

# Research on Multibeam Bathymetric System Based on Geometrical Relation Mo Model

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**Abstract:** In this paper, the basic principles of multibeam bathymetry system are discussed in depth, the development of which originates from the single-beam bathymetry technology. Through profound mathematical modelling and geometric relationship derivation, a systematic and detailed analysis is carried out for the coverage width of multibeam bathymetry and the overlap rate between two adjacent bands in the case that the survey line is parallel to the horizontal plane. Adopting the idea of combining numbers and shapes, combined with the triangle side angle relationship, we established a geometrical-mathematical model with an  $\alpha$ -slope slant line, which lays a solid theoretical foundation for solving the problem. In this study, we successfully solved the expression of seawater depth  $D$  of the multibeam bathymetric system in the case that the direction of the survey line is parallel to the horizontal plane by the method of listing relations. At the same time, we make full use of the sine-cosine theorem of triangles to derive the coverage width of the bathymetric strip in depth. Combining these two organically, a complete and detailed expression for the coverage width is formed, which provides a powerful mathematical tool for the further study of deep-sea bathymetry technology. In addition, by applying the mathematical model to the vacant data in Table 1, we successfully fill in this missing information, demonstrating the feasibility and accuracy of the model in practical applications. This study not only makes remarkable progress in theory, but also provides strong support for practical applications in the field of ocean bathymetry.

**Keywords:** Multibeam bathymetric system, Mathematical modelling, Geometrical relationships, Line direction, Horizontal plane parallelism.

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## 1. Introduction

With the rapid development of multibeam bathymetry technology and the increasing demand for high-precision seabed topography measurements, ocean bathymetry is experiencing a shift from the traditional discrete, low-precision and low-efficiency to full-coverage, high-precision and high-efficiency. In the current context, the traditional technical design has been difficult to meet the urgent requirements for fine description of seabed topography. The arrangement of the multibeam line measurement system involves the direction of beam emission and the distance between neighbouring lines, while the depth and shallow slope of the seabed terrain significantly affects the measurement data and effect of the multibeam system [1]. Compared with single-beam surveys, multibeam surveys can form efficient profiles under the ocean, and their practicality and efficiency are significantly improved.

The system transmits and receives acoustic waves in a wide range of angles through an array of acoustic transmitter and receiver transducers, forming a strip-type high-density bathymetric data, capable of mapping the three-dimensional topography and geomorphology of the seafloor within a certain width of the strip along the route, taking into account the changes in the seafloor topography around the underwater structure [2]. The existence of beam overlapping areas in multibeam surveys is unavoidable, and it is also necessary to pay attention to the situation of missed measurements. Therefore, the design of the distance between the beams and the measurement angle is extremely critical [3].

When arranging the multibeam system, the emission direction of the beams and the spacing between adjacent survey lines must be fully considered. This involves high-precision measurements of deep-sea topography, and therefore requires sophisticated acoustic wave transmitting

and receiving transducer arrays for wide-angle transmitting and directional receiving of the seafloor [4][8]. By forming a strip of high-density bathymetric data in the vertical plumb plane of the heading, the multibeam system can effectively map the three-dimensional topography and geomorphology of the seafloor within a certain width strip of the course, which provides a reliable data basis for the fine description of deep-sea topography.

It should be emphasised that the existence of overlapping beams in the multibeam survey is an inevitable phenomenon, and full attention should be paid to the leakage in the study. When designing the distance between beams and the measurement angle, scientific and reasonable consideration must be made on the basis of the complexity and variability of the actual seabed topography, so as to ensure that the system is able to obtain accurate and reliable measurement data in various seabed geomorphological situations [9].

Overall, the design and optimisation of multibeam bathymetric systems is a complex issue that takes into account the characteristics of seabed topography, acoustic wave propagation characteristics and system engineering. Through in-depth study of the arrangement principle of the multibeam system, the beam transmitting and receiving technology, as well as the processing method of beam overlapping area, we can better meet the urgent demand for high-precision measurements of seabed topography, and promote the development of ocean bathymetry technology in the direction of more comprehensive, efficient and accurate.

## 2. Related Work

We have conducted an in-depth study on the design of the multibeam bathymetric system, paying special attention to the complex relationship between the coverage width of each strip and the spreading angle of the acoustic wave emitted by the converter, the depth of the ocean and the overlap rate [10].

In order to ensure the accuracy of the measurements, we clearly defined the overlap rate and tried to keep it in the range of 0.1~0.2 in the design to meet the requirements of the title for the accuracy of the measurements.

In the multibeam bathymetry system, the lead is hammered in the plane of the survey line and forms a slanting straight line with an angle of  $\alpha$  with the horizontal plane, where  $\alpha$  is defined as the slope. Considering the characteristics of the multibeam transducer, the opening angle is  $120^\circ$ , the slope is  $1.5^\circ$ , and the depth of seawater at the centre point is 70 m [11]. By establishing a mathematical model of the coverage width of the multibeam bathymetry and the overlap rate between adjacent strips, we are able to calculate the index value of each position in Table 1 more accurately, which can provide a reliable reference for the actual bathymetry.

With the help of Auto CAD software, we drew a simplified schematic diagram to reveal the expression of the water depth  $D$  under every 200m distance of adjacent survey lines. Through the combination of number and shape and the triangle sine theorem, we established the corresponding mathematical model [12]. This model is not only based on the given conditions such as the opening angle, slope, and the depth of seawater at the centre point of the sea, but also takes into account factors such as the distance of the survey line from the centre point, which provides a more comprehensive theoretical support for the comprehensive design of the system [13].

Based on the above theoretical foundation, we carried out specific numerical calculations using the Python programming language to derive key parameters such as the seawater depth, the coverage width, and the overlap rate of the previous lateral line. Such calculations not only fully consider the various complex factors in the system design, but also ensure the accuracy and reliability of the measurement results [14]. This study not only explores the design principle of multibeam bathymetry system in depth theoretically, but also provides effective calculation tools and methods in practical applications.

### 3. Model Establishment and Solution

#### 3.1. Model Establishment

Regarding the establishment and solution of the mathematical model of the overlap rate between the coverage width and adjacent strips of multibeam sounding, the two elements of the beam distribution and the ocean terrain are extremely important, because the terrain is a slanting line with an angle of  $\alpha$  with the horizontal plane, so this paper builds a mathematical model based on the principle of combining planar geometry and triangular numbers and shapes [15].

Firstly, Auto CAD software is used to make a simple drawing, as shown in Figure 1 below:

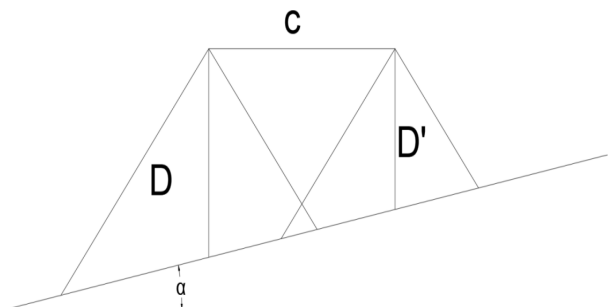


Figure 1. Schematic of ocean depth

(1) The depth of seawater ( $m$ ) can be found relatively easily using simple trigonometric operations:

$$D = D' + c \cdot \tan \alpha \quad (1)$$

In the formula 1,  $D$  is the depth of seawater,  $D'$  is the depth at the centre point of the sea as described in the question,  $c$  is the distance between the measuring line and the centre point, and  $\alpha$  indicates the slope [16].

(2) To solve for the width of coverage ( $m$ ), the equation of the width of coverage and the lead depth of seawater is derived based on the idea of combining numbers and shapes and the sine-cosine theorem of triangles:

$$M = D \times \sin \frac{\theta}{2} \left( \frac{1}{\sin \left( \frac{\pi-\theta}{2} - \alpha \right)} + \frac{1}{\sin \left( \frac{\pi-\theta}{2} + \alpha \right)} \right) \quad (2)$$

In the formula 2,  $D$  is the depth of seawater,  $\theta$  is the opening angle and  $\alpha$  is the slope.

(3) Calculation of the overlap rate, assuming that the survey lines are parallel to each other and the seabed terrain is flat, then the overlap rate between two adjacent strips is:

$$\eta = 1 - \frac{d}{M} \quad (3)$$

In Equation 3,  $d$  is the separation distance between two neighbouring strips and  $M$  is the coverage width of the multibeam strip.

The association leads to Equation 4:

$$\begin{cases} M = D \times \sin \frac{\theta}{2} \left( \frac{1}{\sin \left( \frac{\pi-\theta}{2} - \alpha \right)} + \frac{1}{\sin \left( \frac{\pi-\theta}{2} + \alpha \right)} \right) \\ \eta = 1 - \frac{d}{M} \end{cases} \quad (4)$$

The mathematical model has been established, and the coverage width and the overlap rate of adjacent lines of the multibeam bathymetric system at different positions on the ocean level in Figure 1 can be solved using this model, which can help to establish the optimal measurement interval by judging the overlap rate, and at the same time, validate the data accuracy of the multibeam measurement system and evaluate the measurement efficiency, and the model can be applied to a variety of situations through a simple extension.

#### 3.2. Solving the model

According to the parameters in the problem: the opening angle of the multibeam transducer is  $120^\circ$ , the slope is  $1.5^\circ$ , the depth of seawater at the centre point is equal to 70m, and the distance between the measuring line and the centre point, this paper takes the mathematical model established in 3.1 as the cornerstone, and solves the problem by using the Python language to get the required depth of seawater  $D$ , the coverage width  $M$  and the overlap rate  $\eta$  of the measuring line with the previous one at each point, and puts the operation into the model to solve the model. overlap rate  $\eta$ , and fill in the arithmetic results in Table 1.

**Table 1.** Calculation results

Distance of the line from the centre/m	Depth of sea water/m	Coverage width/m	Overlap with previous line/per cent
-800	90.94874	315.8133	—
-600	85.71155	297.6276	33.64%
-400	80.47437	279.4418	29.59%
-200	75.23718	261.256	25.00%
0	70	243.0703	19.78%
200	64.76282	224.8845	13.78%
400	59.52563	206.6987	6.81%
600	54.28845	188.513	-1.39%
800	49.05126	170.3272	-11.17%

By analysing the data in Table 1, we can observe the following trend: since the distance (m) of the survey line from the centroid corresponds to the first lateral line at -800m, the coverage is null at that location. In the problematic conditions, with the opening ( $^{\circ}$ ) and slope ( $^{\circ}$ ) kept constant, the seawater depth and coverage width show an increasing trend as the distance from the centroid increases, i.e., in the direction of the slope descending; on the contrary, in the direction of the slope ascending, the seawater depth and coverage width show a decreasing trend.

To ensure the convenience and adequacy of multibeam measurements, the overlap between neighbouring strips should be kept between 10-20%. However, among the nine points in the table, the overlap rate did not meet the specified standard when the distance of the survey line from the centre point was -200m, -400m,  $\pm 600$ m and 800m. Specifically, when the value of  $\eta$  is less than 0, it indicates that there is a gap between the two neighbouring strips at that point, i.e., there is a leakage of measurements.

## 4. Conclusion

In the design and analysis of the multibeam bathymetric system, firstly, with the increase of the distance of the survey line from the centre point, i.e. in the direction of the slope descent, the seawater depth and coverage width show an increasing trend; on the contrary, in the direction of the slope rise, the seawater depth and coverage width show a decreasing trend. This indicates that the measurement results of the multibeam bathymetric system at different locations have a certain regularity, which is closely related to the changes in seabed topography. Secondly, in order to ensure the convenience and completeness of multibeam measurements, the overlap rate between adjacent strips should be maintained between 10 and 20 per cent. However, in some locations, particularly where the distance of the survey line from the centre point is -200m, -400m,  $\pm 600$ m and 800m, the overlap rate fails to meet the required standard and there are cases of missed measurements. This emphasises the need for careful adjustment of parameters in the system design to ensure accurate and comprehensive measurements. Therefore, combining the mathematical model and the analysis of actual data, we recommend that the beam arrangement scheme be re-optimised in the system design, especially in the locations where there are missed measurements, to ensure that the coverage width and overlap rate are within the specified range of the title. Such optimisation will help to improve the performance of the multibeam bathymetric system under different seabed topographic conditions and provide more reliable data

support for marine geological mapping and resource exploration.

## 5. Discuss

In discussing the design and analysis of the multibeam bathymetric system, we need to focus on some key aspects to gain a deeper understanding of the system performance and suggest further improvements. Firstly, we observed that there were leakages in the measurements, especially at locations where the survey line was at -200m, -400m,  $\pm 600$ m and 800m from the centre point. This may be due to misconfiguration of the system parameters or lack of optimisation of the layout scheme. The specific reasons for these positions, such as the presence of obstructions or the complexity of the underwater terrain, should be explored in depth during the discussion in order to target system improvements. Secondly, for cases where the required overlap rate is not achieved in the system, the discussion should focus on how to adjust the beam arrangement and launch angle to ensure sufficient overlap between neighbouring strips. Possible improvement options include adjusting the angle at which the beams are fired or changing the operating parameters of the system to better accommodate variations in different seabed topographies. In addition, practical application scenarios of multibeam bathymetric systems, such as marine geological mapping and resource exploration, should be considered in the discussion. For different application scenarios, the system design may need to be differentiated to meet the needs of different fields. For the complexity of seabed topography, the discussion may also include how to optimise the system to adapt to more complex underwater geomorphology. Finally, the discussion could explore the comparison and integration of multibeam bathymetry with other bathymetric technologies (e.g., single-beam bathymetry, sonar, etc.). This helps to assess the advantages and limitations of multibeam bathymetry in different scenarios and provides guidance for future development of deep-sea measurement technologies.

In summary, by discussing these key aspects in depth, a more comprehensive and in-depth understanding of the optimisation and future development of multibeam bathymetric systems can be provided. It also provides useful guidance for wider application of this technology in the field of marine science.

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