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Quantile Regression Model and Its Application Research

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Abstract: Quantile regression is a regression analysis method that estimates the parameters of a model by minimizing the weighted sum of absolute residuals. It was introduced by Roger Koenker in 1978. As a complementary and extended approach to the traditional regression method, namely least squares method, quantile regression addresses the limitations of least squares method in the presence of heteroscedasticity and ensures the robustness of quantile regression through its robustness to outliers, which compensates for the weakness of least squares method in dealing with outlier data. In practical applications, quantile regression can provide a more comprehensive reflection of data information and capture the tails of the distribution of the dependent variable, thus overcoming the limitation of least squares method that can only estimate the central tendency of the dependent variable distribution. Moreover, quantile regression provides more reasonable interpretations and prevents biased or even erroneous estimations that can occur when using least squares method. Overall, quantile regression offers a novel regression method that addresses many shortcomings of least squares method. By combining quantile regression with least squares method, we can understand both the central tendency and tail behavior of the dependent variable distribution. The fitting results obtained from quantile regression can also provide insights into the suitability of least squares estimation. Briefly, the combination of both methods yields better results in statistical problems. This paper introduces the principles of quantile regression and further discusses its scope and application, aiming to provide a preliminary summary for a better understanding of quantile regression.

Keywords: Quantile regression, Panel data model, Penalized quantile regression estimation.

1. Introduction

Traditional panel data models are mostly based on the basic assumptions of mean regression, where the regression results only reflect the structural relationship between data near the mean. They are not accurate in characterizing the relationship between variables in the upper and lower tails. Moreover, traditional panel data models assume that the error term follows a normal distribution. When the sample data does not satisfy the classical assumptions, such as having heavy tails or outliers, the estimation results are often not robust and efficient. Due to the limitations of mean regression, some scholars have turned to the study of quantile regression.Compared to mean regression models, quantile regression can more comprehensively reflect the information in the data and is not affected by contaminated data. Currently, the most researched quantile regression model is linear quantile regression, which assumes a linear relationship between the variables and provides a comprehensive analysis of the data from various quantiles. However, as the complexity of modern data increases, the advantages and limitations of parametric quantile regression models become more apparent. Parametric quantile regression relies on the establishment of a parameter model and is limited by the given model assumptions, lacking flexibility and adaptability to the data. Therefore, the application of nonparametric quantile regression and Bayesian perspectives in quantile regression has gradually extended and deepened the original regression forms to find the best approaches to handle complex data.

Since Koenker and Bassett (1978) introduced the theory of linear quantile regression, quantile regression (QR) has become a rapidly developing and widely applied regression model method in recent decades. It not only deepens the understanding of traditional regression models but also expands the types and applications of regression models, making the fitting of statistical data more accurate and detailed[1]. Quantile regression models are developed based on robust estimation models, including M-estimation theory based on general convex loss functions, R-estimation theory based on sample rank statistics, and L-estimation theory based on sample order statistics. Quantile regression emphasizes estimating the quantiles of the dependent variable using quantiles of the explanatory variable. This is achieved by establishing quantile estimation equations and using linear programming or nonparametric estimation methods to estimate the coefficients or unknown parameters corresponding to different quantiles. Quantile regression is an extension of median regression and mean regression. Specific quantile regression models include quartile regression, decile regression, percentile regression, LOGIT quantile regression, scrutiny quantile regression, and other models. These models not only overcome the limitations of error restrictions and outlier interference but also solve the problem of modeling data from multiple perspectives.

Recent developments in quantile regression research mainly focus on the technical methods and applications of quantile regression. Foreign scholars such as Koenker and Zhijie Xiao have addressed specific inference problems in quantile regression. Kim and Muller have studied the asymptotic properties of two-step quantile regression. Tasche has examined the unbiasedness of least quantile regression. Chernozhukov and Han Hong have proposed a three-step evaluation method for scrutiny quantile regression. Wu Jiannan, Bret-schneider et al. have compared the advantages and disadvantages of significant weight analysis methods and quantile regression using Monte Carlo methods with 100 random data sets. Koenker (2004) first applied quantile regression methods to panel data models and proposed panel data quantile regression methods, which are powerful complements and extensions to traditional panel data analysis methods[2]. They can fully utilize the characteristics of large sample panel data, accurately describe the impact of explanatory variables on the conditional distribution of the dependent variable, relax the assumptions of error distribution, and improve the interpretability, robustness, and efficiency of the model's estimators. Research has found that under non-normal assumptions, quantile regression models are more stable than traditional mean regression. Carloslamarche further explores the PQR estimation method and combines it with empirical analysis using real data. Cai (2010) discusses the analysis of nonparametric spline quantile regression for ordinary bivariate data from a Bayesian perspective, estimates the mean of the parameters using Monte Carlo Markov Chain, and studies the optimal selection of the proposed distribution[3]. The conclusion is that Bayesian nonparametric quantile regression is more flexible than parametric quantile regression, as it combines nonparametric with quantile regression and applies it to ordinary data. Later, scholars extended this from ordinary data to panel data, fully utilizing the sample information and combining Bayesian methods with quantile regression to handle panel data, resulting in beneficial conclusions. Yeh et al. (2011) also provide empirical likelihood estimates for panel quantile regression models. Before this, many scholars introduced empirical likelihood functions, but Yeh et al. argue that for panel data quantile regression models, considering the correlation within each cross-section, the likelihood ratio mentioned in the literature is no longer applicable to this model, which diminishes the charm of the empirical likelihood as a statistical quantity[4]. In this study, to adapt to the correlation within cross-sections and obtain more accurate estimates, the authors replaced the quantile score function with smooth empirical likelihood estimates, thus obtaining empirical log-likelihood and maximum empirical likelihood functions for further research. Antonio and Galvao (2011) studied fixed-effects dynamic panel models and found that in general, the time dimension T is smaller than the individual dimension N, making it difficult to estimate individual fixed effects[5]. In the least squares estimation of dynamic panel data quantile regression models, the unobserved initial values cause biases in the dynamic process of quantile regression fixed effects estimation. Therefore, the authors used instrumental variable methods to estimate the dynamic panel data quantile regression model. Koenker (2011) first considered quantile regression methods for additive models and used piecewise linear bases to find the smoothing parameters that minimize the model's SIC value[6]. They then constructed a confidence band for the nonparametric function using the Sandwich method. Waldmann (2013) studied additive quantile regression models from a Bayesian perspective by transforming the asymmetric Laplace distribution using a scale-mixture Gaussian distribution to establish a Bayesian hierarchical quantile regression model. Finally, they used the Markov Chain Monte Carlo (MCMC) algorithm to quickly solve the problems of nonparametric function estimation and construction of confidence bands[7]. Kengo and Antonio (2012) studied panel data models with individual fixed effects and established the consistency and asymptotic normality of quantile regression estimators as the number of individuals and time periods tends to infinity[8]. Considering the non-smoothness of the true function, the authors imposed strong restrictions on time to prove asymptotic normality. Aghamohammadi and Mohammadi (2015) analyzed longitudinal data models with random effects from a Bayesian perspective using penalized quantile regression methods[9]. Galvao and Kato (2016) applied smooth quantile regression methods to panel data with fixed effects, smoothed the indicator function using kernel estimation, and concluded that the fixed effects estimator has a limiting normal distribution. They also proposed a first-order bias-corrected estimator to reduce bias, although the bias correction increased the variability of small samples[10].

Domestic scholars Luo Youxi and Tian Maozai studied the quantile regression method for panel data and considered three methods for handling fixed effects. Through Monte Carlo simulation, they showed that the panel data quantile regression model is more stable and effective than mean regression when the error does not follow a normal distribution. Zhang Yuanjie and Tian Maozai discussed the panel data quantile regression model with only fixed effects and established a linear quantile regression model. To eliminate fixed effects, they used the k-step differencing method for the same individuals before conducting the quantile regression. Monte Carlo simulation showed that this two-stage method has good accuracy and stability, but it is only suitable for linear models, and further improvements are needed for nonlinear models.Wang Na studied the panel data quantile regression model and considered the estimation of individual effects in panel data. They proposed a pattern recognition method to simultaneously estimate the coefficients of the independent variables and the individual fixed effects. They introduced a Copula structure and proposed a maximum likelihood estimation method for panel data quantile regression with random effects. Xu Jie extended the research to composite quantile regression method on individual fixed effects panel models to achieve more efficient estimation of regression coefficients by integrating multiple quantiles. This method retains the robustness of quantile regression and improves estimation efficiency through composition. By introducing a specific idempotent matrix to eliminate the individual effect term, it avoids the problem of parameter curse and transforms the panel data model into a linear model. Then, the composite quantile regression method is used to construct the objective function of regression coefficients. The results show that, compared to quantile regression, composite quantile regression has better stability under non-normal conditions. This method has greatly promoted the development of quantile regression, but it is limited to linear models and further research is needed for panel data with non-linear relationships.Kong Hang compared the classical kernel estimation, polynomial estimation, and k-nearest neighbor estimation methods with a Bayesian nonparametric quantile regression model. They obtained useful conclusions: quantile regression can set quantile points according to needs, and determine the optimal quantile points through model evaluation; they also proposed a new simplified Bayesian method, which greatly improves computational efficiency by reducing the likelihood function from .Through Gibbs sampling for model calibration, the accuracy is improved. He Jing et al. also studied cross-quantile regression curve models of additive models and applied them to housing price research. They analyzed the impact of various variables on housing prices under different quantiles. However, the above methods rely heavily on the large sample properties of estimates in the model testing and confidence interval construction processes. In addition, in the analysis of panel data quantile regression models, scholars have also devoted themselves to variable selection. By applying penalties to variables to select them and considering the influence of random effects on covariates, they used a Bayesian hierarchical model and used lasso and adaptive lasso penalties in the quantile regression test function to reduce this influence. This method directly assumes a linear model and has good performance in dealing with data with linear relationships proposed a double-regularized quantile regression method for high-dimensional mixed effects models. By simultaneously applying L1 regularization penalties to both random and fixed effects coefficients, important explanatory variables can be selected, and bias caused by individual random fluctuations can be eliminated. Luo Youxi and Li Hanfang studied longitudinal data models with multiple random effects and proposed two new penalty quantile regression methods. By applying Lasso and adaptive Lasso penalties to the quantile regression coefficients, both methods can automatically select independent variables in the model. The proposed methods not only accurately estimate and select quantile regression coefficients but also have strong robustness to random error distributions.

Furthermore, there are studies on the application of quantile regression methods. In this regard, Barnes and W. Hughes (2002) used quantile regression to analyze the return of cross-sector public bond markets. Buhai (2004) studied its application in duration models and cyclic structure equation models. Xu and Lin (2016) studied the carbon dioxide emissions at the provincial level in China and attempted to identify differences among different quantiles[11]. These studies applying quantile regression to real data have yielded valuable conclusions. Roca-Pardiñas and Ordóñez (2019) studied sulfur dioxide pollution using a semiparametric additive quantile regression model, and solved the model using the two-stage Backfitting algorithm[12]. The model was tested with simulated and real data, confirming its effectiveness. However, this algorithm is dependent on initial values and has high computational complexity. To avoid relying on large sample properties and complex iterative algorithms, Fredj and Nabila (2017) studied the impact of geographical environment on Islamic banking performance[13], and Chang et al. (2018) examined whether governmental ideology affects environmental pollution[14]. Ž Šarić et al. (2018) used quantile regression to study the relationship between road signs placed on state roads in 13 states over four years and accident rates[15].

The results indicated that low sign visibility, fewer mandatory signs, and an increase in ineffective signs all increased accident rates. This has important decision-making implications for reducing traffic accidents.There are also many domestic scholars who have applied quantile regression estimation methods to medical and health research, public management research, and other statistical data research with extreme distribution characteristics. With the matured and widespread theoretical development of quantile regression methods, their applications in various fields such as economics, finance, environmental studies, and management are becoming increasingly widespread. The most common application in economic research is the study of factors influencing income levels. Liu Shengqiao (used the quantile regression model to analyze the role of education and experience factors in Chinese residents' income and found that both education and experience promote income growth. The regression coefficient for education changes inversely with income level, with a decrease in education return rate as income level increases, while the regression coefficient for experience follows the same trend as income level. Wei Xiahai and Yu Lingfeng used this model to empirically analyze the income difference between regular and non-regular employment within urban areas. The results showed that the return rate of education exhibits an increasing-then-decreasing pattern under both scenarios as wage levels increase. On the other hand, the wage-efficiency line varies significantly among different samples, exhibiting a linear relationship and monotonically increasing trend in regular employment but a U-shaped relationship in non-regular employment. Jiang Liqing and Qian Wenrong studied the wage gap between public and non-public sectors and found that the wage difference does exist, with public sector wages being significantly higher. However, as the quantile increases, the income difference between the two sectors tends to reduce. Kou Enhui and Liu Baihui estimated the wage function of migrant workers and analyzed the wage gap between short-term and long-term migrant workers using the MM method. The results showed that the main factors influencing migrant workers' wages are regional factors and education factors. At low quantiles of the wage variable, there is a significant difference in wage income between short-term contract workers and long-term contract workers. Guo Yifu discussed the main factors affecting the income levels of urban and rural residents and their relative impacts on the urban-rural income gap based on endowment composition and individual return rates of urban and rural residents. Wu Yanke and Tian Mao first used factor analysis to measure household socioeconomic status based on variables representing the highest education level of fathers and mothers, and family social class. Then, using the FFL unconditional quantile regression method according to different family socioeconomic statuses, they conducted unconditional quantile regression analysis of education return rates. Quantile regression models can also be used to analyze the main factors influencing consumption levels. Chen Juan, Lin Long, etc. added household income and government expenditure factors to the consumer utility function to explore the inherent relationship among consumption, production, and government behavior. The empirical results show that as the consumption level changes, the influence of various factors on consumption also varies. The impact is also significantly different between urban and rural areas. Zhao Xindong and Li Lin used 2007 China Household Income Project survey data to simultaneously consider the linear and nonlinear effects of household economic factors and household population characteristics on consumption levels. They constructed an additive semiparametric quantile regression model and analyzed the contributions of household economic factors and household population characteristics to consumption levels, as well as the changes in their effects at different consumption levels.In other economic research fields, quantile regression methods can be applied to total factor productivity analysis, exchange rate fluctuations, network commodity pricing, and others. Mei Bo and Tian Maozai constructed a spatio-temporal quantile regression model based on spatio-temporal models and the ALD distribution. They used thin plate regression splines to expand the spatial domain and proposed a hierarchical Bayesian quantile regression model based on the relationship between the mixed model and splines. Using this model, they studied the major factors influencing PM2.5 concentrations in Beijing and explained the spatial distribution characteristics of the impact of meteorological conditions on PM2.5 concentrations. Li Zedou et al. applied quantile regression using representative data on indicators of national image and empirically studied the major factors influencing national image. The estimation and analysis of education return rates, labor discrimination factors, and more have been conducted. It can be seen that the theoretical development of quantile regression has become more mature, and there is an increasing demand for the application of quantile regression models to solve practical problems in life. However, the biggest limitation of quantile regression in panel data research is its reliance on parametric models. Parametric models are models formulated by researchers based on a rough analysis of data using structured expressions and parameter sets. When modeling, certain prior assumptions need to be made about the data, which may not fully fit the pre-given model and may not satisfy the researcher's predetermined assumptions. Parametric models have relatively poor flexibility and strong data requirements.

2. Principle of Quantile Regression Model

2.1. The basic principle of quantile regression

The general linear regression model can be set as follows:

$$y = a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k + u$$

Under the premise of satisfying the Gauss-Markov hypothesis, it can be expressed as follows:

$$E\left(y|x\right) = a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k$$

Where, $a_0, a_1, \ldots, a_k$ is the coefficient of explanatory variable to be estimated.

The above model is the expression of the mean reversion model, which is the result of taking the mathematical expectation on both sides of the equation. Similar to the mean regression model, the median regression model can also be set as follows:

$$M\left(y|x\right) = a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k + M\left(u\right)$$

Where, $M\left(y|x\right)$ is the conditional median about x, and $M\left(u\right)$ is the median of the random disturbance term. The quantile regression model is as follows:

$$Q_y\left(\tau|x\right) = a_0 + a_1x_1 + a_2x_2 + \cdots + a_kx_k + Q_u\left(\tau\right)$$

For the mean regression model, the least squares method (OLS) can be used to estimate the unknown parameters. For the median regression model, the least square method (LAD) can be used; For quantile regression model, the linear programming method (LP) can be used to estimate the minimum weighted absolute deviation, and the regression coefficient of explanatory variables can be obtained. They can be expressed as follows:

OLS Approach: $\min E\left(y - a_0 - a_1x_1 - a_2x_2 - \cdots - a_kx_k\right)^2$

Be solved: $\hat{E}\left(y|x\right) = \hat{a}_0 + \hat{a}_1x_1 + \hat{a}_2x_2 + \cdots + \hat{a}_kx_k$

LAD Approach: $\min E\left|y - a_0 - a_1x_1 - a_2x_2 - \cdots - a_kx_k\right|$

Be solved: $\hat{M}\left(y|x\right) = \hat{a}_0 + \hat{a}_1x_1 + \hat{a}_2x_2 + \cdots + \hat{a}_kx_k$

QR Approach: $\min E\rho_\tau\left(y - a_0 - a_1x_1 - a_2x_2 - \cdots - a_kx_k\right)$

Be solved: $\hat{Q}_y\left(\tau|x\right) = \hat{a}_0 + \hat{a}_1x_1 + \hat{a}_2x_2 + \cdots + \hat{a}_kx_k$

Where, $\rho_\tau\left(t\right) = t\left(\tau - I\left(t < 0\right)\right), \tau \in \left(0,1\right)$.

2.2. Panel data model and quantile regression method

Considering the general panel data model, the expression is as follows:

$$y_{it} = x_{it}^{'}\beta + \alpha_i + u_{it}, i = 1,2,\cdots,N.t = 1,2,\cdots,T.$$

Where, i represents different sample individuals, t represents different sample observation time points, u represents the random error term, β represents the coefficient vector of the explanatory variable, and α_i represents the unobservable random effect of the i th sample.

$$x_{it} = \left(1, x_{it1}, x_{it2}, \cdots, x_{itp}\right)^{'},$$

In the fixed effect case, the estimator of beta is

$$\min_{\alpha,\beta}\left\|y - X\beta - Z\alpha\right\|^2$$

$$\hat{\beta} = \left(X'MX\right)^{-1}X'My, M = I - P, P = Z\left(Z'Z\right)^{-1}Z'$$

Assumptions: u ~ N(0,R), α~N(0,W)， V=αZ+u

Then $E\left(vv'\right) = \left(ZWZ' + R\right) = V$

GLS estimation method and penalty least square method (PLS) reflecting individual influence can be used to estimate regression coefficient β for panel data model in this case, respectively expressed as follows:

GLS method: $\min_{\beta}\left\|y - X\beta\right\|_{V^{-1}}^2$

PLS method: $\min_{\alpha,\beta}\left\|y - X\beta\right\|_{R^{-1}}^2 + \left\|\alpha\right\|_{W^{-1}}^2$

The common solution of both is: $\hat{\beta} = \left(X'V^{-1}X\right)^{-1}X'V^{-1}y$.

Quantile regression method can also be used to estimate the parameters of the panel data model. For that. The quantile equation under the following conditions is established:

$$Q_{y_{it}}\left(\tau_j|x_{it},\alpha_i\right) = x_{it}^{'}\beta\left(\tau_j\right) + \alpha_i,$$

$$i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

The quantile equation above assumes that the individual effects are fixed. For this equation, Koenker (2004) pointed out that when the number of individuals N is large and the number of observations contained by each individual is relatively small, appropriate contraction control of the individual effect can effectively reduce the variance α_i due to estimation. For the linear quantile loss function $\rho_T(\mu)$, in order to maintain the linear characteristics of the objective function, l_i a linear penalty term can be considered, that is

$$P(\alpha) = \sum_{i=1}^{n}|\alpha_i|$$

A penalty quantile regression (PQR) method is proposed for estimation. The details are as follows:

$$\left\{\left\{\hat{\beta}\left(\tau_j,\lambda\right)\right\}_{j=1}^{J}, \left\{\hat{\alpha}_i\left(\lambda\right)\right\}_{i=1}^{N}\right\} = \arg\min_{\alpha,\beta}\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{i=1}^{N}w_j\rho_{\tau_j}\left(y_{it} - x_{it}^{'}\beta\left(\tau_j\right) - \alpha_i\right) + \lambda\sum_{i=1}^{N}|\alpha_i|$$

Where, wj is the weight corresponding to each quantile and λ is the adjustment coefficient. If λ=0, it is a fixed effect quantile regression estimator (FEQR). If λ>0, it is a penalty quantile regression estimator (PQR). In addition, Koenker also investigated the asymptotic properties of quantile regression and penalty quantile regression estimators in detail. On this basis, Monte Carlo simulation method is used to compare and analyze the effect of different regression estimation methods under small sample Settings.

3. Conclusion

On the basis of the comprehensive analysis of the meaning and basic principle of quantile regression method, the application of quantile regression method in panel data model is deeply analyzed, and the estimation effect of different regression estimation methods in panel data model is compared and analyzed. In general, quantile estimation methods have certain advantages in estimating error terms with non-normal distributions or unobservable random effects. Through the review and summary of the existing literature, we can get the following enlightenment: On the one hand, the theory and method of quantile regression have been recognized, popularized and applied by many economists, and the development of its related theories tends to be mature and perfect. However, there are still gaps in the extended quantile regression model, and it is possible to improve the existing parameter solving methods. For example, there are few scholars involved in the study of Logistic quantile regression model and the comparative study of unconditional quantile regression and conditional quantile regression. On the other hand, the research on quantile regression model of panel data is in its infancy. Since Koenker (2004) proposed this method, there are still many areas worth exploring and improving in the research of such models. Firstly, for the parameter estimation problem of fixed effect and random effect models, how to improve the existing methods to get a simpler and easier solution is one of the problems to be solved[1]. Secondly, the literature on nonlinear quantile regression, spatial quantile regression and quantile autoregressive models of panel data is relatively lacking, which needs further research. Thirdly, the advanced quantile regression methods, such as variable coefficient quantile regression, cointegrated quantile regression and structural mutation test of quantile regression, are extended to panel data samples, which is also a further improvement of the theoretical system of such models. Finally, for the cutting-edge theoretical methods in such models, mature application software modules or function commands have not been developed, and it is difficult to simply implement them through the interface operation of measurement software such as Eviews or Stata, which brings great inconvenience to parameter estimation and statistical analysis of quantile regression of panel data, and is also an urgent problem to be solved in the future. To sum up, the development of quantile regression model has gradually matured, and the research on panel quantile regression model has also made great progress. The two types of models have been applied more and more widely in social life, and their research results and value have been recognized by more and more scholars and government decision-making departments. However, there are still many problems to be further solved, especially for the relatively lacking research field in quantile regression model of panel data, it is urgent for more econometricians to make contributions and breakthroughs in some aspects.

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