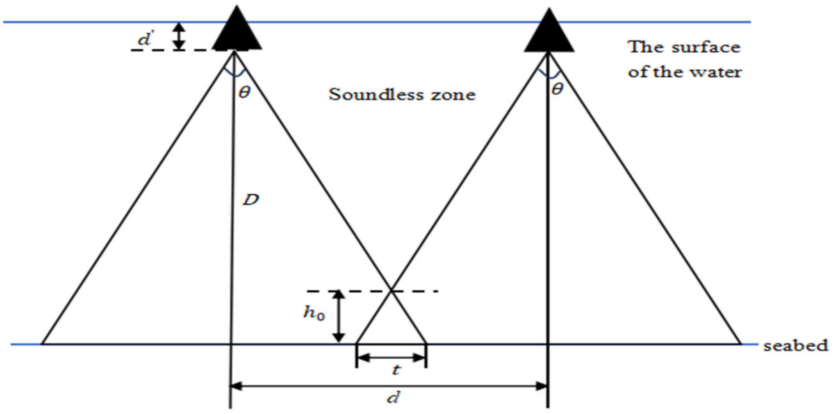

**Figure 3.** Schematic diagram of the multibeam acoustic static area

The transducer opening angle is a fixed value，Then the minimum resolved target height is shown in Figure 3, the adjacent line spacing $d$ is:

$$d=[2(D-d^{'}-h_0)]\tan\frac{\theta}{2} \tag{6}$$

$D$ :Water depth, $d^{'}$ :The water depth of the ship, $h_0$ :Minimum distinguishable heightTherefore, the overlapping coefficient $\eta_i$ :

$$\eta_i=\frac{t}{d}=\frac{2h_0\tan\theta}{[2(D-d^{'}-h_0)]\tan\theta}=\frac{h_0}{D-d^{'}-h_0} \tag{7}$$

$t$ :Overlap width， $d$ :Measure the line spacing

This paper assumes that the spacing of the measurement lines is the same, and the error obtained by formula (6) is large, so the definition, the overlap rate:

$$\eta_i=\left(1-\frac{d}{W_i}\right)\times100\% \tag{8}$$

$d$ :Spacing between the two adjacent test lines, $W_i$ :The coverage width of the article $i$ measurement line.

### 3.1.2. Result

According to the model established in 3.1.1. Meanwhile, assuming the center point is 0, the corresponding seawater depth is 70m, and the distance between the center point is 200,400,600,800. The results are shown in Table 1:

Table 1. Calculation results of coverage width, seawater depth and overlap rate changing with the distance at the center point of the test line

| Distance of the line from the central point/m | -800 | -600 | -400 | -200 | 0 | 200 | 400 | 600 | 800 |
|---|---|---|---|---|---|---|---|---|---|
| Depth/m | 90.95 | 85.71 | 80.47 | 75.24 | 70.00 | 64.76 | 59.53 | 54.29 | 49.05 |
| Cover width/m | 315.06 | 296.91 | 278.77 | 260.63 | 242.49 | 224.34 | 206.20 | 188.06 | 169.92 |
| The rate of overlap with the previous test line/% | — | 34.64 | 30.52 | 25.84 | 20.50 | 14.32 | 7.10 | -1.45 | -11.74 |

Note: ①The results retain two decimal places; ②The overlap rate is less than 0.

③$\theta=120^{d}$，$\alpha=1.5^{d}$，$D=70$，$d=200$

## 3.2. Model establishment and solution of multi-band coverage width and overlap rate based on slope and line direction Angle

### 3.2.1. Model establishment and analysis

On the basis of section 3.1, considering a rectangular sea area to be measured, the study dimension is extended to 3 d, setting the projection angle between the measurement line direction and the normal direction of the seabed slope in the horizontal plane, so as to establish a mathematical model of multi-beam depth coverage width.

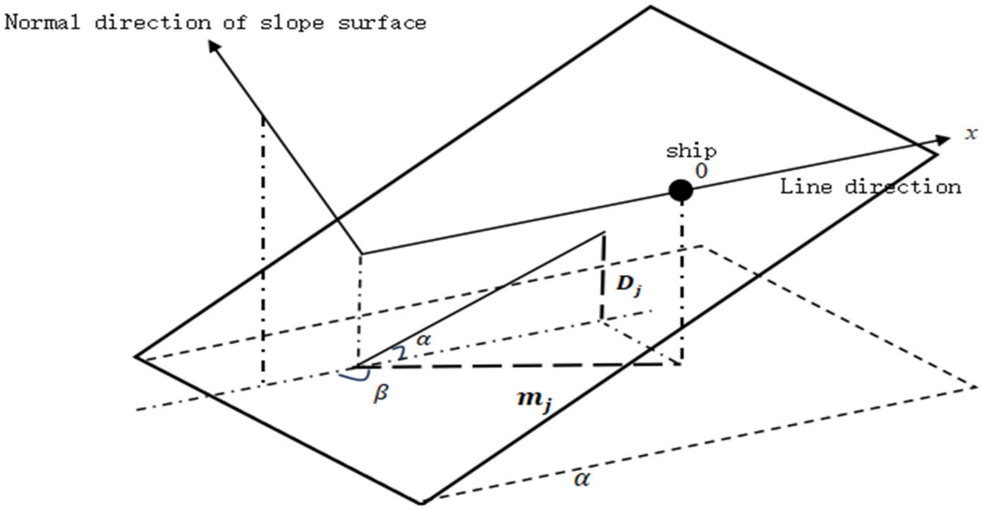

**Figure 4.** 3 D geometric diagram of multiple beam analysing

Step1 Calculate the coverage width of the strip $W_j$

Based on the analysis of 3.1, the coverage width can be expressed as:

$$W_j = \left( \frac{D_j \sin\frac{\theta}{2}}{\sin(90-\frac{\theta}{2}-\alpha)} + \frac{D_j \sin\frac{\theta}{2}}{\sin(90-\frac{\theta}{2}+\alpha)} \right) \cdot \cos\alpha \quad (9)$$

$D_j$:Water depth, $\theta$:Transducer Angle, $\alpha$ :slope.

Step2 Calculate seawater depth $D_j$

Due to the influence of the slope and the direction angle of the line, the seawater depth is slightly different from the solution of section 3.1, which is corrected by the geometric relationship in Figure 4. There are:

$$D_j = (D - m_j \tan\alpha) - d\cos(\pi - \beta)\tan\alpha \quad (10)$$

$D$ :Sea water depth at the central point of the sea area, $\alpha$ :slope,

$d$ :Spacing between the two adjacent test lines.

$\beta$ :The projection Angle between the normal direction and the seabed slope in the horizontal plane

In summary, the coverage width is integrated as follows:

$$W_j = \left( \frac{[D - m_j \tan\alpha - d\cos(\pi-\beta)\tan\alpha]\sin\frac{\theta}{2}}{\sin(90-\frac{\theta}{2}-\alpha)} + \frac{[D - m_j \tan\alpha - d\cos(\pi-\beta)\tan\alpha]\sin\frac{\theta}{2}}{\sin(90-\frac{\theta}{2}+\alpha)} \right) \cdot \cos\alpha \quad (11)$$

$\theta$:Transducer Angle, $\alpha$ :Slope

### 3.2.2. Result

According to the above mathematical model, the specific results are shown in Table 2:

**Table 2.** 3.2 Example calculation results

| Cover width/m | | Measure the distance / nautical mile of the ship from the center point of the sea area | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0.3 | 0.6 | 0.9 | 1.2 | 1.5 | 1.8 | 2.1 |
| Measure the direction of the line /° | 0 | 434.73 | 384.23 | 333.72 | 283.22 | 232.72 | 182.21 | 131.71 | 81.21 |
| | 45 | 429.40 | 378.90 | 328.40 | 277.90 | 227.39 | 176.89 | 126.39 | 75.88 |
| | 90 | 416.55 | 366.05 | 315.54 | 265.04 | 214.54 | 164.04 | 113.53 | 63.03 |
| | 135 | 403.69 | 353.19 | 302.69 | 252.19 | 201.68 | 151.18 | 100.68 | 50.17 |
| | 180 | 398.37 | 347.87 | 297.36 | 246.86 | 196.36 | 145.86 | 95.35 | 44.85 |
| | 225 | 405.79 | 356.68 | 301.73 | 251.91 | 203.82 | 156.74 | 103.79 | 50.98 |
| | 270 | 414.76 | 363.13 | 317.89 | 263.28 | 218.38 | 162.94 | 116.95 | 66.27 |
| | 315 | 427.97 | 376.83 | 329.76 | 275.06 | 229.38 | 178.19 | 127.43 | 78.19 |

Note: 1)Both of the results are kept in two decimal places.

$\theta = 120^{d}$, $\alpha = 1.5^{d}$, $D = 120$

## 3.3. Design of the shortest measuring line of the measuring length

### 3.3.1. Model establishment and analysis

Suppose a rectangular sea area with a length of 2 nautical miles from north to south and 4 nautical miles wide from east to west, the seabed slope is 1.5 (deep and shallow in west and east), and the depth of the sea water at the central point of the sea area is 110m. Now the survey line with the shortest length is measured according to the design of this sea area, as shown.in.Figure5:

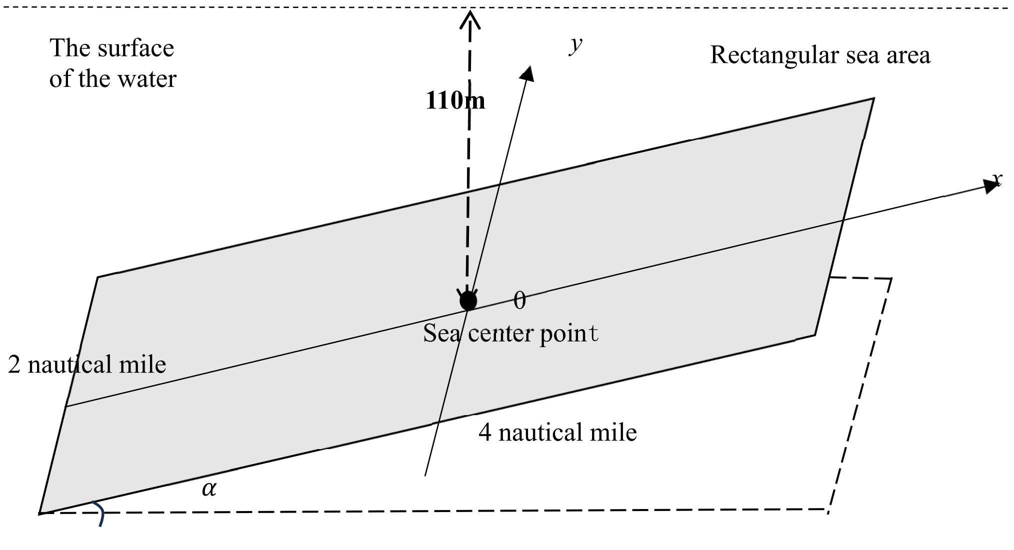

**Figure 5.** Schematic diagram of the restricted sea area

First, the north-south survey line is selected for analysis. In addition, the measurement line spacing should be appropriate: the excessive spacing will not cover the whole sea area to be measured, creating blind spots leading to the target missed test; overdensity will increase the workload, resulting in the waste of manpower and material resources.

The following figure 6 is the flow chart of the design measurement line:

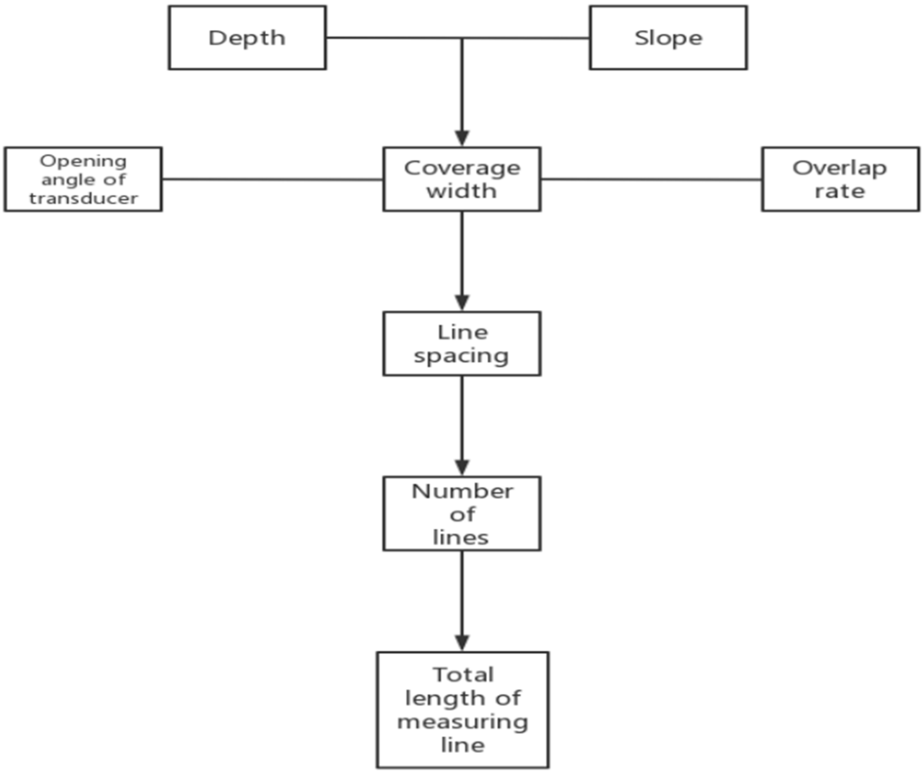

**Figure 6.** Flow chart of multi-beam measuring line layout

Step1 Calculate coverage width $W_k$

By consulting the reference [9,10], there are relevant measurement specification requirements: the measuring line of the multi-beam sounding system should be in the same direction as that of the isotonic line, which is mainly reflected in the downstream direction layout in this question. The width of multi-beam scan is related to the depth of sea water. The deeper the depth, the wider the width of the sweep. Therefore, the spacing of multi-beam test lines should consider the influence of water depth.

The depth at the four boundary angles of the sea area is first calculated( $D_k$ ), according to the known conditions of FIG. 6, Then calculate the coverage width $W_k$ , The analysis process is the same as that in 3.1, so there are:

$$D_k = D - m_k \cdot \tan\alpha \tag{12}$$

$$W_k = \left( \frac{D_k \sin\frac{\theta}{2}}{\sin(90 - \frac{\theta}{2} - \alpha)} + \frac{D_k \sin\frac{\theta}{2}}{\sin(90 - \frac{\theta}{2} + \alpha)} \right) \cdot \cos\alpha \tag{13}$$

$D$ :Sea water depth at the central point of the sea area

$m_k$ :The distance between the measuring line and central point

$\alpha$ :Slope, $\theta$:Transducer Angle

Another: Take positive integer $k$ , Represents the article k line.

Step2 Calculate the spacing of the adjacent test lines

When the open Angle of the transducer is constant, the larger the spacing between adjacent lines, the smaller the overlap rate, that is, the line spacing is inversely proportional to the overlap rate. By consulting the reference [11], the relationship between the test line spacing $d$ and the overlap rate $\eta_k$ is as follows:

$$d = W_k(1 - \eta_k) \tag{14}$$

$W_k$ :Cover width

It is necessary to meet that the overlap rate between adjacent bands is between 10% and 20%, and the line spacing should be as small as possible, so the maximum overlap rate is 20%, so that the measurement length can be the shortest.The minimum line spacing can be obtained by adding the formula $\eta_k = 0.2$ .

Step3 Calculate the number of measured lines $n$

The number of measuring lines can be analyzed by the east-west width of the rectangular sea area and the spacing of the measuring lines:

$$n = \frac{4 \times 1852}{d} + 1 \tag{15}$$

Among them, the molecular part is the value of the east-west width of the sea area into units (meters), and the denominator is the spacing of adjacent measuring lines.

Step4 Calculate the total length of the line

Because there is a measuring line of n, so there is a spacing of n-1, therefore:

$$s = d(n-1) \tag{16}$$

$n$ : Number of lines measured, $d$ :Spacing between adjacent measuring lines.

The following verifies whether the survey line can fully cover the whole sea area to be tested: the north-south survey line is distributed from east to west, and it needs to cover the whole east-west width of the rectangular sea area, that is, 4 nautical miles (7408m). If the total measurement length is greater than or equal to 7408m, the designed line scheme is reasonable; Otherwise, it should be appropriately downsized $\eta_k$ ,Then the model is added into the above model to solve until the total measurement length is greater than or equal to 7408m, indicating that the measurement line can completely cover the whole sea area to be tested and meet the constraints of the problem. In summary, the model is established as follows:

$$\min s = d(n-1)$$

$$s.t.\begin{cases} d = W_k(1-\eta_k) \\ 0.1 < \eta_k < 0.2 \\ s \geq 7408 \end{cases} \tag{17}$$

$s$ :The total length of the line, $d$ :Adjacent line spacing, $n$ :Number of lines measured, $W_k$ :Cover width, $\eta_k$ :Overlapping coffiecient

### 3.3.2. Results

According to the above model, first, substitute the known conditions into equations (12) and (13) in Figure 5, write programs, and calculate with Python software. The specific results are shown in Table 3:

**Table 3.** Calculation results of problem 3 water depth and coverage width

| Distance from the center point of the sea area/m | -2(West) | -1(South) | 0 | 1(North) | 2(East) |
|---|---|---|---|---|---|
| Depth/m | 206.00 | 158.50 | 110.00 | 61.50 | 13.00 |
| Cover width/m | 718.52 | 550.18 | 381.84 | 213.49 | 45.15 |

Note: ①Both of the results are kept in two decimal places.

② $\theta = 120^{c}$, $\alpha = 1.5^{c}$, $D = 110$

## 4. Conclusions

By abstract the practical problem into a geometric model, it is concluded that with the change of the distance between the center of the measuring line, the closer the bottom of the slope, the deeper the depth of the sea water, the greater the coverage width of the multi-beam sounding strip, and the higher the overlap rate between the measuring line and the previous measuring line. At the same time, the results are extended to the 3 D situation, and after the corresponding optimization, the coverage width decreases with the increase of the distance from the center point of the sea area. According to the design of the optimal number of measurement lines in the sea area, combined with the limited conditions given in this paper, the spacing of adjacent measurement lines is 36.37m, the number of measurement lines is 206.09 (206 here), and the total length of the measurement line is 7455.85m. According to the model analysis in 3.3.1, the total measurement length calculated is 7455.85m more than 7408m. Therefore, in this paper, a set of north-south measuring line with a spacing of 36.37m is designed, which meets the requirement of fully covering the whole sea area to be tested and the overlap rate of 10%~20%, and finds that the shortest measurement length of the measuring line is 7455.85m. In this paper, the problem of multi-beam measuring line and overlap rate is studied by establishing geometric mathematical model. The established model describes the problem accurately and skillfully to some extent, and reflects the corresponding variation law of coverage width and overlap rate in multi-beam sounding, which is easy to understand and operate.

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