

Prediction of Medical Trauma Rehabilitation by GM (1,1) Model

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Abstract: With the continuous development of science and technology, scholars at home and abroad have adopted different methods to predict the recovery of trauma patients. GM(1,1) gray model was constructed to predict the trauma recovery of patients with anal fistula. For the anal fistula trauma data of small samples, the model constructed in this paper can still meet the prediction demand. The constructed prediction model can realize the multi-stage prediction of the wound condition of the patient, and the accuracy of the model is high, which is helpful for the medical staff to adopt the appropriate treatment strategy according to the rehabilitation condition of the patient.

Keywords: GM(1,1); Prediction; Trauma; Test of model.

1. Introduction

Trauma is mainly a type of damage caused to organs and tissues of the human body by various external forces or mechanical factors, including cuts, punctures, contusions, etc[1]. Trauma not only exists during times of war, but also frequently occurs during times of peace. Trauma not only has a high incidence, but also varies greatly in severity, making it a prominent global issue. Traumatic patients often have complex injuries, rapid disease progression, and a risk of death[2]. According to incomplete statistics, approximately one million people worldwide die from trauma every year, and in China, only a few hundred thousand people die from trauma each year[3]. Trauma has become the leading cause of death for young adults, attracting widespread attention from countries around the world and various sectors of society, and becoming a focus of discussion among many scholars[4].

Scholars have established dynamic trauma prediction models based on machine learning algorithms such as random forests and support vector machines. There are also scholars who use texture synthesis technology to simulate wound healing image prediction algorithms for myofibroblasts. Guo Chengyu and others developed a trauma bleeding volume grading prediction model based on deep learning methods to assist in predicting trauma bleeding volume[5].

The GM (1,1) model is suitable for predicting small sample data and meets the needs of predicting trauma rehabilitation, so this prediction model was adopted in the study. Predicting the trend of changes in patient trauma area and circumference over time can help medical staff evaluate the degree of recovery from trauma, further implement predictive treatment for patients, and reduce medical safety risks. At the same time, by predicting and reducing trauma mortality and disability rates, we can reduce the harm that trauma brings to humanity[6].

2. Experimental Data

The entire experimental data was collected from 4 patients with 5 stages of anal fistula medical trauma data, as shown in Table 1.

Table 1. Medical trauma data of patients with anal fistula

Patient	Number of periods	Area of trauma/cm ²	Perimeter of trauma/cm
1	1	0.7235	4.6694
	2	0.6497	4.2183
	3	0.5581	3.7236
	4	0.4873	3.2941
	5	0.4024	2.8493
2	1	1.0510	5.7021
	2	1.0034	5.4276
	3	0.9525	5.1318
	4	0.9049	4.8589
	5	0.8550	4.5472
3	1	0.8574	4.6941
	2	0.7998	4.3983
	3	0.7369	4.0816
	4	0.6731	3.7514
	5	0.6233	3.4456
4	1	0.5948	4.0537
	2	0.5322	3.6614
	3	0.4487	3.2018
	4	0.3806	2.8135
	5	0.3071	2.3631

3. Basic Theories and Methods

3.1. Basic Principles

In the late 1970s and early 1980s, Professor Deng Julong, a renowned scholar in China, proposed the Grey System Theory, which is an original discipline in China and an important discipline in the fields of deterministic theory and systems science[7]. Usually, a system where some information is known but some information is unclear is called a grey system. Grey system theory is used to solve problems of data scarcity and uncertainty. The grey system theory model, also known as the grey model or grey dynamic model, abbreviated as the GM model.

Grey prediction is a method of establishing a GM model that extends from the past to the future based on known or uncertain information from the past, in order to understand the development trend of events and provide a basis for planning decisions and state evaluation. Grey prediction is

mainly built on the basis of modeling. In addition to predicting future values such as population and food consumption, it can also predict the time when outliers occur, that is, disaster prediction[8].

3.2. Research Method

The grey system theory solves the problems of data scarcity and uncertainty. The GM (1,1) model is the most widely used grey dynamic prediction model in grey system theory. Due to its advantages of requiring less raw data and high accuracy, it has been widely applied in the field of medical research in recent years [9]. In the GM (1,1) model, G represents Grey and M represents Model. The first 1 in parentheses represents the first order of the differential equation, while the second 1 represents a variable in the differential equation. Its basic method is to accumulate the initial data to generate a data sequence, and then modify the accumulated sequence to a nearest neighbor generated sequence. The corresponding curve of the new data sequence can be approximated infinitely by a specific curve. The approximation curve is used as the basic model, and the predicted values are rolled and accumulated several times to predict the development trend. This study used the GM (1,1) model to construct a trauma prediction model, predicting the trauma of patients, and used the small residual probability P and a posterior difference ratio C value to evaluate the accuracy of the model.

3.3. Construction of GM (1,1) Model

3.3.1. Feasibility Analysis of Modeling

The rank ratio formula is $\sigma^{(0)}(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)}, k = 2, 3, \dots, n$, if all $\sigma^{(0)}(k)$ are within the interval $(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}})$ (e is the natural logarithm, n

is the sample size analyzed), then through the rank ratio test, the original time series can be predicted using the model GM (1,1) for grey prediction. If the level ratio is outside the interval, it does not meet the grey prediction condition and needs to be processed.

3.3.2. Data Preprocessing

(1) Record non negative original time series $X^{(0)}(k) = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}, k = 1, 2, \dots, n;$

(2) Record the one-time accumulation of the original time series to generate a sequence

$$X^{(1)}(k) = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}, k = 1, 2, \dots, n;$$

(3) Revise a sequence generated by accumulation to a sequence generated by the nearest mean $z^{(1)}(k) = \{z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n)\}$,

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, \dots, n.$$

3.3.3. Modelling Principles

The GM (1,1) model constructs differential equations on preprocessed data sequences, obtains the time response function of the differential equations, and then uses the cumulative method to perform reverse operations to obtain predicted values. The specific operations are as follows:

(1) The grey differential equation of the GM (1,1) model is $x^{(0)}(k) + az^{(1)}(k) = b$. The parameter in the formula is the development grey number, and parameter b is the endogenous

control grey number, $x^{(0)}(k)$ is grey derivative, $z^{(1)}(k)$ is whitening background value;

$$(2) \text{ Introducing a data matrix } B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \dots & \dots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \text{ data}$$

$$\text{vector } Y_n = \begin{bmatrix} x^0(2) \\ x^0(3) \\ \dots \\ x^0(n) \end{bmatrix}, \text{ parameter vector } u = \begin{bmatrix} a \\ b \end{bmatrix};$$

(3) Estimating the Parameter Vector of GM (1,1) Model Using Least Squares Method

$$\hat{u} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (B^T B)^{-1} B^T Y_n;$$

(4) Substitute the estimated values of parameters a and b into the whitening equation

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b, \text{ the solution to the equation, also}$$

known as the time response function, is obtained as

$$x^{(1)}(t) = (x^{(1)}(0) - \frac{b}{a})e^{-at} + \frac{b}{a}, x^{(1)}(0) = x^{(0)}(1);$$

(5) The time response sequence of GM (1,1) grey differential equation is:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}, k = 1, 2, \dots, n;$$

(6) The original data sequence is $\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), k = 1, 2, \dots, n$.

3.3.4. Accuracy Inspection

The GM (1,1) model may have certain errors in practical application, so after selecting the model, the accuracy of the model needs to be verified before the problem can be predicted and analyzed. The GM (1,1) model generally adopts three testing methods, namely residual test, correlation test, and posterior test. Only when all three tests are passed, can the constructed model meet the requirements and can be used for problem prediction and analysis.

(1) Residual Test

Set absolute residuals $\Delta(k) = |x^{(0)}(k) - \hat{x}^{(0)}(k)|$, relative

residual $\varphi_k = \frac{\Delta(k)}{x^{(0)}(k)}$, Average relative residual

$$\bar{\varphi}_k = \frac{1}{n} \sum_{k=1}^n \varphi_k, k = 1, 2, \dots, n, \text{ when the average relative}$$

residual value is less than 0.2, the model is called a qualified residual model.

(2) Correlation Test

Correlation coefficient

$$\eta(k) = \frac{\min\{\Delta(k)\} + \rho \max\{\Delta(k)\}}{\Delta(k) + \rho \max\{\Delta(k)\}}, k = 1, 2, \dots, n, \rho$$

is resolution coefficient, the smaller the value of ρ , the greater the difference between the correlation coefficients,

and the stronger the discriminative ability, $\rho = 0.5$, correlation $r = \frac{1}{n} \sum_{k=1}^n \eta(k)$, when the correlation is greater than 0.6, the correlation test is passed.

(3) Posterior Error Test

Posterior difference test is a test of the statistical characteristics of residual distribution, recording the mean square deviation of the original time series

$$S_1 = \left(\frac{\sum_{k=1}^n [x^{(0)}(k) - \bar{x}^{(0)}]^2}{n-1} \right)^{\frac{1}{2}},$$

Mean squared deviation of residuals

$$S_2 = \left(\frac{\sum_{k=1}^n [\Delta(k) - \bar{\Delta}(k)]^2}{n-1} \right)^{\frac{1}{2}}, k=1,2,\dots,n, \bar{x}^{(0)} \text{ is the}$$

average of the original data, $\bar{\Delta}(k)$ is the mean of absolute residuals.

By calculating the posterior difference ratio

$$C = \frac{S_2}{S_1}, \text{Small residual probability}$$

$P = p(0.6745S_1 > |\Delta(k) - \bar{\Delta}(k)|)$ determine the accuracy of the constructed model, the smaller the C value, the higher the credibility of the constructed model. The larger the P value, the better the accuracy of the model fit. The reference table for model accuracy level discrimination is shown in Table 2:

Table 2. Reference Table for Precision Level Discrimination of GM (1,1) Model

Model Accuracy level	P	C
Excellent	$P \geq 0.95$	$C \leq 0.35$
Qualified	$0.80 \leq P < 0.95$	$0.35 < C \leq 0.50$
Barely Qualified	$0.70 \leq P < 0.80$	$0.50 < C \leq 0.65$
Unqualified	$P < 0.70$	$C > 0.65$

4. Construction of Predictive Models

After calculation, the maximum level ratio of the original time series was 1.2393, and the minimum level ratio was 1.0474, both falling within (0.7165, 1.3956). The level ratio passed the test. Build a differential equation model and calculate the development grey number a, endogenous control grey number b, correlation degree r, and posterior error ratio C for each prediction model, as shown in Table 3. The predictive equations of the GM (1,1) model for the recovery of medical trauma in four patients are:

$$x_1^{(1)}(k+1) = -4.5407e^{-0.1547k} + 5.2642;$$

$$x_1^{(1)}(k+1) = -34.9547e^{-0.1288k} + 39.6241;$$

$$x_2^{(1)}(k+1) = -19.4366e^{-0.0530k} + 20.4876;$$

$$x_2^{(1)}(k+1) = -95.9526e^{-0.0583k} + 101.6547;$$

$$x_3^{(1)}(k+1) = -9.9269e^{-0.0840k} + 10.7843;$$

$$x_3^{(1)}(k+1) = -56.4519e^{-0.0813k} + 61.1460;$$

$$x_4^{(1)}(k+1) = -3.2713e^{-0.1780k} + 3.8661;$$

$$x_4^{(1)}(k+1) = -27.7773e^{-0.1418k} + 31.8310.$$

$x_1^{(1)}(k+1)$ represents the predictive equation for the

trauma area of the first patient; $x_1^{(1)}(k+1)$ represents the predictive equation for the circumference of the first patient's trauma and so on.

5. Discussion and Conclusion

Research has shown that the area and circumference of anal fistula wounds in four patients gradually decrease over time, and the wounds heal steadily. At present, scholars mostly predict trauma complications, trauma outcomes, and trauma stress disorders. Lee Na Hyeon, Kim Seon Hee et al. used chest computed tomography to quantify lung contusion volume and predict respiratory complications in patients with chest trauma [10]. Scholars have used injury severity score risk assessment tools to predict trauma outcomes [11]. In order to scientifically grasp the trend of trauma development, this article establishes a trauma GM (1,1) prediction model. The establishment of a trauma prediction model can improve the effectiveness of trauma treatment and increase the success rate of trauma treatment. Therefore, the study of trauma prediction models has important practical significance.

Although this study has made certain progress and achievements, there are still some shortcomings. The GM (1,1) model has a strong dependence on historical data, and the established model does not consider the relationship between various factors, resulting in a large error; Based on the research results of this article, the next step will be to optimize the initial conditions of the GM (1,1) prediction model and improve its prediction accuracy.

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