

# Model-Driven Multibeam Line Measurement System Design

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**Abstract:** Multibeam bathymetric system is a complex combination of multi-sensor system. The multibeam bathymetric system is mainly used in the measurement of water depth of waterways, and the scientific layout of the survey line can improve the measurement efficiency, and at the same time, truly and perfectly reflect the topography of the waterway. This paper establishes a mathematical model of coverage width and overlap rate between adjacent strips for multibeam bathymetry, and calculates the parameters such as water depth, coverage width and overlap rate between adjacent strips at the center point of different survey lines. Firstly, a single slope multibeam line model was established to calculate the coverage width and the overlap rate between adjacent strips of the multibeam bathymetric system, taking into account the influence of the slope of a single slope. Then, considering the influence of the angle between the direction of the survey line and the projection of the normal direction of the seafloor slope on the horizontal plane, a mathematical model of multibeam bathymetry with variable survey line direction is established by downscaling the geometrical relationship in the three-dimensional space to the two-dimensional plane. Again, based on the principle of line design, all the lines with overlap rate between 10% and 20% are comprehensively screened, and the enumeration method is adopted to find out the optimal lines under each group of conditions. Finally, a three-dimensional seabed model is constructed based on the actual sea depth data, and the optimal measurement direction and spacing of the survey lines are determined based on the real seabed topographic features, which achieves the goal of minimizing the length of the survey lines, and at the same time, ensures that the survey lines can cover the entire sea area to be surveyed, and the iterative method is used to search for the final direction of the survey lines and find the optimal solution.

**Keywords:** Multibeam Line Measurement System, Full-Coverage Bathymetry, Visualization, Enumeration Method.

## 1. Introduction

In recent years, ocean exploration has received increasing attention from States. The tools for ocean exploration have also been constantly updated, and multibeam sounding technology has emerged. Multibeam bathymetric system is a system based on the development of single-beam bathymetric technology, which uses multiple transducers to simultaneously transmit acoustic signals and receive signals reflected back from the seabed on the receiving transducer. The multibeam bathymetric system overcomes the shortcomings of single-beam bathymetry and is able to realize full-coverage bathymetry [1-4]. In multibeam bathymetric systems, the coverage width  $W$  varies with the transducer opening angle  $\theta$  and water depth  $D$ . If the survey lines are parallel to each other and the seafloor terrain is not clear, the coverage width  $W$  varies. If the survey lines are parallel to each other and the seafloor topography is flat, the overlap rate between neighboring strips is defined as  $\eta = 1 - d/W - d/W$ , where  $d$  is the spacing between two neighboring survey lines and  $W$  is the coverage width of the strip. If  $\eta < 0$ , then it indicates a missed measurement. In order to ensure the convenience of measurement and data integrity, the overlap rate between neighboring strips should be 10% to 20%. Meanwhile, due to the large undulation of the real seabed topography, if the average bathymetry of the sea area is used to design the line spacing, although the average overlap rate between strips can meet the requirements, there may be leakage of measurements in the shallower places; if the shallowest bathymetry of the sea area is used to design the line spacing, although the overlap rate of the shallowest places can meet the requirements, there may be too much

overlap in deeper places, which will result in a large amount of redundancy of the data, affecting the efficiency of the measurements [5-8].

For this reason, this paper addresses the problem of multibeam line sounding, establishes a mathematical model of the coverage width and the overlap rate between adjacent strips for multibeam sounding, and calculates the parameters of water depth, coverage width and the overlap rate between adjacent strips at the center point of different sounding lines from the center point. Firstly, a single slope multibeam line model was established to calculate the coverage width and the overlap rate between adjacent strips of the multibeam bathymetric system, taking into account the influence of the slope of a single slope. Then, considering the influence of the angle between the direction of the survey line and the normal direction of the seafloor slope projected on the horizontal plane, a mathematical model of multibeam bathymetry with variable survey line direction is established by downscaling the geometric relationship in the three-dimensional space to the two-dimensional plane. Again, based on the principle of line design, all the lines with overlap rate between 10% and 20% are comprehensively screened, and the enumeration method is adopted to find out the optimal lines under each group of conditions. Finally, a three-dimensional seabed model was constructed based on the actual bathymetric data, and the optimal measurement direction and spacing of the survey lines were determined based on the real seabed topographic features to achieve the goal of minimizing the length of the survey lines, and at the same time to ensure that the survey lines could completely cover the entire sea area to be surveyed, so that an iterative method could be used to find the final direction of the survey lines and find the optimal

solution.

## 2. Model Formulation and Solving

### 2.1. Single-slope multibeam bathymetric modeling

The seawater coverage of the survey ship's bathymetry is given. The coverage width of the multibeam bathymetry is affected by the seafloor slope, and the bathymetry model is first established at the distance of the survey line from the center point, as shown in Figure 1.

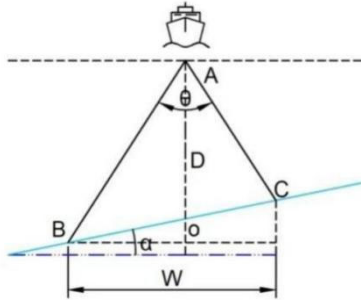


Figure 1. Schematic diagram of bathymetry at the center point

The survey vessel conducts multibeam bathymetry at point A. The beams are reflected as they propagate down the slope of the seafloor, and the reflection points are noted as B and C. From the trigonometric relationship

$$\angle AOC = 90^\circ - \alpha \quad (1)$$

$$\angle AOB = 90^\circ + \alpha \quad (2)$$

Thus the remaining two corners can be derived:

$$\angle ACO = 180^\circ - \angle AOC - \frac{\theta}{2} \quad (3)$$

$$\angle ABO = 180^\circ - \angle AOB - \frac{\theta}{2} \quad (4)$$

Knowing the depth of the sea water  $D = 70m$ , the lengths of OC and BO can be found by the sine theorem, viz:

$$OC = \frac{D}{\sin \angle ACO} \sin \frac{\theta}{2} \quad (5)$$

$$BO = \frac{D}{\sin \angle ABO} \sin \frac{\theta}{2} \quad (6)$$

$$BC = BO + OC \quad (7)$$

W is the multibeam strip coverage width and the solution equation is:

$$W = BC \cdot \cos \alpha \quad (8)$$

Through the above system of equations, it can be concluded that the seawater coverage width W is affected by the seawater depth D at the current location of the measuring vessel, i.e., if the seawater coverage width measured at a certain location is required, the seawater depth at the current location needs to be clarified.

Measurement of seawater depths under different survey

lines with positional relationships as shown in Figure 2.

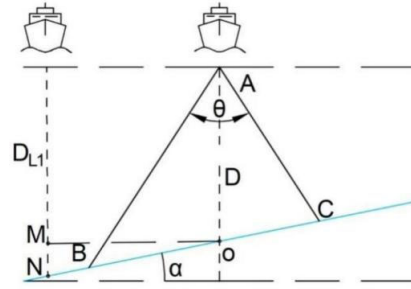


Figure 2. Measured coverage area solving diagram

From the geometric relation:

$$MN = \tan \alpha \cdot d \quad (9)$$

Therefore  $D_{L1} = D + MN$ , which can be further found as  $W_{L1}$  and similarly one can find  $D_{Li} (i = 2,3,4)$  and  $W_{Li} (i = 2,3,4)$ .

The seawater coverage width is the projection of the beam to the seafloor level after propagation to the seafloor slope, as shown in Figure 3.

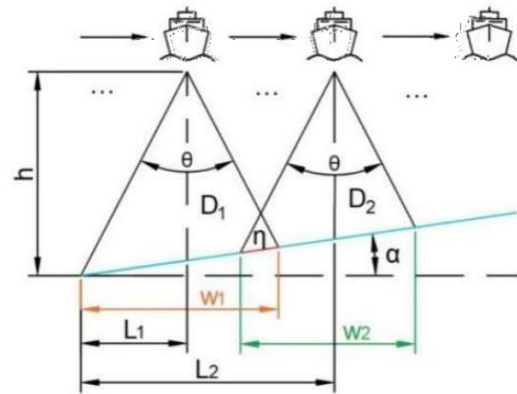


Figure 3. Schematic diagram of multibeam bathymetry

Also the relationship between coverage and coverage width between neighboring strips is:

$$\eta = 1 - \frac{d}{w} \quad (10)$$

As the survey vessel takes depth measurements towards positions of increasing slope, there will be moments when the survey lines do not overlap.

The solution of the equations is carried out to vary the distance of the survey line from the center point to obtain the depth of seawater D at different locations, the coverage width W, and the coverage  $\eta$  between the current survey line and the previous one.

According to the analysis of single-slope multibeam seawater bathymetry data, it can be found that with the center point as the starting point, when the bathymetry is carried out to the left, the width of the sea area covered by the beam gradually increases, and the overlap rate between the adjacent lines shows a trend of gradual increase; when the bathymetry is carried out to the right of the center point, due to the influence of the seafloor slope, the beams of the right side will be reflected earlier than the beams of the left side, which results in the coverage of the seawater width becomes smaller

and the overlap rate between the adjacent lines gradually decreases. The overlap rate between neighboring lines gradually decreases. The specific results are shown in Figure 4. The overlap rate of the survey line is negative when the distance from the center is 600m and 800m, that is, the phenomenon of missing side. According to the theoretical analysis of the average measuring line distance, it can be inferred that the model is basically consistent with the theoretical model, so it can be considered that the model is in line with the actual situation.

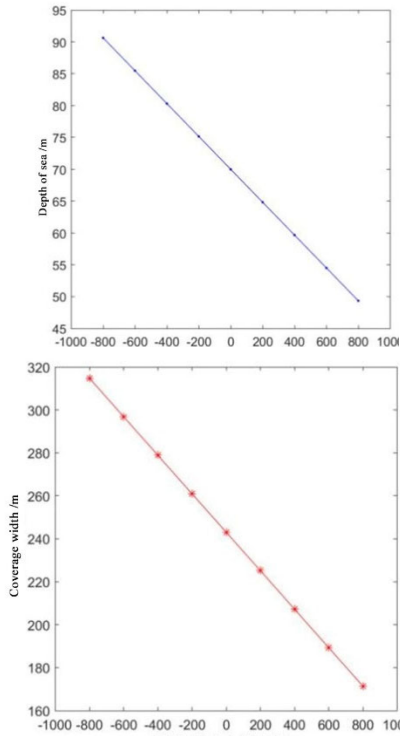


Figure 4. Schematic diagram of the analysis of the results at different bathymetric positions

## 2.2. Multibeam bathymetric model with variable line direction

Multibeam bathymetry system is developed on the basis of single-beam bathymetry system, which can launch multiple beams for bathymetry at a time in the plane perpendicular to the trajectory. Now, according to the question two, assuming that a vessel conducts a line survey from point P to point Q on the sea level, i.e., the direction of PQ is the direction of the line survey, the projection of the slope surface at point P is intersected by the slope surface of the seabed at point P', and the projection of the horizontal plane is intersected by the horizontal plane at point P'' point, the same way as the projection of point Q in the seabed slope surface and the projection of the horizontal plane were intersected with the seabed slope surface and the horizontal plane at point Q' and Q'', from the title we can know that the angle  $\beta$  for the direction of the line of measurement and the seabed slope surface of the normal projection between the horizontal plane, so the normal in the horizontal surface of the projection is perpendicular to the seabed slope and the horizontal plane of the intersection of the line MN. MN. By the working principle of multibeam bathymetry, it can be obtained that multiple beams can be sent out when the survey ship works in the plane perpendicular to the track, thus the vertical plane of the course track PQ is made to intersect the horizontal plane at the point S, and P''S is connected to it, and the positional relationship

in the three-dimensional space is shown in Figure 5.

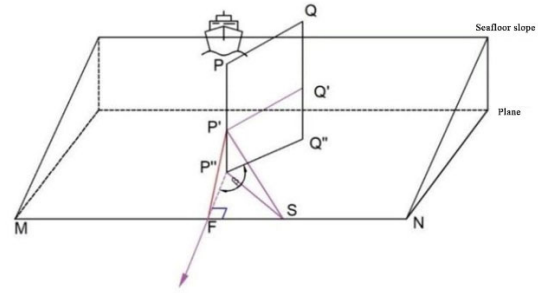


Figure 5. Measured position relationship in three-dimensional space

During the work of the survey vessel, it can be derived that  $\beta$  is the projection of the normal vector of the slope on the horizontal plane  $P''F$ . The angle between  $P''F$  and the line connecting the projection of the course PQ on the horizontal plane, i.e.  $P''Q''$  is the angle between the projection of the normal vector of the slope on the horizontal plane and  $P''Q''$  is the angle between.

As a result, the measured position relationship under three-dimensional space is analyzed and solved, due to the spatial complexity of the three-dimensional relationship, this question is solved by first adopting the dimensionality reduction processing, so that the three-dimensional spatial relationship is projected into a two-dimensional position relationship, first of all, we can learn that the point P' is the projection of the point P on the seabed slope, then in the  $\Delta P'P''F$  center. We can see that  $LP'FP''$  is equal to the slope angle  $\alpha$  of the seabed slope, assuming that the distance between  $P'P''$  is X, i.e., the distance between the slope projection of P point and the horizontal projection is X.

Therefore, in  $\Delta P'P''F$  in, it follows from the sine theorem:

$$P''F = \frac{X}{\tan \alpha} \quad (11)$$

Up to this point, the  $\Delta P''FS$  downscaled to a two-dimensional graph.

The positional relationship between the angle  $\beta$  between the direction of the survey line and the projection of the normal direction of the seafloor slope in the horizontal plane can be derived from the fact that the angle  $\beta$  is affected by the direction of the survey line, and because multibeam bathymetry works by transmitting beams under the vertical plane of the course. It can be derived from the triangular relationship:

$$P''S = \frac{P''F}{\cos(90^\circ - \beta)} \quad (12)$$

From the above equation, we can derive the  $P''S$ . The geometric relationship between the angle  $\beta$  and the angle  $\beta$ , at the same time  $P''S \perp P''Q''$ . The projected line of two points P, Q in the horizontal plane, and when the measuring ship is sailing, because the multi-beam needs to transmit beams in the course vertical plane, so the course vertical plane will intersect the seabed slope in a line, and the angle between the intersecting slope and the horizontal plane is affected by the  $\beta$ -angle, so the slope of the intersecting slope is assumed to be y, and at the same time, the intersecting slope is downgraded to make it in the two-dimensional plane for the Solving. It is obtained by the sine theorem:

$$\gamma = \arctan\left(\tan \frac{P'P''}{P''S}\right) \quad (13)$$

Here, we have utilized the dimensionality reduction technique to simplify the working position relationship of the survey vessel, which was originally in three-dimensional space, to a planar position relationship. Therefore, we can construct the multibeam bathymetry coverage model of this question based on the model conclusion in question one. According to Fig. 6-6, we derive the relationship expression for the seawater coverage width as:

$$W = \left(D \frac{\sin 60^\circ}{\sin(30+\gamma)} + D \frac{\sin 60^\circ}{\sin(30-\gamma)}\right) \cos \gamma \quad (14)$$

The expression for the overlap rate relationship between neighboring bands is:

$$\eta = 1 - \frac{d}{W} \quad (15)$$

where  $\theta$  is the transducer opening angle,  $d$  is the spacing between neighboring strips, and  $\gamma$  is the slope.

The above system of equations was solved, and the coverage width  $W$  was calculated using the multibeam bathymetric coverage model by calculating the angle  $\beta$  between different survey lines and the projection of the seafloor slope normal in the horizontal plane, and by taking into account how much distance the survey vessel is from the center point, and the results were obtained for different angles and positions.

### 2.3. Line design model for the sea area to be surveyed

The application of bathymetric modeling to the design of survey lines in different directions is explored. By rationally planning a specific set of survey lines, the aim is to achieve full coverage of a rectangular sea area with a north-south length of 2 nautical miles and an east-west width of 4 nautical miles. In doing so, we minimized the total length of the lines and ensured that the overlap between adjacent strips was kept in the range of 10 to 20%. To achieve this overlap rate requirement, we traversed and tested all possible overlap rates. Based on the model in Problem 2, we can derive the coverage width calculation formula as:

$$W = 2 \cdot D \cdot \tan\left(\frac{\theta}{2}\right) \quad (16)$$

The strip spacing can be derived from the formula for the overlap rate:

$$d = W \cdot (1 - \eta) \quad (17)$$

Based on the strip spacing and the length of the sea area to be measured, we can derive the number of strips needed to measure the sea area to be measured:

$$n = \text{ceil}\left(\frac{L}{d}\right) \quad (18)$$

where  $\text{ceil}$  is the upward rounding function;  $L$  is the length of the sea area to be measured.

At the same time, since the coverage width  $W$  is affected by the seafloor slope, for which we establish an adjustment factor correction:

$$D' = D - d \cdot \tan(\alpha) \quad (19)$$

At this point, the survey line design model is completed, according to the listed system of equations to solve the number and location of the survey line, which ensures that the survey line can cover the entire sea area to be measured.

Solving the equations yields the relationship between the depth of seawater at different distances, as shown in Figure 6.

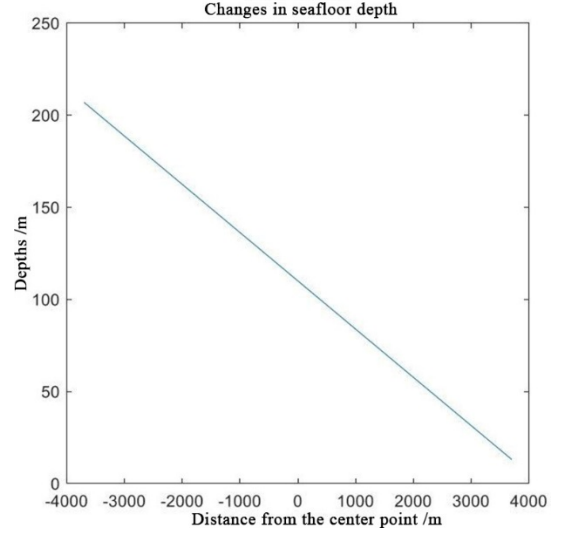


Figure 6. Map of changes in seafloor depth

At this point, it is necessary to ensure that the overlap of the lines is maintained at between 10 and 20 per cent, and the lines are traversed at 1 per cent intervals to obtain the line spacing and the number of lines required. It is necessary to minimize the total length of the lines and to ensure that the lines completely cover the sea area to be measured. After calculation, the coverage width of the target line is 381.05 meters, and the line spacing is 323.89 meters. In order to cover the entire sea area to be surveyed, 23 lines need to be laid.

Based on the spacing of the survey lines and the coverage width of the survey lines, in order to express the measurement effect of the survey lines more intuitively, we visualize all the survey lines, and the obtained results are shown in Figure 7.

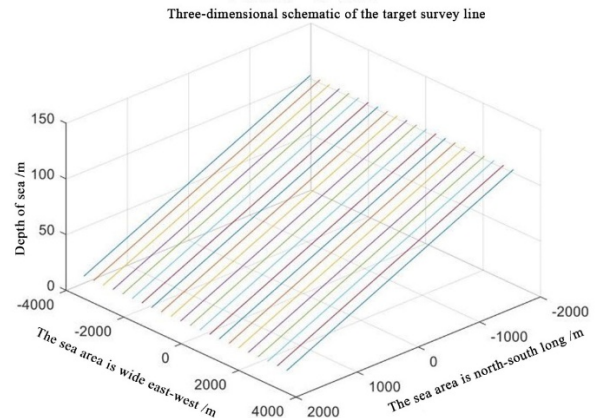


Figure 7. Three-dimensional schematic diagram of line measurement

By solving the survey line design model, we can conclude that the optimal coverage width of the survey line is: 381.05 meters Spacing of the survey line is: 323.89

meters Number of survey lines: 23 Compared with the survey lines in other directions, it saves the total length of the survey, and at the same time the overlap rate of the survey line is: 15%, which is in line with the overlap range of the survey line from 10% to 20%, and the working length of the survey line is: Spacing of the survey line - Number of survey lines, which is greater than the length of the sea area to be surveyed, then it is considered that the line can completely cover the entire sea area to be surveyed. The working length of the line is: line spacing - number of lines, which is greater than the length of the sea area to be measured, then it is considered that the line can completely cover the entire sea area to be measured.

## 2.4. Measurement wiring design for multibeam wave measurement ships

To solve the ideal survey line based on the specific bathymetric data map, the 3D spatial model of the bathymetric data was firstly visualized based on the actual bathymetric data as shown in Figure 8.

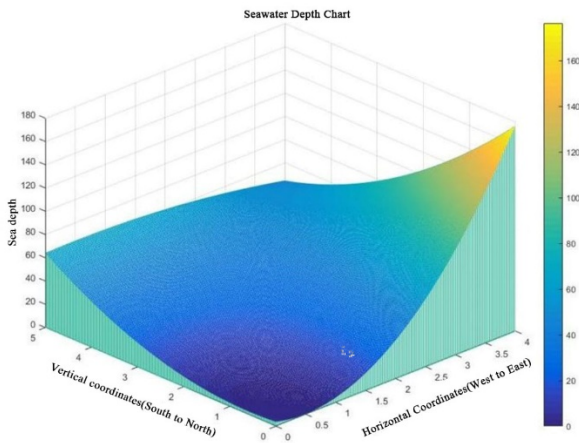


Figure 8. 3D simulation of sea depth

Based on this, we optimize all feasible styli to obtain the optimal stylus. Through the enumeration method, we traverse all the survey lines and find out the optimal survey line that meets the requirements by experimenting with different spacing and direction of the survey line, and then calculate the total length of the survey line by using the geometric relationship. By solving the problem, the optimal line can be obtained from the actual bathymetry data, and the optimal line is visualized as shown in Figure 9.

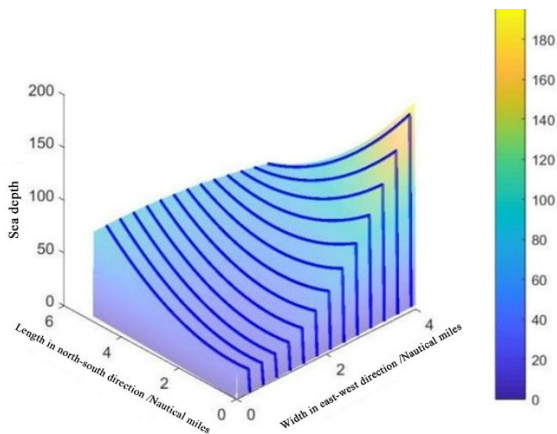


Figure 9. Simulation of the optimal measurement line

Based on the solution analysis, the relevant parameters of the optimal survey line are obtained:

The angle between the optimal survey line and the projection of the slope normal in the horizontal plane is:  $0^\circ$ .

The optimum line spacing is 0.3 nautical miles.

The total length of the shortest survey line is: 85,192 meters.

Define the total length of the survey line, i.e. the sum of the lengths of all the survey lines; the spacing of the survey lines  $d_c L_c$ . This is the distance between two neighboring lines; the number of lines, i.e. the number of lines needed to cover the sea area to be measured.  $n_c$ . The quantitative relationship can be derived:

$$L_c = d_c \cdot n_c \quad (20)$$

We first define the area of the uncovered sea water region as  $W_a$ , the total area of seawater to be measured is  $W$ . The percentage of the missed sea area is denoted as  $Q\%$ , so it follows from the physical relationship:

$$Q\% = (W_a/W_c) \cdot 100\% \quad (21)$$

First of all, according to the requirements of the topic, we need to calculate the width covered by each line, and then calculate the overlap area and overlap rate of the lines based on the coverage width of these lines, thus defining the overlap rate as  $P\%$ , the width of the overlapped area as  $N$ , and the coverage area of the lines as  $M$ , which can be derived from the physical relationship:

$$P\% = (N/M) \cdot 100\% \quad (22)$$

The total length of the portion of the overlap area in which the overlap exceeds 20% is calculated, so based on the portion of the above equation where  $P$  is greater than 20, i.e.:

Excess of 20% over overlap =  $(P\% - 20\%) -$  width of line coverage =  $(P\% - 20\%)$

The required portion is the sum of the portion that exceeds the 20% overlap rate.

Calculated based on the above physical equation:

The total length of the minimum survey line is 85,192 meters; the percentage of the omitted sea area to the total sea area to be surveyed is 1.49 per cent; and the total length of the overlapping area with an overlap of more than 20 per cent is 74,302.24 meters.

## 3. Conclusion

This paper addresses the problem of multibeam bathymetry, establishes a mathematical model of coverage width and overlap rate between adjacent strips for multibeam bathymetry, and calculates the water depth, coverage width and overlap rate between adjacent strips at the center of different survey lines. Firstly, a single slope multibeam line model was established to calculate the coverage width and the overlap rate between adjacent strips of the multibeam bathymetric system, taking into account the influence of the slope of a single slope. Then, considering the influence of the angle between the direction of the survey line and the normal direction of the seafloor slope projected on the horizontal plane, a mathematical model of multibeam bathymetry with variable survey line direction is established by downscaling the geometrical relationship in the three-dimensional space to the two-dimensional plane. Again, based on the principle of

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