

First-principles Calculations of The Elastic Properties, Electronic Structure and Optical Properties of Hexagonal Al₄SiC₄

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Abstract: The crystal structure, elastic constants, electronic structure and optical properties of Al₄SiC₄ have been systematically investigated using the first principles calculation method based on density functional theory. The calculated lattice constants and elastic constants are consistent with the experimental and calculated values, and it shows that the hexagonal Al₄SiC₄ crystal structure is stable. The calculated bulk, shear, Young's modulus and Poisson's ratio of hexagonal Al₄SiC₄ are in accordance with the values in the literature. The calculated band gap of the Al₄SiC₄ is 1.076 eV. The changes of optical response functions of hexagonal Al₄SiC₄ in the (100) and (001) directions with photon energy are calculated, including complex dielectric function, complex refractive index, absorption spectrum and reflection spectrum. In the (100) and (001) directions, the static dielectric constants are 7.74 and 8.96, respectively, and the refractive indexes are 2.78 and 2.99, respectively. The calculated results can provide theoretical basis for related applications.

Keywords: Al₄SiC₄; Electronic structure; Optical properties; First-principles.

1. Introduction

Al₄SiC₄ ceramic is characterized by its low density (3.03 g/cm³), high melting point (> 2700°C), high chemical stability, high strength, low thermal expansion coefficient and excellent oxidation and hydration resistance. It has become a promising high-temperature structural material and high-performance refractory material to be developed, and has gradually attracted people's attention in the fields of aerospace structural ceramics and high-temperature electronic semiconductors [1][6]. Since Barczak et al. [7] first discovered Al₄SiC₄, several research groups [2][5][8][10] have focused on the synthesis and preparation of Al₄SiC₄ (such as solid-state reaction sintering method, chemical vapor deposition). Experimental study on the properties and applications of volume-solid phase sintering method, hot pressing sintering method, high temperature self-spreading method, penetration method, mechanical alloying method, etc. [3][11][20]. Inoue et al. [1] used a mixture of aluminum, silicon and carbon and a mixture of kaolin, aluminum and carbon as raw materials to synthesize Al₄SiC₄ powder by two-step method, and then conducted pulse current sintering to synthesize massive Al₄SiC₄. Yamamoto et al. [2] prepared Al₄SiC₄ powder with aluminum, silicon and carbon black as raw materials. Huang et al. [3] studied the high temperature oxidation resistance and high temperature mechanical properties of Al₄SiC₄ prepared by reactive hot pressing sintering, and obtained that the bending strength increased with the increase of temperature within the test temperature range of 1000 ~ 1300°C. Wen et al. [4] used Al, graphite powder and polycarbosilane (PCS) as raw materials to prepare Al₄SiC₄ bulk ceramics by reactive hot pressing method, and studied the mechanical properties, thermal properties and oxidation behavior of Al₄SiC₄ ceramics. Solozhenko et al. [14] synthesized Al₄SiC₄ polycrystalline samples by reacting aluminum with graphite and silicon, and fitted the experimental data of pressure and

volume (p-V) with Birch equation of state to obtain a volume modulus of 180.2 GPa. In recent years, theoretical studies on Al₄SiC₄ have also been reported, but not many [21][27], mainly focusing on the calculation of material properties based on first principles, such as lattice constants, elastic properties, band structure, optical properties and thermodynamic properties [21],[22],[24],[28]. Li et al. [24] calculated the phonon spectrum, chemical bond, thermal and thermodynamic properties of Al₄SiC₄ using density functional theory, and obtained that the linear thermal expansion coefficient of Al₄SiC₄ in the direction [001] was slightly higher. In the direction [100]. Liao et al. [25] have studied the bonding properties, elastic stiffness and ideal strength of Al₄SiC₄, and the results show that the mechanical properties of Al₄SiC₄ are closely related to the crystal structure, and the volume modulus is 179 GPa, the shear modulus is 140 GPa, and the lattice parameter is a = 3.222 Å. c = 21.352 Å. So far, there are few theoretical studies on the elastic properties, electronic structure and optical properties of Al₄SiC₄ including complex dielectric function, complex refractive index, absorption spectrum and reflection spectrum. Therefore, in this paper, the elastic properties, electronic structure and optical properties of Al₄SiC₄ are calculated using the mode conservation pseudopotential based on density functional theory, and the relationship between them is analyzed theoretically, which provides a theoretical basis for the practical application of Al₄SiC₄.

2. Computational Methods

This paper adopts the first principles calculation method based on density functional theory, and all the calculations are completed on the Cambridge Sequential Total Energy Package (CASTEP) [29] module. In the calculation process, Perdew-Burke-Ernzerhof (PBE) method on General gradient approximation (GGA) is selected for exchange correlation terms [30]. The interaction potential between ions and valence electrons is described by the mode conservation

pseudopotential. The valence electrons involved in the calculation are Al: $3s^23p^1$, Si: $3s^23p^2$, C: $2s^22p^2$. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm was used to optimize the structure of the cell model, and the truncation energy was set to 750 eV. The self-consistent cyclic convergence accuracy is 5.0×10^{-7} eV/atom, the stress error between atoms is less than 0.02 GPa, the maximum displacement convergence error between atoms is less than 5.0×10^{-4} Å, and the total energy convergence standard of the system is 5.0×10^{-6} eV/atom. The k point grid in Brillouin zone is $5 \times 5 \times 2$.

3. Result and Discussion

3.1. Lattice parameter

Al_4SiC_4 is a hexagonal system with space group P63mc [11], group number 186, lattice constant $a = b = 3.2771$ Å, $c = 21.676$ Å, $V = 201.60$ Å³, $\alpha = \beta = 90^\circ$, $\gamma = 120^\circ$. The crystal structure of Al_4SiC_4 is shown in Figure 1, where the purple is Al atom, the light gray is C atom, and the yellow is Si atom. Through the geometric structure optimization of Al_4SiC_4 cell, the lattice constant after optimization is obtained, as shown in Table 1. Compared with the experimental value [11], the difference between a and c is -0.63% and 0.31%, indicating that the model and calculation method adopted are credible.

Table 1. Calculated lattice parameters a, c (in Å), and volume V (in Å³) with the literature results for comparison

Method	a	c	V
Pres.	3.2562	21.6097	198.427
Expt. [11]	3.2771	21.676	201.60
Expt. [9]	3.2812	21.7042	—
Expt. [14]	3.277	21.74	201.60
Calc. [25]	3.222	21.352	—
Calc. [24]	3.282	21.796	—

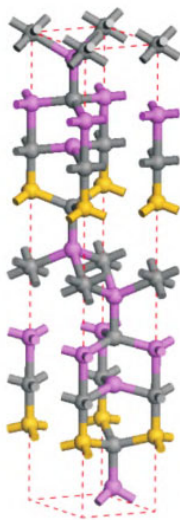


Figure 1. Crystal structures of Al_4SiC_4 . The purple, gray and yellow spheres represent the Al, C and Si ions, respectively.

3.2. Elastic property

For the hexagonal crystal phase Al_4SiC_4 , there are five independent constants, respectively C_{11} , C_{12} , C_{13} , C_{33} , C_{44} , $C_{66} = (C_{11} - C_{12}) / 2$, and the calculation results are shown in Table 2. As can be seen from Table 2, $C_{33} > C_{11} > C_{12} > C_{44} > C_{13} > 0$. According to the elastic stability criterion of hexagonal crystal structure [31]: $C_{11} - |C_{12}| > 0$, $C_{44} > 0$, $(C_{11} + C_{12}) C_{33} - 2(C_{13})^2 > 0$, it can be seen that the calculated elastic constant of Al_4SiC_4 satisfies the above stability conditions, so the crystal structure of Al_4SiC_4 is stable.

Table 2. Calculated second order elastic constants (C_{ij} in GPa) of Al_4SiC_4

Method	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}	C_{66}
Pres.	389.1	127.9	46.8	404.4	110.7	130.6
Calc. [25]	386	118	50	409	122	134
Calc. [24]	369.5	117	52.9	383.5	110.2	121.2

Based on the calculated elastic constant C_{ij} , the volume modulus and shear modulus can be obtained according to the Reuss-Voigt-Hill [32] model:

$$G_V = \frac{1}{15}(2C_{11} + C_{33} - C_{12} - 2C_{13}) + \frac{1}{5}(2C_{44} + \frac{C_{11} - C_{12}}{2})$$

$$G_R = 15 [4(2S_{11} + S_{33}) - 4(S_{12} + 2S_{13}) + 6(S_{44} + S_{11} - S_{12})]^{-1}$$

$$B_V = \frac{1}{9}(2C_{11} + C_{33}) + \frac{2}{9}(C_{12} + 2C_{13})$$

$$B_R = [2S_{11} + S_{33} + 2(S_{12} + 2S_{13})]^{-1}$$

$$G_H = (G_R + G_V) / 2$$

$$B_H = (B_R + B_V) / 2$$

Where S_{ij} is the elastic flexibility tensor, Poisson's ratio and Young's modulus can then be calculated:

$$\nu_X = \frac{3B_X - 2G_X}{2(3B_X + G_X)}$$

$$E_X = \frac{9G_X B_X}{3B_X + G_X}$$

Where X = Reuss, Voigt or Hill. The calculation results are shown in Table 3. The calculated bulk moduli of Reuss, Voigt and Hill are 186.0, 179.7, 182.2 GPa, respectively, which are consistent with the calculated values (179 GPa [25], 171.9 GPa [26]) and experimental value (182.0 GPa [14]) agree well, indicating that the calculation method is reasonable. The shear modulus is 134.5, 129.3, 131.9 GPa, which is close to the value in the literature (140 GPa [25]). Bulk modulus B and shear modulus G represent the ability of materials to resist compressive deformation and shear strain, respectively. The ratio of shear modulus and bulk modulus G/B is widely used to characterize the ductility and brittleness of materials. According to an empirical criterion proposed by Pugh [33], when $G/B > 0.57$, the material is brittle, and when

$G/B < 0.57$, the material is ductile. In this paper, the G/B value of Al_4SiC_4 calculated by GGA-PBE method is 0.73, which is greater than the critical value of 0.57, indicating that

Al_4SiC_4 is brittle, which is consistent with the predicted results of literature^[26].

Table 3. Calculated bulk modulus B_X ($X = V, R, H$) (in GPa), shear modulus G_X (in GPa), Young's modulus E_X (in GPa), Poisson's Ratio ν_X with the literature results for comparison

Method	B			G			E			ν_X		
	B_V	B_R	B_H	G_V	G_R	G_H	E_V	E_R	E_H	ν_V	ν_R	ν_H
Pres.	180.6	179.7	180.2	134.5	129.3	131.9	323.2	312.8	318.0	0.202	0.210	0.206
Calc.	172.0 ^[26]	171.7 ^[26]	179 ^[25] 171.9 ^[26]	127.0 ^[26]	123.6 ^[26]	140 ^[25] 125.3 ^[26]			302.4 ^[26]			0.206 ^[26]
Expt.		182.0 ^[14]			—				—			—

3.3. Electronic property

Band structure is an important basis for material analysis. The band structure of Al_4SiC_4 is shown in Figure 2. The dotted line in the figure represents the Fermi level. It can be seen from the figure that the conduction band base of Al_4SiC_4 is located at the high symmetric point M in the first Brillouin region, and the valence band top is located at the high symmetric point G in the first Brillouin region, so Al_4SiC_4 has an indirect band gap, which is consistent with the results reported by Forster et al.^[21]. The energy gap $E_g = 1.076\text{eV}$ of Al_4SiC_4 is consistent with the calculated value (1.05eV^[22], 1.12eV^[26]), but lower than the experimental value (2-2.5eV^[5]). Due to the deficiency of the calculation method in solving the excited state energy, the obtained band gap will be smaller than the experimental value, so the calculated value is smaller than the experimental value, but it does not affect the analysis of the band structure.

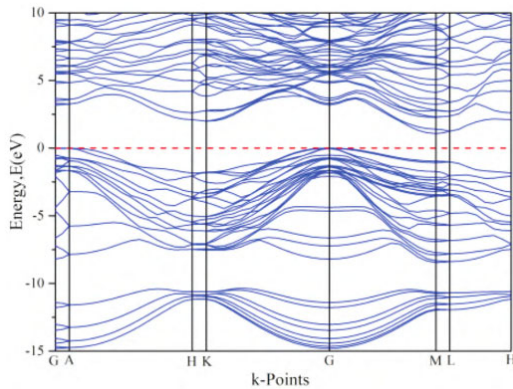


Figure 2. The band structure of Al_4SiC_4

Figure 3 shows the total state density and fractal density of Al_4SiC_4 . As can be seen from Figure 3, it can be roughly divided into three regions: $-15\text{ eV} \sim 10\text{ eV}$, $-8.8\text{ eV} \sim 0\text{ eV}$, and $0\text{ eV} \sim 19.1\text{ eV}$. In the low cost zone ($-15\text{ eV} - 10\text{ eV}$), it is mainly composed of Si-2s and C-2s electronic states, and the electron orbitals of the two atoms are hybridized, indicating that there is a strong covalent bond between the two atoms, so that the system has a more stable structure. The high valence zone ($-8.8\text{ eV} - 0\text{ eV}$) is mainly composed of SI-2P and C-2P electron states, and the density of states near the Fermi plane is mainly composed of p electron contributions of Al, Si and C atoms, and s electron contributions are less. The conduction band is mainly composed of Al-2p and Si-2p electron states.

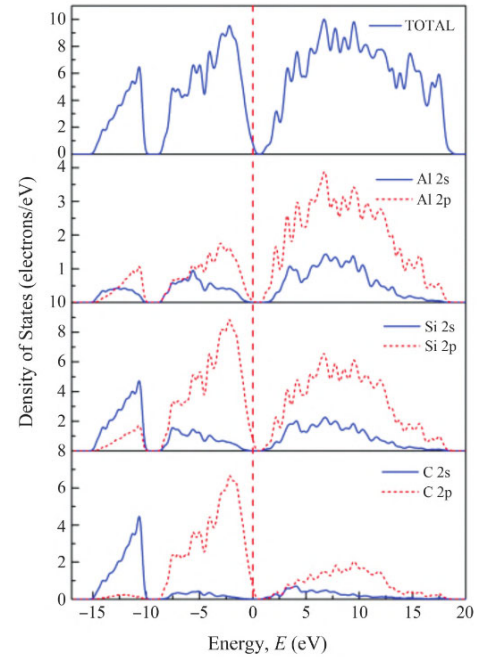


Figure 3. The total density of states and partial density of states of Al_4SiC_4

3.4. Optical property

3.4.1. Complex dielectric function

The macroscopic optical response of a solid can usually be described by the complex dielectric function $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$, where ϵ_1 and ϵ_2 are the real and imaginary parts of the complex dielectric function, respectively, and ω is the frequency of the photon. The calculated complex dielectric function curves of hexagonal crystal phase Al_4SiC_4 in the polarization direction of incident light along the a axis (100) and the c axis (001) are shown in Figure 4.

As can be seen from Figure 4, the static dielectric constant of Al_4SiC_4 in the direction of (100) and (001), that is, the value of zero frequency, is 7.74 and 8.96, respectively. In the direction of (100), in the low energy region, both the real and imaginary parts of the dielectric function increase with the increase of photon energy. The maximum value of the real part of the dielectric function at 4.38eV is 14, and the maximum value of the imaginary part at 5.86eV is 13.8, which is consistent with the results reported in the literature (the real part is 4.38eV^[22], the imaginary part is 5.82eV^[22]). Subsequently, the virtual part of the dielectric function decreases and approaches 0 after about 20 eV. The real part of the dielectric function decreases first and then rises, reaching the lowest value -4.12 at 9.07eV. The real part of the dielectric function increases slowly within the range of about 9 eV to

35 eV. In the direction of (001), the real part of the dielectric function reaches a maximum of 15.7 at 3.71 eV, and the imaginary part reaches a maximum of 15.3 at 5.32 eV.

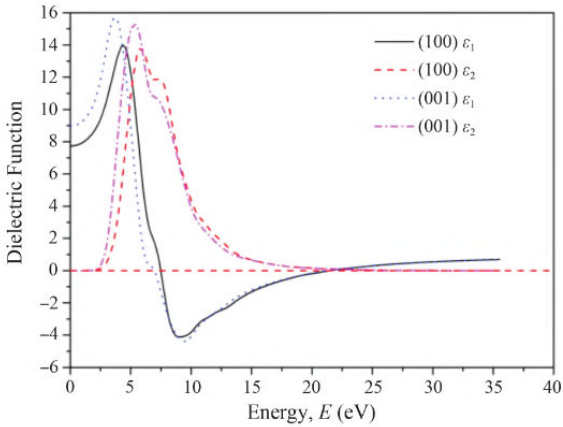


Figure 4. The complex dielectric functions of Al₄SiC₄

3.4.2. Complex refractive index

According to the relationship between dielectric function and complex refractive index: $\epsilon_1 = n^2 - k^2$, $\epsilon_2 = 2nk$, (n is the refractive index, k is the extinction coefficient), the refractive index n and extinction coefficient k of the hexagonal crystal phase Al₄SiC₄ can be calculated along the polarization direction of the incident light along the a axis (100) and the c axis (001) with the photon energy. As shown in Figure 5.

As can be seen from Figure 5, the refractive index of Al₄SiC₄ at zero frequency in the direction of (100) and (001) is 2.78 and 2.99, respectively, which is consistent with the results reported in the literature (2.73 [22]). With the increase of photon energy, the refractive index first increases, then decreases and then increases. The refractive index along the direction (100) reaches a maximum value of 3.85 at 4.63 eV and a minimum value of 0.164 at 20.62 eV. The extinction coefficient starts to increase significantly at 2.02 eV, reaches a maximum value of 2.61 at 8.2 eV, and then decreases. The refractive index along the direction (001) reaches a maximum value of 4.06 at 3.94 eV and a minimum value of 0.159 at 19.58 eV. The extinction coefficient starts to increase significantly at 2.04 eV, reaches the first peak at 2.46 eV at 5.95 eV and then declines, then continues to rise to 8.43 eV, reaches the second peak at 2.50 eV, and then slowly decreases.

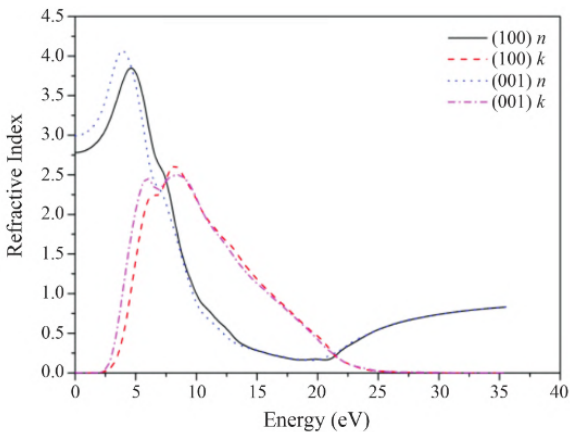


Figure 5. The complex refractive indexes of Al₄SiC₄

3.4.3. Absorption coefficient

Fig.6 shows the absorption coefficients of the hexagonal crystal phase Al₄SiC₄ along the polarization direction of the incident light along the a axis (100) and the c axis (001) with the photon energy. It can be seen that in the direction of (100) and (001), with the increase of photon energy, the absorption coefficient of Al₄SiC₄ first increases and then decreases, and the absorption coefficient is close to 0 after (0-2.5 eV) and 30 eV, respectively. The absorption coefficient reaches a peak value of $3.58 \times 10^5 / \text{cm}$ at 8.98 eV in the direction of (100), and $3.62 \times 10^5 / \text{cm}$ at 9.46 eV in the direction of (001).

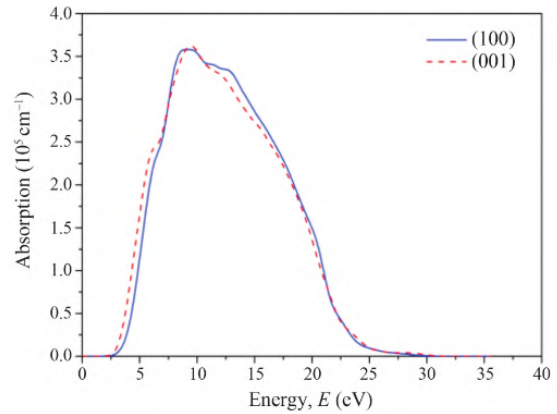


Figure 6. The absorption coefficients of Al₄SiC₄

3.4.4. Reflectance

Based on the relationship between reflectivity and rereflectivity [34]: $R(\omega) = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$, n is refractive index, k is extinction coefficient. The reflectance curve of Al₄SiC₄ along the a axis (100) and c axis (001) of the incident light polarization direction along with the photon energy can be obtained, and the calculation results are shown in Fig.7. As can be seen from Fig.7, the reflectance of Al₄SiC₄ in the direction of (100) and (001) generally increases with the increase of photon energy, at first shows a wave rise and then a rapid decline, and reaches a maximum value of 0.642 and 0.628 at 18.0 eV and 17.8 eV, respectively. When the photon energy is greater than 22 eV, the reflectance in both directions is almost the same. In the visible light energy range of 1.6eV ~ 3.2eV, the reflectance of (100) direction is between 24% ~ 29%, and the reflectance of (001) direction is between 27% ~ 35%, but the latter is slightly larger than the former. Materials with reflectance greater than 25% are generally considered to have a metallic luster. As a result, Al₄SiC₄ has a metallic luster.

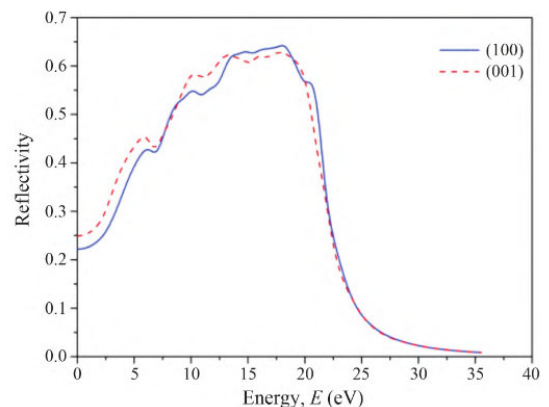


Figure 7. The reflectivities of Al₄SiC₄

4. Conclusion

In this paper, the geometric structure parameters of the hexagonal crystal phase Al_4SiC_4 are optimized by using the generalized gradient approximate mode conservation pseudopotential method based on the framework of density functional theory. The elastic constant, band structure, state density, complex dielectric constant, complex refractive index, absorption coefficient and reflectivity of the hexagonal crystal phase Al_4SiC_4 are calculated and analyzed. It provides theoretical support for the practical application of hexagonal phase Al_4SiC_4 .

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