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A Method for Calculating the Optical Efficiency of Solar Tower Heliostats

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Abstract: Solar tower heliostat is one of the highlights of China's new energy development in recent years, the Optical Efficiency is an important index for evaluating the field distribution of heliostat, but the calculation of Shadow Blocking Efficiency and Truncation Efficiency is more complicated. In this paper, through the ray tracing method, based on the idea of Monte Carlo algorithm, combined with the calculation method of spatial analytic geometry, a more complete and operable method of calculating Optical Efficiency is proposed.

Keywords: Tower heliostat, Light tracing method, Coordinate transformation, Monte Carlo algorithm.

1. Introduction

The Shadow Blocking Efficiency and Truncation Efficiency of heliostat mirrors have been studied many times before. As early as 1980, SASSI proposed to determine the projected area directly from the projected positions of the mirror vertices of the heliostat[1], and then to find the Shadow Blocking Efficiency, but this method is not easy to operate in programming. Later, some researches proposed the approximate calculation formulas of shadow Shadow Blocking Efficiency and Truncation Efficiency through relatively reasonable approximation[2]-[4], and Zhang et al. proposed a more accurate and more complete calculation method through the ray tracing method[5]. In this paper, on the basis of these studies, the calculation method is improved even further by comprehensively considering the incident and reflected light.

2. Device Structure and Working Principle of Heliostat

The basic component of the tower solar thermal power plant is the heliostat, and a large number of basic components constitute the heliostat field. The base of the heliostat consists of a longitudinal rotating axis and a horizontal rotating axis, which control the azimuth and pitch angles of the mirrors respectively. The horizontal axis is equipped with a plane reflector.

The heliostat converges sunlight by reflection to the collector on the absorption tower, which heats up the heat-conducting medium and converts light energy into heat energy for storage.

3. Computational Reserve

3.1. Establishment of the coordinate system

3.1.1. Mirror field coordinate system

Take the location of the absorption tower in the fixed-sun mirror field as the origin O, set the positive direction of x-axis as the east direction, the positive direction of y-axis as the north direction, and the positive direction of z-axis as the direction perpendicular to the ground upwards, and establish the coordinate system of the mirror field.

3.1.2. Mirror coordinate system

To take the outer normal direction of the mirror for the z-axis positive direction, parallel to the mirror on the surface, the lower two sides of the mirror for the mirror (along the z-axis negative direction) to the right for the x-axis positive direction, the y-axis parallel to the mirror on the left, the right two sides of the mirror, the establishment of a mirror coordinate system, as shown in the Figure 1 below.

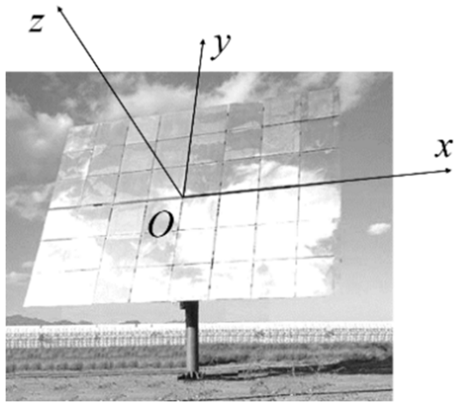

Figure 1. Mirror coordinate system

3.1.3. Sunlight-cone coordinate system

Sunlight is not a parallel ray, but a conical ray with a certain cone angle as shown in **Figure 2**, so the reflected ray of the sun's incident ray through any point of the heliostat is also a conical ray. The axis of the solar cone is called the "**main ray**".

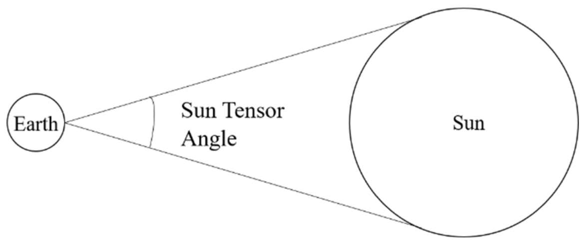

Figure 2. Sunlight cone and solar tensor angle

The *z-axis* positive direction is taken as the direction along the main ray of light pointing to the centre of the sun, the *x-axis* is parallel to the ground, and the *y-axis* positive direction can be determined by the following formula to establish a light-cone coordinate system.

$$\hat{y} = \hat{z} \times \hat{x} \tag{1}$$

Denote the angle between the rays in the cone of sunlight and the main rays as σ, and let the projection of the sunlight rays in the *xOy* plane make an angle with the *x-axis* in the positive direction as τ, as shown in the **Figure 3** below.

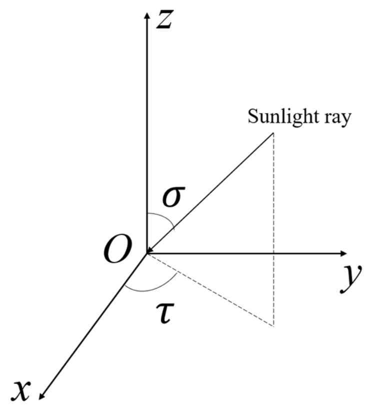

Figure 3. Sunlight-cone coordinate system

3.2. Determination of the normal vector for each heliograph

The solar altitude angle and solar azimuth angle of each point in the heliostat field are approximately equal. The control system of the heliostat field controls the normal direction of the heliostat in real time so that the light emitted from the centre of the sun is constantly directed to the centre of the collector after its reflection.

In the mirror field coordinate system, the centre point of the collector is known to be $O'(0,0,H)$, and the centre point of the mirror of the fixed-sun mirror *j* is assumed to be $O_j(x_j, y_j, h_j)$. Therefore, the direction vector of the reflected ray is $(-x_j, -y_j, H-h_j)$ and its unit vector:

$$\boldsymbol{V_{R,j}} = \left(\frac{-x_j}{\sqrt{x_j^2 + y_j^2 + (H-h_j)^2}}, \frac{-y_j}{\sqrt{x_j^2 + y_j^2 + (H-h_j)^2}}, \frac{H-h_j}{\sqrt{x_j^2 + y_j^2 + (H-h_j)^2}} \right) \tag{2}$$

The incident ray direction vector is:

$$\boldsymbol{V_{in,j}} = (\cos\alpha_s \sin\gamma_s, \cos\alpha_s \cos\gamma_s, \sin\gamma_s) \tag{3}$$

where angle α_s is the solar altitude angle and angle γ_s is the solar azimuth angle.

By the law of reflection, we can get the incident ray, the reflected ray, the normal ray in the same plane, the angle of incidence and the angle of reflection are equal. Therefore, we can get the unit normal vector of the fixed-sun mirror *j* in the mirror coordinate system as follows

$$\boldsymbol{n}_j = \frac{\boldsymbol{V_{in,j}} + \boldsymbol{V_{R,j}}}{\left|\boldsymbol{V_{in,j}} + \boldsymbol{V_{R,j}}\right|} \tag{4}$$

3.3. Coordinate system conversion

3.3.1. Coordinates of a point in the mirror coordinate system transferred to the mirror field coordinate system

The transformation matrix $\boldsymbol{T}$ from the transformed mirror coordinate system to the mirror field coordinate system is as follows:

$$\boldsymbol{T} = \begin{bmatrix} l_x & l_y & l_z \\ m_x & m_y & m_z \\ n_x & n_y & n_z \end{bmatrix} \tag{5}$$

where, (l_x, m_x, n_x), (l_y, m_y, n_y), (l_z, m_z, n_z) are the unit vector representations of the mirror *x-axis*, *y-axis* and *z-axis* in the mirror field. Obviously, (l_z, m_z, n_z) is the unit normal vector of the jth fixed-sun mirror in the mirror field coordinate system $\boldsymbol{n}_j$.

Since the *x-axis* of the mirror coordinate system is parallel to the *xOy* plane of the mirror field coordinate system, $l_z = 0$; and since (l_x, m_x, n_x) is perpendicular to $\boldsymbol{n}_j$, we can list the following system of equations:

$$\begin{cases} l_z = 0 \\ l_x^2 + l_y^2 = 1 \\ (l_x, m_x, n_x) \cdot (l_z, m_z, n_z) = 0 \end{cases} \tag{6}$$

It can be calculated as (l_y, m_y, n_y), and then the coordinates of the point in the mirror coordinate system of the fixed-heaven mirror *j* (x_j, y_j, z_j) can be converted to the coordinates of the mirror field coordinate system (x, y, z) according to the following equation.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \boldsymbol{T} \cdot \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} + \boldsymbol{O_j} \tag{7}$$

where $\boldsymbol{O_j}$ are the coordinates of the mirror centre of the heliostat *j* in the mirror field coordinate system.

3.3.2. Coordinate Representation of Vectors in the Mirror Field Coordinate System Transmirror Coordinate System

Transformation of Eq. (7) yields the formula for the transformation of vectors in the mirror-field coordinate system to for the mirror-field coordinate system.

$$\begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} = \boldsymbol{T^T} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{8}$$

where $\left(x, y, z\right)$ is the representation of the vector in the mirror field coordinate system, $\left(x_j, y_j, z_j\right)$ is the coordinate representation of the vector in the mirror coordinate system of the fixed-heaven mirror *j*, and $\boldsymbol{T}^{\boldsymbol{T}}$ is the transpose matrix of the matrix ***T***.

3.3.3. Transferring the coordinates of a point in the mirror field coordinate system to the mirror coordinate system

Rewriting Eq. (7) gives the following coordinates of the point in the mirror field coordinate system transferred to the mirror coordinate system:

$$\begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} = \boldsymbol{T}^{\boldsymbol{T}} \cdot \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \boldsymbol{O}_{\boldsymbol{j}} \right) \tag{9}$$

where $\left(x, y, z\right)$ are the coordinates of the point in the mirror field coordinate system, $\left(x_j, y_j, z_j\right)$ are the coordinates of the point in the mirror coordinate system of the heliostat *j*, and $\boldsymbol{O}_{\boldsymbol{j}}$ are the coordinates of the mirror centre of the heliostat j in the mirror field coordinate system.

3.3.4. Coordinate representation of vectors in the light-cone coordinate system transferred to the mirror-field coordinate system

The representation of the vector converted to the mirror field coordinate system in the light cone coordinate system can be obtained by the following equation

$$\boldsymbol{V}_{\boldsymbol{S}} = \boldsymbol{T} \cdot \boldsymbol{V}_{\boldsymbol{S}}' \tag{10}$$

where $\boldsymbol{V}_{\boldsymbol{S}}'$ is the direction vector of the incident light in the light cone coordinate system, $\boldsymbol{V}_{\boldsymbol{S}}$ is the direction vector of the incident light in the mirror field coordinate system, and the matrix ***T*** is as follows:

$$\boldsymbol{T} = \begin{bmatrix} \cos\gamma_s & -\sin\alpha_s \sin\gamma_s & \cos\alpha_s \sin\gamma_s \\ -\sin\gamma_s & -\sin\alpha_s \cos\gamma_s & \cos\alpha_s \cos\gamma_s \\ 0 & \cos\alpha_s & \sin\alpha_s \end{bmatrix} \tag{11}$$

3.4. Core Ideas of the Monte Carlo Algorithm

Inside a region of area *S* in the plane there is a small region of area S', now take if *N* points randomly inside the region *S*. If there are N' points inside the region S', then it can be approximated:

$$\frac{N'}{N} = \frac{S'}{S} \tag{12}$$

and the larger *N* is, the more accurate the approximation is.

4. Calculation of Optical Efficiency

The most important indicator of whether a mirror field arrangement is reasonable is the **Optical Efficiency** of the mirror field. The Optical Efficiency of the mirror field takes into account the cosine loss caused by the angle between the mirrors and the incident sun line, the attenuation loss of sunlight in the process of propagation, the reflectivity of the mirrors (including the reflective properties of the mirrors themselves and the staining of the mirrors exposed to the air), the loss of shadows caused by the mutual blocking of the fixed-sun mirrors, and the spillover of the sunlight reflected on the absorber, and other factors.

4.1. Calculation of Cosine Efficiency $\eta_{\cos}$

Since the direction of incidence of sunlight is not parallel to the normal direction of the mirror's lighting port, the effective area of light received by the heliostat will be smaller than the actual area of the heliostat. **Cosine Efficiency** is the value of the cosine of the angle between the normal direction of the mirror and the direction of incoming sunlight.

Set the direction vectors $\boldsymbol{V}_{\boldsymbol{in,j}}$, $\boldsymbol{V}_{\boldsymbol{R,j}}$ of the incident and reflected rays of the heliostat *j*. Set the angle between the incident and normal vectors of the heliostat *j* to be θ_j, then the angle between the incident and reflected rays is $2\theta_j$, and the rest of the chord values are calculated by the following formula:

$$\cos 2\theta_j = \frac{\left|\boldsymbol{V}_{\boldsymbol{in,j}} \cdot \boldsymbol{V}_{\boldsymbol{R,j}}\right|}{\left|\boldsymbol{V}_{\boldsymbol{in,j}}\right| \cdot \left|\boldsymbol{V}_{\boldsymbol{R,j}}\right|} \tag{13}$$

From the half angle formula:

$$\cos\theta_j = \sqrt{\frac{1 + \cos 2\theta_j}{2}} \tag{14}$$

The Cosine Efficiency of the fixed-sun mirror j is obtained:

$$\eta_{\cos, j} = \cos\theta_j \tag{15}$$

4.2. Calculation of Atmospheric Transmittance η_{at}

Atmospheric Transmittance is an important parameter of the space environment that strongly influences solar energy, and refers to the proportion of energy in a certain wavelength of light that passes through the Earth's atmosphere. As solar light travels through the air, it is attenuated by airborne dust, particles, etc., so that the losses in the light path from the mirror to the heat absorber are slightly different depending on the distance between the mirror and the reflection target point.

Distance from the centre of the mirror O_j to the centre of the collector $O'\left(0, 0, H\right)$ in the mirror field coordinate system of the heliostat mirror *j*:

$$d_{HR,j} = \left|O_j O'\right| \tag{16}$$

When $d_{HR,j} \leq 1000$ m is used, the relationship between Atmospheric Transmittance and d_{HR} is given by the following equation:

$$\eta_{at,j} = 0.99321 - 0.0001176 \times d_{HR,j} + 1.97 \times 10^{-8} \times d_{HR,j}^2 \quad (17)$$

4.3. Specular Reflectance η_{ref}

The **Specular Reflectance** of a heliostat j $\eta_{ref,j}$ can be measured physically or averaged over a number of heliostats as the specular reflectance of all heliostats η_{ref} .

4.4. Calculation of Shade Blocking Efficiency η_{sb}

In a mirror field the shadow of a heliostat falls into the mirror of another heliostat, or the reflected light shines onto the back of another heliostat, etc., which results in a loss of the sun's rays received by the mirror field, and a loss of shadow blocking light. In the actual tower heliostat field, neighbouring heliostats are so close that the mirrors can be approximated as parallel to each other.

The basic idea of the calculation of the **Shading Blocking Efficiency** of the fixed-sun mirror *j* is: take a number of points on the mirror, determine whether each point is "effective" (i.e., incident light and reflected light will not be blocked by the other fixed-sun mirror, the same below), according to the idea of the Monte Carlo algorithm, the effective number of points accounted for the ratio of the total number of points taken can be regarded as the Shadow Blocking Efficiency.

Here are the steps to determine whether a point *M* on the mirror of a fixed-sun mirror *j* is valid:

Step1: Select a certain heliostat *i* that is adjacent to heliostat *j*.

Step2: Represent the point $M_j\left(x_j, y_j, z_j\right)$ in the mirror coordinate system of the fixed-sun mirror *j*, change the point *M* to the coordinate $\left(x, y, z\right)$ in the mirror field coordinate system by coordinate transformation, and then change it to the coordinate $M_i\left(x_i, y_i, z_i\right)$ in the mirror coordinate system of the fixed-sun mirror *i* by coordinate transformation.

Step3: If $z_i > 0$, it means that the point *M* taken on the mirror of heliostat *j* is in front of the mirror of heliostat *i* (i.e., between the mirror of heliostat *i* and the sun), then neither the incident nor reflected rays of the point will be blocked by heliostat *i*. Go back to Step1 and select another neighbouring heliostat *i*. If $z_i < 0$, proceed to Step4.

Step4: Since the direction vector of the incident light in the mirror field coordinate system is $\boldsymbol{V_{in,j}}$ and the direction vector of the reflected light is $\boldsymbol{V_{R,j}}$, combined with the coordinates of the point *M* in the mirror field coordinate system, it is not difficult to find the two projection points of the point *M* along the incident light and the reflected light projected onto the plane where the surface of the mirror *i* is located in the fixed-heaven mirror, respectively. If one of the projection points is inside the mirror, it means that this point taken on the mirror *j* is invalid; if neither of the two projection points is inside the mirror, it means that both the incident and reflected rays of the point *M* will not be blocked by the heliostat mirror *i*. Go back to Step1 and choose another neighbouring heliostat mirror *i*.

Step5: When all the fixed-sun mirrors adjacent to the fixed-sun mirror *j* have been judged and the incident and reflected rays of the point *M* will not be blocked, then the point *M* on the mirror of the fixed-sun mirror *j* is valid.

Recall that a total of n_j points are taken on the heliostat *j*, and the effective number of points is $n_{eff,j}$, then the Shadow Blocking Efficiency of the heliostat *j* can be approximated as:

$$\eta_{sb,j} = \frac{n_{eff,j}}{n_j} \quad (18)$$

4.5. Calculation of Truncation Efficiency η_{trunc}

The **Truncation Efficiency** is defined as the percentage of the energy intercepted by the heat absorber as a function of the energy converged by the mirror field.

Calculation of the Truncation Efficiency of a heliostat can be approximated by calculating the proportion of light that can be received by the collector from the light uniformly reflected by the mirror of the heliostat. However, the sunlight is not parallel to the earth, but has a certain cone angle of a beam of conical light, so the sun's incident light reflected by any point of the heliostat is also a beam of conical light. Check the information can be known, the distance between the sun and the earth is about 150 million kilometres, the sun's diameter of about 1.3 million kilometres, so the sun tensor angle of about 0.53 degrees, the sun's cone of light half-angle spread $\delta = 4.65$ mrad .

The basic idea of calculating the Truncation Efficiency of a heliostat *j* is to take the "effective points" of the heliostat *j* in 3.4, and for each effective point, randomly take a number of rays in the cone of sunlight directed to these points, and judge whether or not the reflected rays of each of the incident rays will be received by the collector. The ratio of the total number of all rays that can be received by the collector to the total number of rays taken can then be approximated as the Truncation Efficiency of the fixed-sun mirror *j*.

Specifically, in the light-cone coordinate system, the direction vector of the incident light is:

$$\boldsymbol{V_S}' = \left(\sin\sigma\cos\tau, \sin\sigma\sin\tau, \cos\sigma\right) \quad (19)$$

In the above equation, σ is the angle between the light ray and the main light ray, which is taken in $\left[0, 4.65\right]$ (unit: mrad); τ is the angle between the projection of the light ray on the *xOy* plane of the light cone coordinate system and the *x-axis*, which is taken in $\left[0, 2\pi\right]$.

For a definite line of incident light, it is converted to a representation in the mirror field coordinate system, $\boldsymbol{V_S}$, and then to the unit vector $\boldsymbol{V_{S,unit}}$.

From the law of reflection of light and geometrical relations, it is easy to conclude that the unit direction vector of the corresponding reflected ray in the coordinate system of the mirror field is:

$$\boldsymbol{V_R} = 2\left(\boldsymbol{V_{S,unit}} \cdot \boldsymbol{n}\right)\boldsymbol{n} - \boldsymbol{V_{S,unit}} \tag{20}$$

Combined with the coordinates of the effective point, the position, height and size of the collector, the calculation method of spatial analytical geometry can be used to determine whether the reflected light is received by the collector or not.

Remembering that a total of m_j rays are taken on the heliostat j and the number of rays that can be picked up by the collector is $m_{rec,j}$, the Truncation Efficiency of the heliostat j can be approximated as:

$$\eta_{trunc,j} = \frac{m_{rec,j}}{m_j} \tag{21}$$

4.6. Calculation of Optical Efficiency η

In summary, the Optical Efficiency of the heliostat j is

$$\eta_j = \eta_{\cos,j} \cdot \eta_{at,j} \cdot \eta_{ref,j} \cdot \eta_{sb,j} \cdot \eta_{trunc,j} \tag{22}$$

Then the Optical Efficiency of the heliostat field

$$\eta = \frac{1}{N}\sum_{j=1}^{N}\eta_j \tag{23}$$

where N is the number of heliostats.

5. Conclusion

Solar tower heliostat fields help humans make full use of solar energy, but previous methods for calculating the optical efficiency of heliostats are difficult to implement programmatically or are not precise enough. In this paper, we mainly use the ray tracing method, combined with the operations of spatial analytic geometry, to derive a method to calculate the shadow shading efficiency and truncation efficiency by analysing the ray trajectory, which is highly accurate and not difficult to implement in programming, and has certain application value in evaluating the distribution of the heliostat field.

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