

# The Underwater Gravity Matching Navigation Adaptation Area Is Preferred Based on The Multi-Starting Point TERCOM Algorithm

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**Abstract:** This paper focuses on the selection of matching area in gravity matching navigation system of underwater submersible. To solve the adaptation of the study region by using the TERCOM algorithm, this paper proposes the trajectory matching with three equal starting points within the study area. The experimental results show that this method can divide the adaptation area of the study area more comprehensively.

**Keywords:** TERCOM algorithm, Gravity matching navigation, Multi-starting point.

## 1. Introduction

Due to the rich marine resources and its important geographical location, countries' investment in the development and utilization of marine resources is increasing, which has greatly promoted the development of marine science and technology. The navigation and positioning function of underwater vehicle has gradually become one of the main directions of scientific research. At present, the most commonly used navigation system for underwater vehicle navigation and positioning is the inertial navigation system (INS). INS is an autonomous passive navigation and positioning system, which can give a variety of information such as position and attitude simultaneously, with high short-term navigation accuracy. However, there is a problem that the positioning error accumulates with time. When it is applied to long voyage, its divergent error must be corrected by other navigation methods. Presently, geophysical information is mainly used to correct the INS. Geophysical information measurements do not rely on external signals or emit energy externally, demonstrating a passive nature. Such passive navigation systems mainly include three types of matching navigation methods combining underwater terrain, geomagnetic field and gravity field information. Gravity matching navigation is positioned according to the changes of the earth's gravity field, does not need to transmit and receive signals. It is less susceptible to external interference, offering advantages such as concealment, real-time performance, and high accuracy. Consequently, gravity-matching navigation stands out as one of the most suitable navigation methods for assisting underwater inertial navigation [1-4].

The matching effectiveness of gravity-aided inertial navigation system (GAINS) is closely related to the distribution of gravity field in the navigation area. The same matching algorithm yields varying results when applied in regions with different gravity characteristics. The matching effect is generally better in areas with obvious gravity field characteristics, that is, when the navigation trajectory passes through the area where the gravity field changes more violently, the gravity sequence collected in real time will produce more obvious fluctuations, and it is easy to find the

trajectory consistent with it in the gravity map. However, there is no accurate correspondence between the gravity field distribution and the suitability. Therefore, it is of great significance to study the suitability of the gravity field in the navigation area to further improve the accuracy of gravity-assisted inertial navigation and plan the route [5,6].

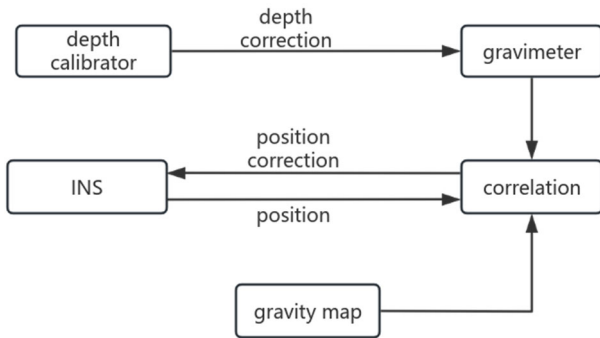
At present, there are a large number of studies on underwater matching navigation based on the TERCOM algorithm. In 2020, Mingda Ouyang studied the advantages and disadvantages of TERCOM algorithm and ICCP algorithm for underwater gravity matching navigation [7]. In 2021, Jiasheng Zou et al. proposed a new gravity matching navigation algorithm based on the combination of ICCP and TERCOM, which can greatly improve the accuracy of gravity matching [8]. In 2021, Zhensheng Xu et al. adopted the combination matching algorithm composed of TERCOM method and PMF algorithm to solve the problem that the current underwater terrain assisted navigation performance cannot be affected by multiple factors under a single algorithm. The results show that the combined algorithm can effectively reduce the matching error and improve the navigation performance under complex conditions [9]. In 2023, Ruiying Wu et al. proposed a two-step improved TERCOM algorithm for affine transformation. The results show that the matching accuracy of the improved algorithm is nearly 50% higher. In the case of mismatching, due to the large difference between coarse and fine matching results, it can play a certain warning role for the mismatching situation [10]. In 2023, Dan Wang et al. proposed a terrain-assisted navigation algorithm based on improved TERCOM. Compared with the traditional TERCOM algorithm, the proposed algorithm can reduce the heading error by 84% and improve the matching accuracy by more than 4 times [11].

Different from previous studies, because the results of the calibration adaptation zone are closely related to the selection of the start point, in order to make the calibration results more accurate, we use the TERCOM algorithm to divide the adaptation region.

## 2. TERCOM Algorithm

TERCOM The algorithm is mainly used in the navigation process of unmanned aerial vehicles, and it is a navigation

system used by comparing the pre-recorded topographic contour map with the measurement made by the airborne radar altimeter in flight. In addition to its application in unmanned aerial vehicles, the TERCOM algorithm is also applied in the underwater environment, which is currently widely used in the matching function of underwater terrain, geomagnetic and gravity auxiliary systems. The principle of the TERCOM algorithm is shown in Figure 1.



**Figure 1.** Principle of the TERCOM algorithm

The TERCOM algorithm is a relevant extreme value matching. The main function of its application to gravity-assisted inertial navigation is to find a path parallel to the path indicated by the inertial navigation system and closest to the navigation path in the high-resolution gravity anomaly map stored in the computer. When the underwater vehicle sails, the depth meter and gravimeter will collect the gravity data of the depth and current position of the vehicle at certain intervals, and a set of gravity outlier reference sequence can be obtained after depth correction. The inertial navigation system gives the current location of the current vehicle. Due to the influence of the inertial navigation error, the true position of the vehicle is generally not the position given by the inertial navigation system, but the real position is near the position given by the inertial navigation system. Centering on the location estimated by the inertial navigation system, a grid area is determined according to a certain range. We trathrough each grid in the search range and obtain multiple sets of sequences parallel to the track of the inertial navigation system. Each group of sequences is correlated with the reference sequences, and the grid position corresponding to the sequence with the highest correlation can be considered as the true position of the navigation vessel. The commonly used related algorithms are cross-correlation algorithm (COR), mean absolute difference algorithm (MAD), and mean variance algorithm (MSD), whose discrete forms are defined as:

$$J_{COR} = \frac{1}{n} \sum_{i=1}^n g_r(i) \cdot g_m(i) \quad (1)$$

$$J_{MAD} = \frac{1}{n} \sum_{i=1}^n |g_r(i) - g_m(i)| \quad (2)$$

$$J_{MSD} = \frac{1}{n} \sum_{i=1}^n |g_r(i) - g_m(i)|^2 \quad (3)$$

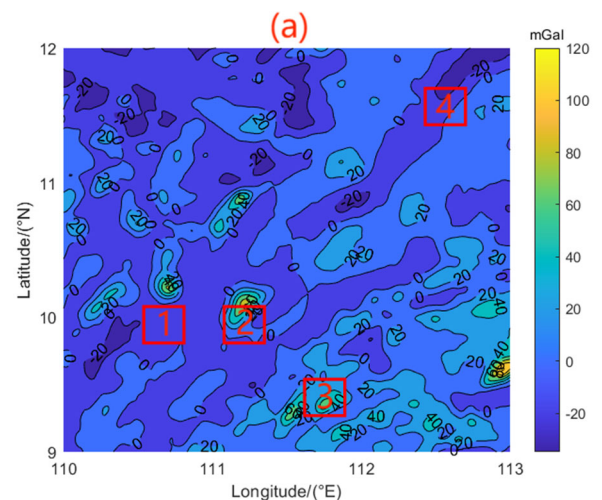
Where  $g_r$  is the measured gravity data,  $g_m$  is the data stored in the digital gravity anomaly map,  $n$  is the measured on the length of the gravity sequence.

The optimal matching path is to maximize the cross-correlation value and minimize the average absolute difference and mean variance value. Among them, the stability and accuracy of the cross-correlation index are the worst and should not be used as the decisive index; the mean absolute difference and mean variance index are close to the cross-correlation index, and the mean variance index is actually the index after amplifying the average absolute difference index, so the mean variance index is used more often in practical application.

TERCOM The algorithm does not require the initial position, is not easily affected by the initial positioning error, and has high reliability when the gravitational field features are rich and the search range is large enough. Moreover, the TERCOM algorithm process is simple and the calculation amount is small. After improvement, it can quickly locate and ensure a certain accuracy.

### 3. Results

In this paper, the gravity anomaly data of the University of California San Diego website (<http://topex.ucsd.edu/>) was used, and the resolution is  $1' \times 1'$ . Some areas of the South China Sea were selected for research, and the latitude and longitude ranges are  $110^{\circ}\text{E}$ - $113^{\circ}\text{E}$  and  $9^{\circ}\text{N}$ - $12^{\circ}\text{N}$ . Then, MATLAB software was employed to perform linear interpolation on the gravity map of this area. The spatial resolution of the interpolated image is approximately  $100\text{m} \times 100\text{m}$ , as shown in Figure 2, which is the two-dimensional gravity anomaly map of the study area with a resolution of  $100\text{m} \times 100\text{m}$ . The maximum value of gravity anomaly in this area is  $120.300 \text{ mGal}$ , the minimum value is  $-39.500 \text{ mGal}$ , and the average value is  $6.309 \text{ mGal}$ . In this paper, the sliding window is used to divide this area into 484 sub-regions. The sliding window size is  $150 \times 150$  grid cells, with the sliding step of 150 cells. The actual size of each sub-region is about  $15\text{km} \times 15\text{km}$ .



**Figure 2.** Marine gravity anomaly map with  $100\text{m} \times 100\text{m}$  resolutions.

In order to contrast and illustrate the relationship between characteristic parameters and positioning accuracy, four regions (1,2,3,4) in Fig.2 were selected for verification. It can be observed that regions 2 and 3 exhibit drastic variations in

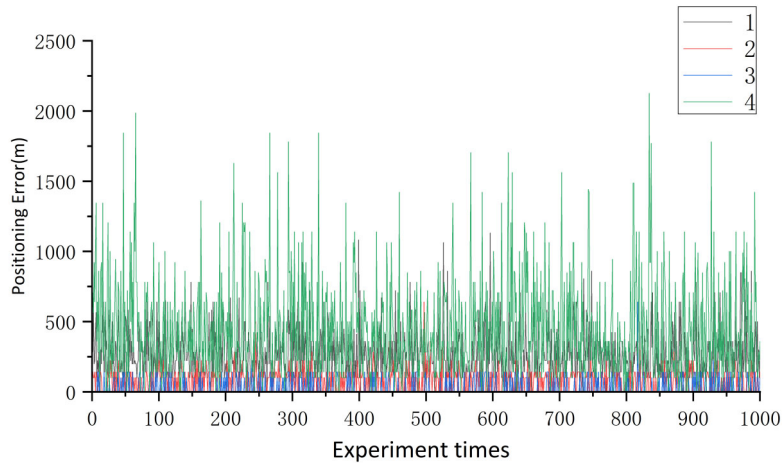
gravity anomalies, and regions 1 and 4 have relatively gentle variations.

The data in Table 1 are the feature values of the four regions(1,2,3,4) after z-score standardization. In theory, the larger parameter values indicate more prominent gravity field features and better matching effects. Upon comparison, it is evident that Region 2 has the largest values for all parameters, and Region 3 has positive values for all parameters, indicating more pronounced gravity anomalies in these two regions. On the other hand, the values of each parameter in regions 1 and 4 are negative, suggesting a relatively gentle distribution of gravity anomalies in these two regions. Overall, there are significant differences in the feature values between Regions 1,4, and Regions 2,3. By comparing with the gravity field distribution of each region in Figure 2, the selected

characteristic parameters in this paper can effectively reflect the gravity field distribution characteristics. Then 1000 trajectory matches were carried out in the selected four regions. The simulation parameters are set as follows: the constant drift of gyroscope in the inertial navigation system was set to 0.05°/h, the constant error of accelerometer was set to 0.04 m · s<sup>-2</sup>, the simulated speed was 10m/s. The number of sampling points was 140, and the sampling period was 10s. The random error with a standard deviation of 1mGal was superimposed on the sampling value as the real-time measurement data of the gravimeter, and the TERCOM algorithm was used for trajectory matching. The distribution of positioning accuracy obtained from the simulation experiments is shown in Figure 3.

**Table 1.** Characteristic parameters of the selected areas.

Area	Range	Pooling difference	Standard deviation of gravity anomaly	Roughness	Correlation coefficient
1	1.1032	-1.0852	-1.1184	-1.0332	-1.1207
2	1.1271	1.3245	1.2524	0.8901	1.2461
3	0.4941	0.0205	0.2262	0.8232	0.2401
4	0.5180	-0.2598	-0.3601	-0.6801	-0.3656



**Figure 3.** The positioning error of areas 1,2,3 and 4

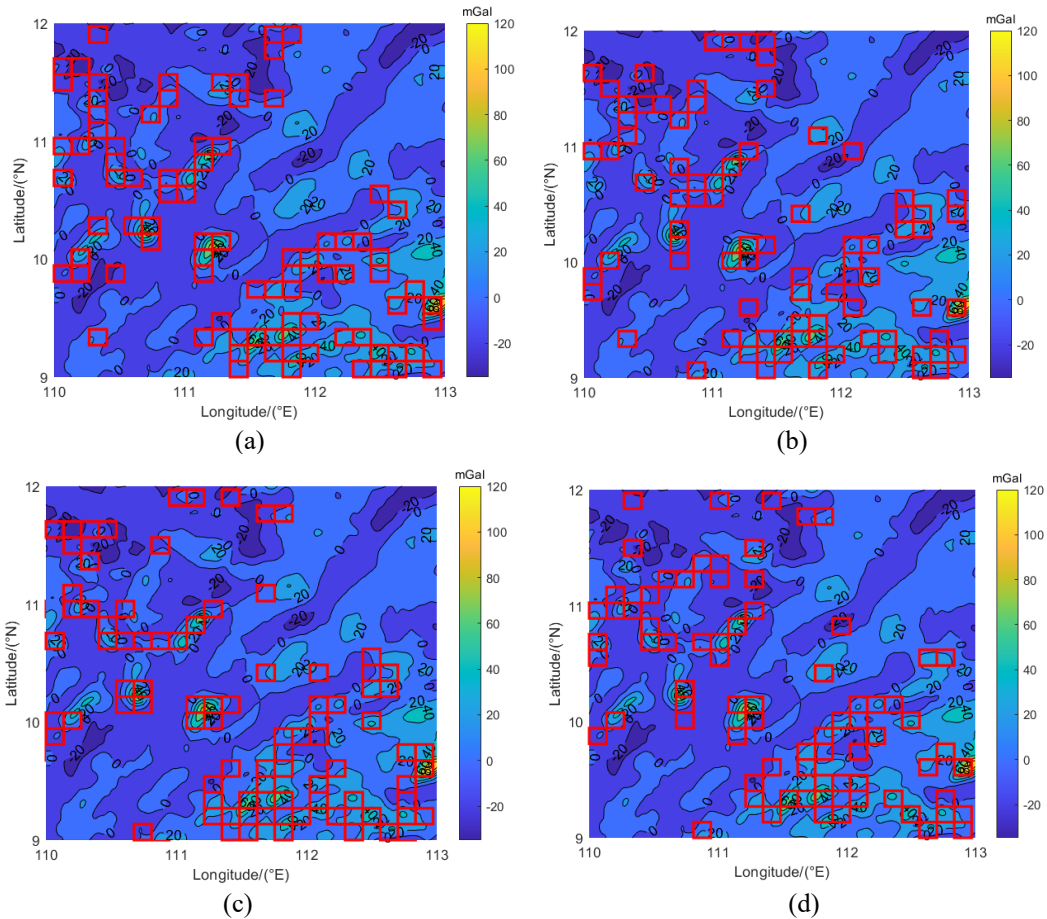
It can be seen from Fig.3 and Table 2 that the positioning accuracy mean for region 1 reaches 460.94 m, and the error fluctuation range is the most severe in 100 experiments, indicating the poorest matching performance in this region. The positioning accuracy of region 4 is smaller than that of region 1 but also reaches 348.77m, and most of the positioning accuracy exceeds 400m, and the matching effect is poor. In contrast, the matching effect of region 2 and 3 is obviously better. In contrast, regions 2 and 3 show significantly better results, with a positioning accuracy mean of 97.40m for regions 2 and 36.43m for regions 3. The positioning accuracy means for both regions are below 100m, and the matching performance is relatively stable.

**Table 2.** Statistics of experimental results of area 1,2,3 and 4.

Area	1	2	3	4
Average Positioning Error/m	460.94	97.40	36.43	348.77

experiments in 484 sample areas. Taking the average positioning accuracy as the standard, the areas with accuracy less than or equal to  $100\sqrt{2}$  m were calibrated as a suitable area, and the others were unsuitable. Due to the close correlation between the calibration results of the suitable areas and the selection of the starting point, in order to make the calibration results more accurate, the experiment selected three starting points for trajectory matching on four equal differences in each direction. The starting point for east-west heading is (3500,3500), (3500,7500), (3500,1150); The starting point for the northeast heading is (3000,3000), (3000,4600), (3000,6200); The starting point for the north-south heading is (3500,3500), (7500,3500), (11500,3500); The starting point for the northwest heading is (9000,3000), (9000,4600), (9000,6200). In the experiment, all the suitable areas for trajectory matching calibration from three starting points are considered to be suitable. Each rectangular border in the figure 4 shows a suitable area, and the remaining area are unsuitable areas.

TERCOM algorithm was used to complete 100 navigation



**Figure 4.** Distribution of suitable areas in application area  
 suitable areas for east-west direction; (b) suitable areas for northeast direction ;  
 (c) suitable areas for north-south direction; (d) suitable areas for northwest direction.

The matching effect of gravity-assisted inertial navigation is closely related to the distribution of gravity field in the navigation area. When the navigation trajectory passes through the area where the gravity field changes more violently, the gravity sequence of the real-time acquisition of gravity will produce more obvious fluctuations, and it is easy to find the corresponding trajectory in the gravity map. That is, the area with severe abnormal changes in gravity is usually the adaptation zone. For trajectory matching using the TERCOM algorithm, the use of the triple starting point allows for a more comprehensive assessment of the fit of the matching regions. As can be seen from Figure 4, the adaptation zones divided by using the three-starting point method are basically in the area where the gravitational field changes more dramatically. The experiment proves that the method proposed in this paper can divide the adaptation region of the study area more comprehensively.

## 4. Conclusions

Because the result of partitioning the adaptation region is closely related to the selection of the starting point, the study matches the three start points in four heading directions. Since the area with violent abnormal change of gravity in the weight map is usually the adaptation zone, it can be seen from the division results of the adaptation zone that the adaptation zones divided by this method are basically in the area of violent gravity field change, which has certain engineering application value.

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