

Investigation of Effect of the Tooth Profile Modification on the Engagement Characteristics of the Gear Pair

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Abstract: As a vital component in the electric vehicle, the higher requirements for the gear precision are urgently needed in the service environment with high speed and heavy load. The tooth profile modification can greatly improve the accuracy and strength of the gear transmissions. Therefore, accurately obtaining the optimum modification parameters of the gear system under the action of specific loads is the premise of the research of gear vibration and noise reduction. The time-varying mesh stiffness calculation model is established in this paper considering the coupling effect of the tooth profile errors and structural coupling effect. The minimum variations of the loaded static transmission error are selected as the objective function to obtain the optimum modification parameters of gear pair. In addition, the effect of torque, modification amount and modification length on the engagement characteristics of the gear pair are studied. The research can provide some references for the design and optimization for the gear transmission system in the electric vehicle.

Keywords: Gear transmission system, Electric vehicle, Tooth profile modification, Static transmission error.

1. Introduction

Safety, comfort, energy efficiency, and environmental friendliness are eternal themes in the development of the automotive industry. In recent years, with increasing environmental awareness and the characteristics of zero emissions during driving, electric vehicles have made significant progress and gradually become a focus of attention. The gear transmission system, as a critical component within electric vehicles, is responsible for transmitting force and motion to drive the vehicle forward. New energy electric vehicle gears operate at high speeds and bear heavy loads, requiring high precision in gear manufacturing. Gear tooth profile modification can greatly improve gear transmission accuracy and enhance gear strength. Therefore, an increasing number of scholars are conducting in-depth research on the meshing characteristics of modified gears in electric vehicles.

Yan Guoping et al. [1] established a finite element model for modified spur gears to analyze the impact of tooth profile modification on gear contact stress. Ge Zixuan et al. [2] investigated the effects of gear tooth profile modification on transmission error and tooth surface load, utilizing genetic algorithms for the optimization design of comprehensive gear modifications. Zou Haoran et al. [3] proposed a time-varying mesh stiffness calculation model for involute gears considering axial forces and modifications, and verified its accuracy through finite element models. Chen et al. [4] developed a time-varying mesh stiffness excitation calculation model for gear pairs considering profile errors and studied the influence of different modification parameters on mesh stiffness. They further incorporated the effects of adjacent tooth elastic coupling during double-tooth meshing [5] and conducted dynamic analysis of the modified gear system [6]. Zhang Xin et al. [7] comprehensively considered the impact of contact thermal deformation in gear systems, calculated gear mesh misalignment, and optimized gear modifications to improve gear transmission stability and reliability. Zhang Liu et al. [8] studied the influence of drum-shaped modifications on normal contact forces and analyzed the effects of different modification curves on gear vibration

noise in automotive transmission systems.

Therefore, this paper focuses on the gear drive system of electric vehicles, determining the optimal amount and length of gear modification under different loads. It analyzes the influence of various modification parameters on the meshing characteristics of gears, providing theoretical guidance for the design optimization and vibration noise reduction of electric vehicle gear drive systems.

2. Meshing Excitation Model for Gears

The spur gear tooth can be simplified as a two-dimensional variable cross-section cantilever beam model, as shown in Figure 1. Based on beam deformation principles, the meshing stiffness of the gear pair is calculated using the potential energy method. This includes bending stiffness (K_b), shear stiffness (K_s), axial compression stiffness (K_a), gear body deformation stiffness (K_f), and Hertz contact stiffness (K_h). The expressions for calculating each stiffness are as follows [5].

$$\frac{1}{K_b} = \int_0^d \frac{(x \cos \alpha - h \sin \alpha)^2}{EI_x} dx \quad (1)$$

$$\frac{1}{K_s} = \int_0^d \frac{1.2 \cos^2 \alpha}{GA_x} dx \quad (2)$$

$$\frac{1}{K_a} = \int_0^d \frac{\sin^2 \alpha}{EA_x} dx \quad (3)$$

$$\frac{1}{K_f} = \frac{\cos^2 \alpha}{WE} \left\{ L \left(\frac{u_r}{S_r} \right)^2 + M \left(\frac{u_r}{S_r} \right) + P (1 + Q \tan^2 \alpha) \right\} \quad (4)$$

$$\frac{1}{K_h} = \frac{4(1-\nu^2)}{\pi EW} \quad (5)$$

In the equations, the specific meanings of the symbols can be referred to in the literature [5]. The expressions for calculating the variable cross-sectional area and moment of inertia of spur gear teeth are as follows.

$$A_x = 2h_x W \quad (6)$$

$$I_x = \frac{2}{3} h_x^3 W \quad (7)$$

In the equations, the specific meanings of the variables are illustrated in Figure 2.

The expression for calculating the single-tooth meshing stiffness (K_m) is as follows [5].

$$\frac{1}{K_m} = \frac{1}{\frac{1}{K_{b1}} + \frac{1}{K_{s1}} + \frac{1}{K_{a1}} + \frac{1}{K_{f1}} + \frac{1}{K_{b2}} + \frac{1}{K_{s2}} + \frac{1}{K_{a2}} + \frac{1}{K_{f2}} + \frac{1}{K_h}} \quad (8)$$

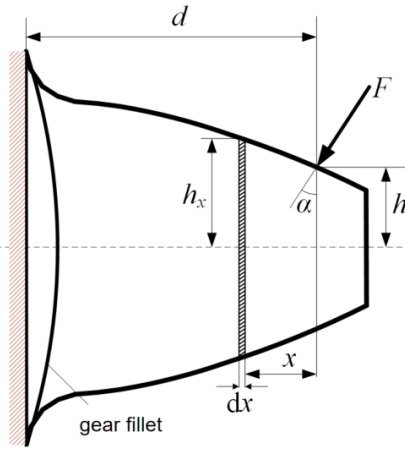


Figure 1. Cantilever beam model of the spur gear

When the gear pair is in the double-tooth meshing region, the two pairs of teeth simultaneously engaged in the meshing process share the same gear body. Simply adding up the single-tooth meshing stiffness calculated from Formula (8) would greatly overestimate the comprehensive meshing stiffness in the double-tooth meshing region. Therefore, Xie et al. [9] derived a fitting formula for the coupling effects of two pairs of meshing teeth in the double-tooth meshing region from the perspective of elasticity. The schematic diagram of structural coupling effects is shown in Figure 2. When gear tooth 1 is subjected to load F_1 , gear tooth 2 undergoes elastic deformation $1/K_{21}$, and similarly, gear tooth 1 also undergoes elastic deformation $1/K_{12}$ due to the load F_2 acting on gear tooth 2. The formula for calculating the effects of inter-tooth elastic coupling is [9].

$$\frac{1}{K_{21}} = \frac{\cos \alpha_1 \cos \alpha_2}{EW} \left[L_1 \frac{u_1}{S_{f1}} \frac{u_2}{S_{f2}} + (M_1 \tan \alpha_2 + P_1) \frac{u_1}{S_{f1}} + (Q_1 \tan \alpha_1 + R_1) \frac{u_2}{S_{f2}} + (S_1 \tan \alpha_1 + T_1) \tan \alpha_2 + U_1 \tan \alpha_1 + V_1 \right] \quad (9)$$

$$\frac{1}{K_{12}} = \frac{\cos \alpha_1 \cos \alpha_2}{EW} \left[L_2 \frac{u_1}{S_{f1}} \frac{u_2}{S_{f2}} + (M_2 \tan \alpha_1 + P_2) \frac{u_2}{S_{f2}} + (Q_2 \tan \alpha_2 + R_2) \frac{u_1}{S_{f1}} + (S_2 \tan \alpha_2 + T_2) \tan \alpha_1 + U_2 \tan \alpha_2 + V_2 \right]$$

In the equation, the specific meanings of the variables can be referred to in the literature [9].

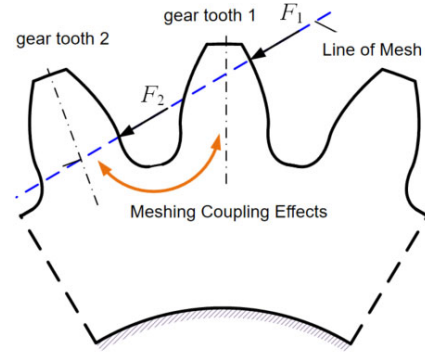


Figure 2. Schematic of the structural coupling effect

Reasonable tooth profile modification can effectively avoid sharp corner contact during gear engagement and disengagement, reduce the vibration noise level of the gear transmission system, improve gear transmission accuracy, and increase gear strength. The schematic diagram of spur gear tooth profile modification is shown in Figure 3, where C_a and L_a represent the amount and length of tooth top modification, respectively. According to the British standard (BS 1970) and international standard (ISO/DIS 1983) specifying the maximum tooth top modification $C_{a_max} = 0.02m$ and the maximum modification length $L_{a_max} = 0.6m$, Chen et al. [4] defined dimensionless modification parameters C_n and L_n . The specific calculation process is as follows [4].

$$C_n = \frac{C_a}{C_{a_max}}, L_n = \frac{L_a}{L_{a_max}} \quad (10)$$

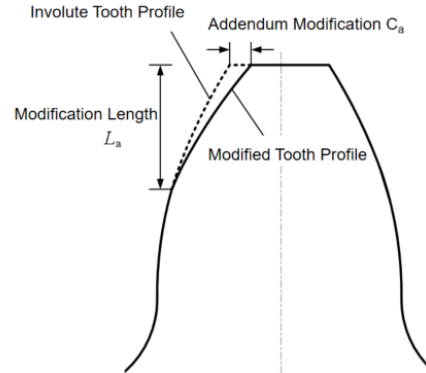


Figure 3. Schematic of the tooth modification

However, excessively large or small tooth profile modification parameters not only fail to improve the meshing state of the gear pair but also introduce adverse effects such as impact. Therefore, determining the optimal modification parameters for tooth profile modification is of great significance for the design and manufacturing of gears. Studies on gear vibration have shown that the amplitude of static transmission error directly affects the vibration level during the meshing process of gear pairs, and smaller static transmission error leads to better meshing behavior. Therefore, this paper takes the minimum value of relative static transmission error fluctuation as the optimization objective to determine the optimal modification parameters. The objective function can be expressed as:

$$R_p = \frac{R_n - R_m}{R_n} \times 100\% \quad (11)$$

In the equation, R represents the RMS value of static transmission error, and subscripts n and m correspond to the cases without and with modification, respectively. It is worth noting that a larger R_p value corresponds to a smaller

fluctuation in static transmission error. Therefore, the optimization objective of this paper is to find the tooth profile modification parameters corresponding to a larger R_p value.

3. Results Analysis

The design parameters of the electric vehicle gear drive system are shown in Table 1.

Table 1. Design parameters of the gear pair in electric vehicle

Name	Large gear	small gear.
Module m (mm)		2
Pressure angle α (°)		20
Addendum coefficient h_a		1
Clearance coefficient c_n		0.25
Tooth width W (mm)	74	37
Number of teeth Z	0.05	0.05
Helix coefficient X_n		0

Optimizing the tooth profile modification parameters under different loads with the minimum value of static transmission error fluctuation as the objective function yields the results shown in Figure 4. The red region in the figure represents a larger R_p value, indicating smaller static transmission error. Selecting the tooth profile modification parameters corresponding to this region for gear pair modification results in smaller static transmission error and reduced vibration level during gear meshing, effectively improving the meshing

performance of the gears. From Figure 4, it is evident that the optimal tooth profile modification amount and modification parameters approximately follow an inverse relationship, i.e., smaller tooth top modification corresponds to larger modification length. As the load increases, the optimal modification region gradually moves towards larger tooth top modification and modification length, and the optimal modification parameter region gradually reduces.

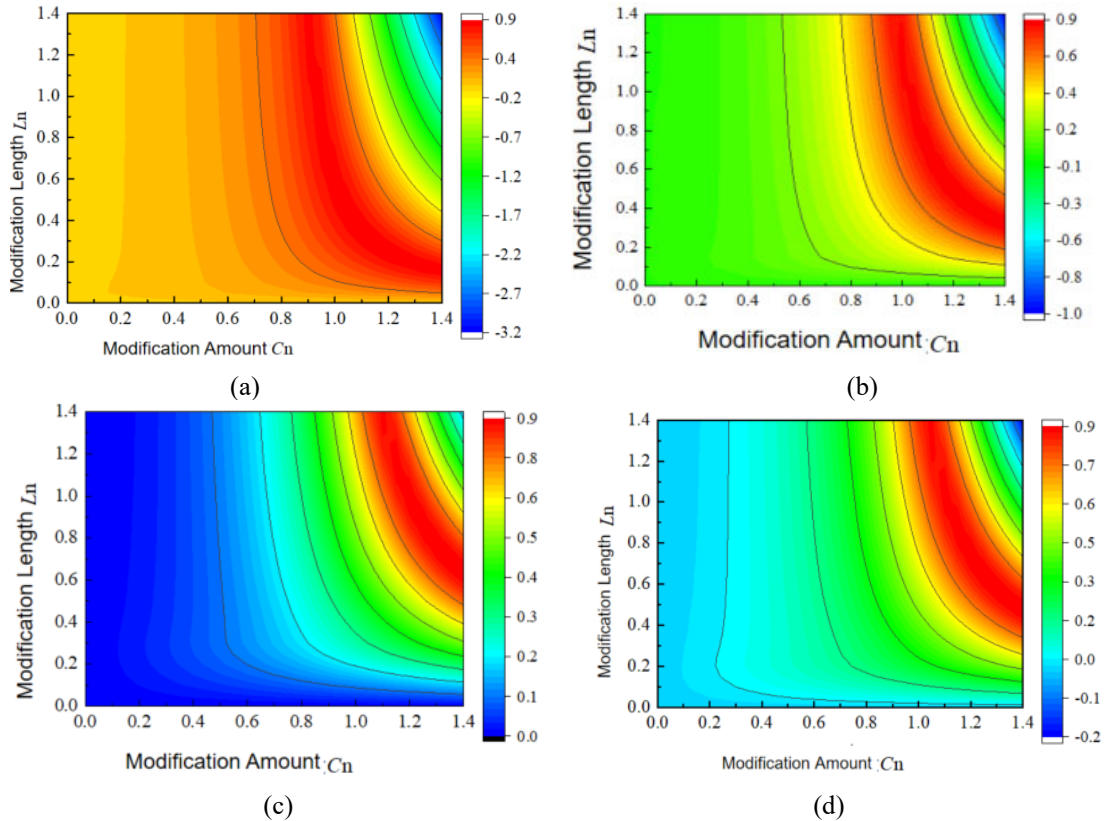


Figure 4. Optimization of the tooth profile under the loads for (a) $T=100\text{N}\cdot\text{m}$, (b) $T=200\text{N}\cdot\text{m}$, (c) $T=300\text{N}\cdot\text{m}$, and (d) $T=400\text{N}\cdot\text{m}$

The influence of different tooth top modification amounts on the meshing characteristics of the gear pair is shown in Figure 5. When the tooth top modification amount varies within the range of 0-0.8, without profile modification, the time-varying mesh stiffness, load distribution coefficient, and static transmission error of the gear pair all exhibit abrupt

changes in the single-double tooth alternating region. With profile modification, the meshing characteristics of the gears show a gradual change in the single-double tooth alternating region, achieving a smooth transition of meshing parameters in this region. Some double-tooth meshing areas are transformed into single-tooth meshing, reducing the

proportion of double-tooth engagement in the meshing cycle. As a result, the overlap of the gear pair decreases, and the load-carrying capacity of the gear system also decreases.

The influence of different modification lengths on the meshing characteristics of the gear pair is shown in Figure 6. The time-varying mesh stiffness, load distribution coefficient, and static transmission error of the gear pair exhibit a smooth

transition in the single-double tooth alternating region as the modification length increases. However, modification leads to the transformation of some double-tooth meshing areas into single-tooth meshing, reducing the load-carrying capacity of the gear pair. This is consistent with the impact of tooth top modification amount on the meshing characteristics of the gears.

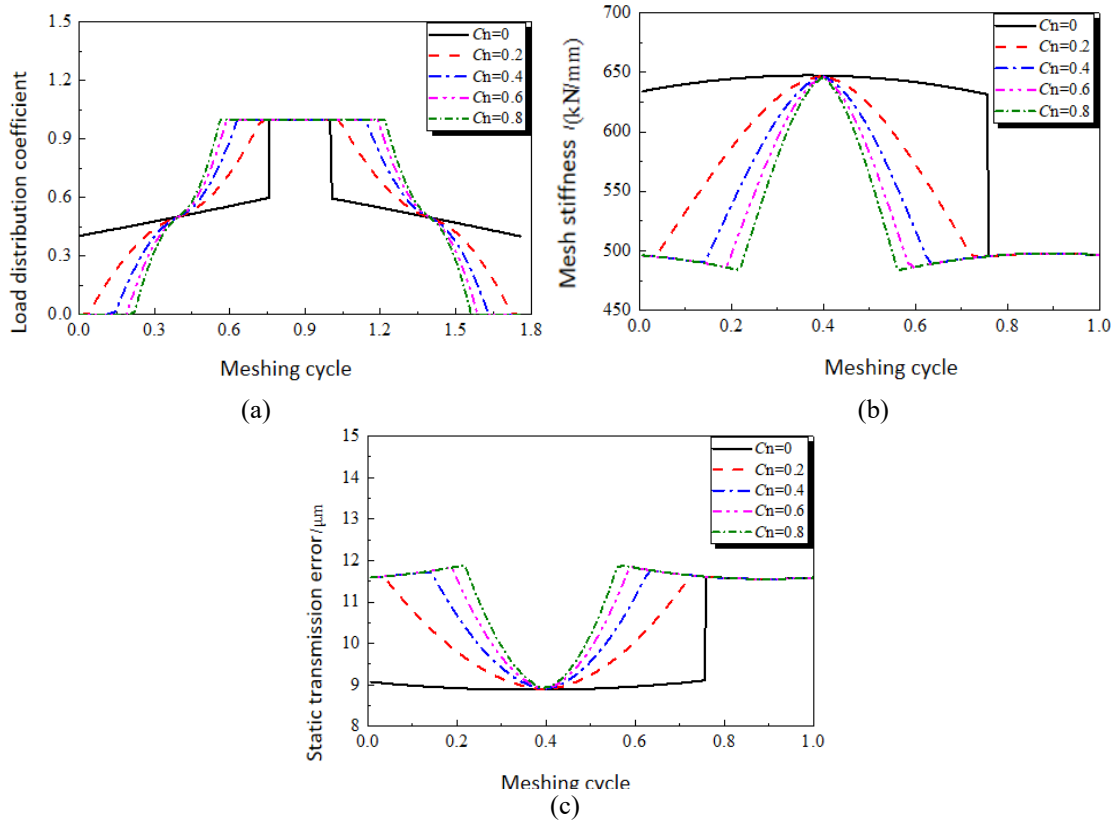


Figure 5. The effect of the modification amount on the mesh characteristics for the gear system ($L_n=0.8$, $T=200$): (a) time-varying mesh stiffness, (b) load sharing ratio, and (c) static transmission error

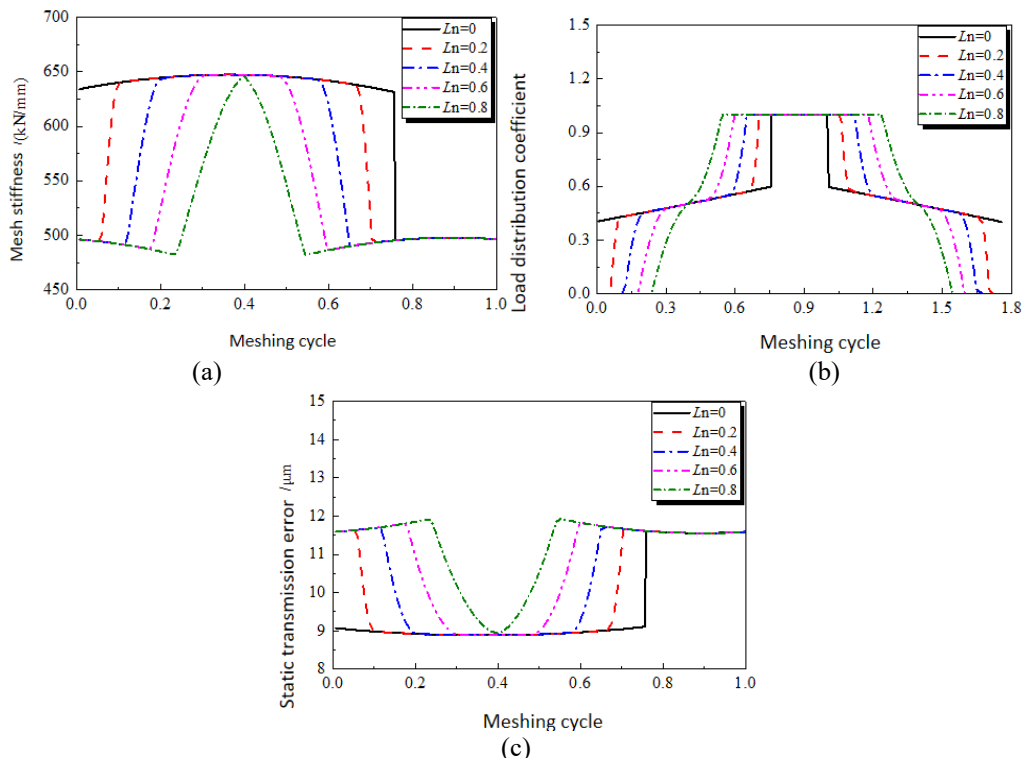


Figure 6. The effect of the modification length on the mesh characteristics for the gear system ($C_n=1$, $T=200$): (a) time-varying mesh stiffness, (b) load sharing ratio, and (c) static transmission error

The influence of different loads on the meshing characteristics of the gears after modification is shown in Figure 7. The time-varying mesh stiffness, load distribution coefficient, and the smoothness of the single-double tooth transition region gradually increase with the increase in load. As the load increases, the proportion of the double-tooth engagement area in the entire meshing cycle gradually

increases, leading to a corresponding increase in the static transmission error of the gears. This results in an intensification of the vibration level in the gear system, as shown in Figure 7c. Therefore, the determination of the modification parameters for the gear pair needs to be considered in conjunction with the actual operating conditions.

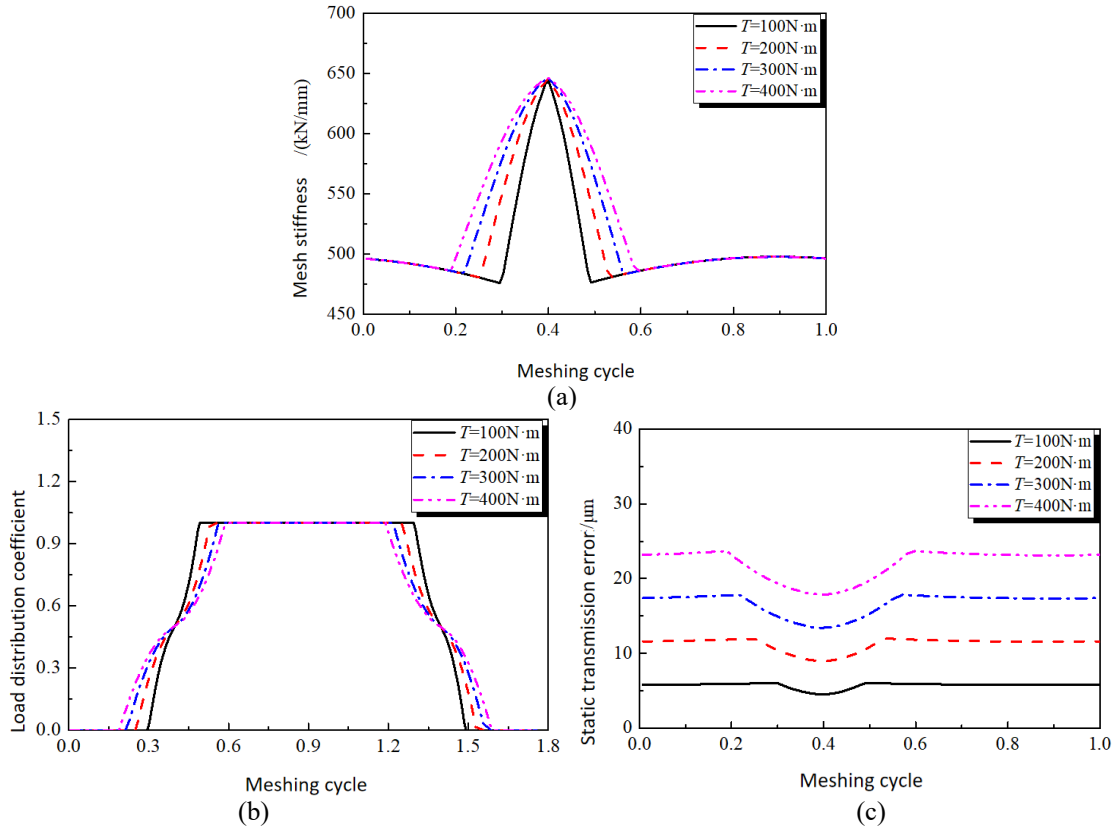


Figure 7. The mesh characteristics for the gear system with profile modification under different loads ($C_n=1.2$, $L_n=0.8$): (a) time-varying mesh stiffness, (b) load sharing ratio, and (c) static transmission error

4. Conclusion

This paper establishes a meshing excitation calculation model for the gear drive system of electric vehicles and conducts an optimization study on the optimal tooth profile modification parameters with the objective of minimizing static transmission error. The main conclusions are as follows:

(1) The optimal tooth profile modification amount and modification parameters approximately follow an inverse relationship. As the load increases, the optimal modification region gradually approaches larger tooth top modification amounts and modification lengths, and the optimal modification parameter region gradually decreases.

(2) Different tooth top modification amounts and modification lengths can improve the abrupt changes in the single-double tooth alternating meshing region, resulting in a smooth transition of the entire meshing process.

(3) Modification leads to the transformation of some double-tooth meshing areas into single-tooth meshing, reducing the proportion of double-tooth engagement in the meshing cycle and consequently lowering the overlap of the gear pair and impacting its load-carrying capacity.

(4) With the increase in load, the proportion of double-

tooth engagement in the entire meshing cycle gradually increases, leading to a corresponding increase in the static transmission error of the gears. This results in an escalation of the vibration level in the gear system.

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