

Study on the Gender Change of the Lamprey in Ecosystem Based on Verhulst and Logistic Models

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Abstract: This paper focuses on the environmental impacts of changes in the sex ratio of the lampreys, the assessment of the strengths and weaknesses of lamprey's adaptations, and their effects on ecosystem stability. In this paper, we first considered Verhulst and Logistic models of ecosystem-sex-ratio interactions, and predicted and analyzed the potential cascading effects of changes in lampreys sex ratio on larger ecosystems under different scenarios. Then, by integrating the Logistic model, we simulated the development and change of the ecosystem under various conditions, identified assessment metrics, and evaluated the relative strengths and weaknesses of the lampreys. The research in this paper provides important evidence for a deeper understanding of the role of the lampreys in ecosystems and the effects of changes in their sex ratio.

Keywords: Verhulst Model, Logistic Model, Dynamic Model.

1. Introduction

In natural ecosystems, the sex ratio of males to females in most species' populations will remain equal at 1:1, but the lampreys are different in that the proportion of sex change may deviate from this even ratio with changes in the external environment [1]. Scott et al [2] showed that the growth rate of the lampreys is affected by food, density, genetics, and the environment, and that the lampreys eel's growth rate determines its sex, so that ecological conditions determine the change of sex ratio of seven-gill populations. In this paper, the correlation between the sex ratio of lampreys and their specific living environment conditions was explored through mathematical modeling.

First, this paper focuses on establishing Verhulst and logistic models for the sex ratio changes of lamprey's populations under different natural marine environmental conditions, predicting and analyzing the potential cascading effects of the changes in the sex ratio of lampreys on larger ecosystems under different scenarios, and comparing the changes in the sex ratio and the environmental factors in the iterative process. The relative strengths and weaknesses of the lampreys were then assessed by modeling and comparatively analyzing food webs in lamprey's habitat. By integrating the logistic model, the development and changes of the ecosystem under various conditions were simulated and assessment indicators were identified. The study in this paper provides new ideas for exploring and analyzing the correlation between population growth and changes in population sex ratio of the lampreys and specific conditions in the ecosystem.

2. Racial Dynamic Model

2.1. Population dynamic models

The dynamic model for the population size of lampreys is established to describe the growth of population size under limited resource conditions. N is defined as the population size of lampreys and the Verhulst model is established [3]. The formula is as follows:

$$\frac{dF}{dn} = r_F P f(n) \left(1 - \alpha_F \frac{P(n)}{P f(n)}\right) \quad (1)$$

$$\frac{dM}{dn} = r_M P(n) \left(1 - \alpha_M \frac{P f(n)}{P(n)}\right) \quad (2)$$

In the above equation, r_F is the inherent growth rate of female lampreys. r_M is the inherent growth rate of male lampreys. $P f(n)$ is the proportion of lamprey's females, $P(n)$ is the proportion of lamprey's males, and K is the maximum average biomass of species within the range of environmental restrictions. α_F is the saturation parameter for the growth of female lampreys, on the contrary, α_M is males. n is the number of iterations.

2.2. Logistic model for lamprey sex ratio

The logistic model for lamprey's sex ratio is established [4], which could analyze the impact of food supply on the developmental rate of juvenile lampreys and changes in the sex ratio of the lamprey's population. The formula is as follows:

$$P(n) = \frac{1}{1 + e^{-r(n-n_0)}} \quad (3)$$

In the above equation, $P(n)$ represents the possibility of juvenile lampreys developing into females. Parameter r is the development rate, and n_0 is the time when the sex ratio of lampreys begins to change.

Considering the complexity of the sex determination mechanism in lampreys, a more detailed model needs to be constructed. The environmental factor T is introduced as a key regulatory parameter to expand the model, which can more comprehensively reveal how environmental conditions affect changes in the sex ratio of lamprey's populations.

$$P(n) = \frac{1}{1 + e^{-r(T)(n-n_0)}} \quad (4)$$

In the above equation, $r(T)$ represents the effect of T on the development rate of lampreys.

2.3. Ecosystem model

Through in-depth research and application of models for the population size and sex ratio of lampreys, it is possible to clarify the changes in population size and sex ratio of lampreys under different specific living environment conditions, as well as how these changes further affect a wider ecosystem.

2.3.1. Changes in the ecological balance

In order to explore the impact of changes in the sex ratio of the lamprey population on the quantity and distribution of other biological groups, the food supply and male sex ratio of the lamprey population were included in the model [5]. The

ratio of initial food resources to the male population of lampreys is defined as the food supply. The formula is as follows:

$$F = \frac{\mu F_f}{M} \quad (5)$$

In the above equation, F is the food supply and influencing factor, and M is the initial amount of food resources. The initial food resource is 100, and the change in male to female sex ratio from 0 to 1 was simulated. The results are shown in Figure 1.

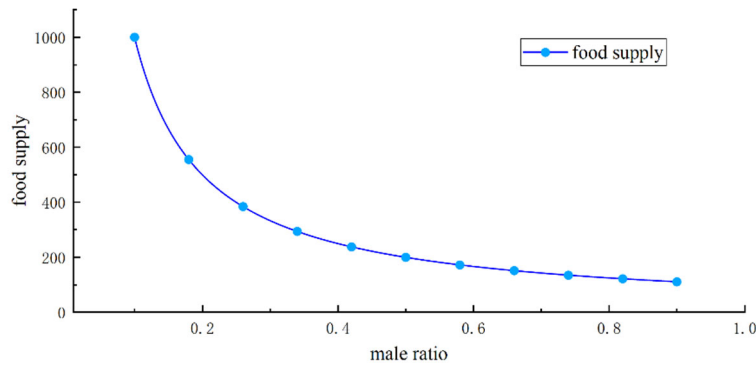


Figure 1. The effect of food supply on lamprey's sex ratio

Figure 1 shows that as the food supply decreases, the proportion of females in the lamprey population significantly decreases. The food supply has a significant impact on the sex ratio of the lamprey population. When the food supply decreases to a certain extent, the population of lampreys is entirely male. In this extreme situation, due to the lack of female individuals, the population is unable to engage in normal reproductive activities, resulting in a sharp decrease in population size.

2.3.2. The impact of the food chain

In order to explore the impact of gender ratio changes in the population of lampreys on the number of predators, the following equation is established:

In the case of sufficient food:

$$P(n) = P(n - 1) + \beta \quad (6)$$

$$Pt(n) = Pt(n - 1) \cdot (1 - \theta) \quad (7)$$

In the case of food shortage:

$$P(n) = P(n - 1) + \gamma \quad (8)$$

$$Pt(n) = Pt(n - 1) \cdot (1 + \xi) \quad (9)$$

In the above formula, $Pt(n)$ is the number of predators. β is the rate of increase in the male proportion of lampreys. γ is the rate at which the male proportion of lampreys decreases. ξ is the rate at which predators increase, and θ is the rate of predator reduction. Assuming 50 iterations, the resulting line graph is in Figure 2.

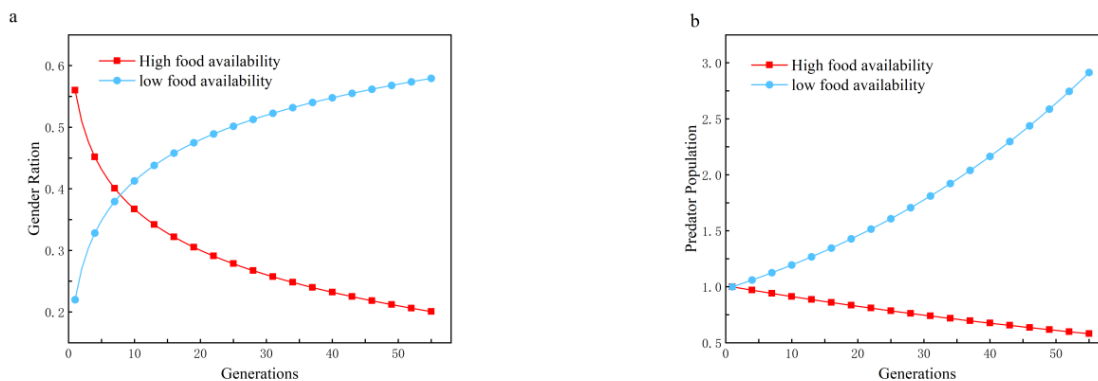


Figure 2. Changes in sex ratio of lampreys and their impact on predator numbers

Figure 2 shows that when food resources are abundant, over time, the male sex ratio of lampreys decreases, and the

number of predators increases. When food resources are scarce, the male sex ratio of lampreys increases, and the number of predators decreases. Therefore, under sufficient food conditions, there is a positive correlation between the proportion of females in the lamprey population and the number of predators.

2.3.3. The impact of reproductive success

Due to the influence of various factors on the sex ratio of the lamprey population during each iteration, the formula is adjusted to:

$$P(n + 1) = \min(1, \max(P(1), P(n) + \varphi)) \quad (10)$$

In the above equation, φ represents a random number with a standard normal distribution and establish a model:

$$Ra(n + 1) = \min(1, \max(Ra(1), Ra(n) + \varphi)) \quad (11)$$

In the above equation, $Ra(n)$ is the resource availability rate, and the success rate of reproduction (Rs) of lampreys is calculated based on the sex ratio of the population. Assuming that the success rate of reproduction of lampreys is directly proportional to the change in population sex ratio, the formula is expressed as:

$$Rs = 1 - abs(P(n) - \vartheta) \quad (12)$$

In the above equation, ϑ Indicates the proportion of male sex of lampreys at the optimal reproductive rate. The final simulation diagram is shown in Figure 3.

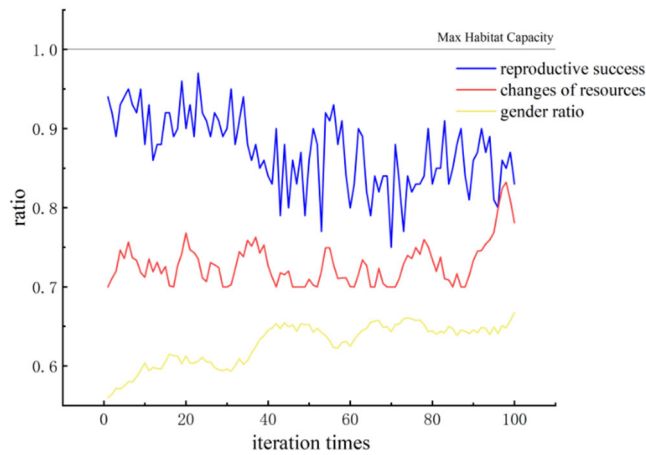


Figure 3. The impact of the change of lamprey's sex ratio on their own reproduction

As shown in Figure 3, the success rate of reproduction of lampreys is closely related to changes in their population sex ratio. As available resources increase, the proportion of females in the lamprey population increases and the success rate of reproduction decreases. Therefore, resource changes can lead to changes in the sex ratio of the lamprey population, thereby affecting the success rate of lamprey reproduction.

2.3.4. Changes in habitat use

Firstly, the sex ratio of the lamprey population is simulated, then the habitat occupation of the lamprey is simulated in each generation. Finally, simulation analysis is conducted by controlling the growth rate of its sex ratio [6]. The formula is as follows:

$$P(n) = 1 - Pf(n) \quad (13)$$

$$Pf(n) = Pf(n - 1) + \varphi \quad (14)$$

$$Op = Hc \cdot Pf(n) + \varepsilon \quad (15)$$

In the above equation, $Pf(n)$ is the proportion of lamprey's females after n iterations. Op is the proportion of habitat occupation, and Hc is the total capacity of the habitat. ε is other influencing factor. The results are shown in Figure 4.

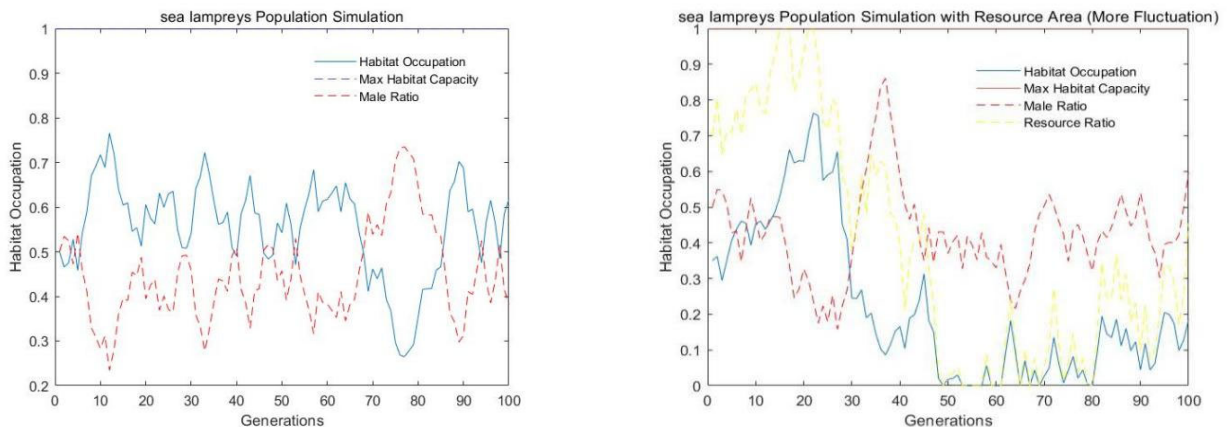


Figure 4. The impact of the change of lamprey's sex ratio on their habitat use

Without limiting resources, Figure 4 shows that the proportion of male sex in the lamprey population increases and the habitat area decreases. When the habitat area reaches its lowest point, the male sex ratio of the lamprey species reaches its maximum habitat capacity.

Assuming limited resources, interspecific competition is also one of the factors affecting habitat area. this situation is simulated. The formula is as follows:

$$Rr(n) = Rr(n - 1) + \varphi \quad (16)$$

$$Op = Hc \cdot Pf \cdot Rr(n) \quad (17)$$

In the above equation, $Rr(n)$ is the proportion of resources. The results showed that the proportion of male population in lampreys increased, while the habitat area and resource share decreased. From the results of 100 iterations, it can be seen that the proportion of male population and resource share of lampreys are generally decreasing. The higher the proportion of male population, the lower the habitat area.

2.3.5. Ecosystem stability

By simulating the change of seven species of lamprey's sex

ratio in ecosystem, the comprehensive index is calculated. the following Figure 5 is obtained.

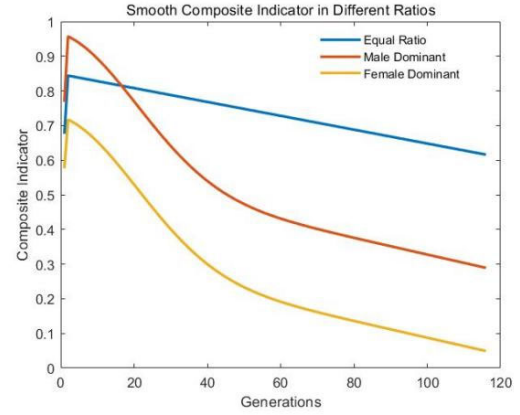


Figure 5. The change of the change of lamprey's sex ratio in ecosystems

Figure 5 shows that when the sex ratio of the lamprey population changes, it will have a chain reaction on the balance of the entire ecosystem.

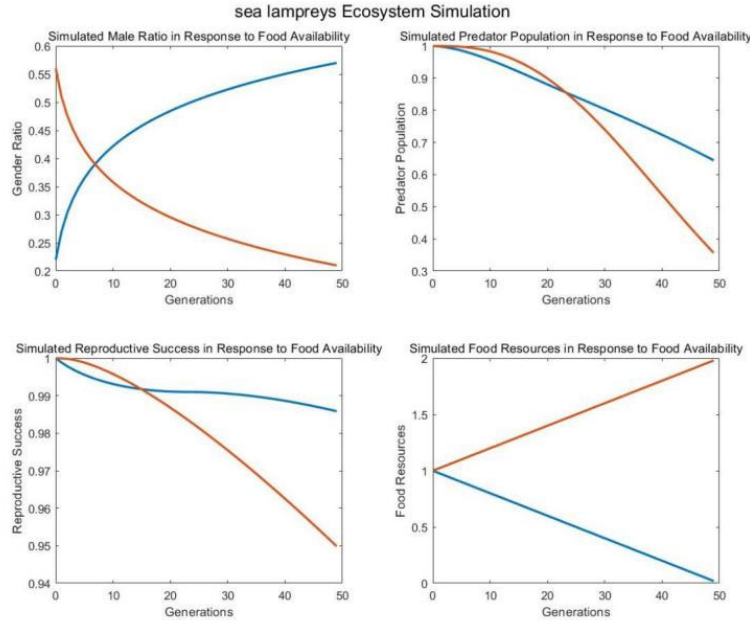


Figure 6. The impact of the change of lamprey's sex ratio on ecosystem stability

Figure 6 shows that the sex ratio of the lamprey population changes, and its population size and reproductive success rate also change. The predator number of lampreys and their own feeding habits will adjust accordingly, which will affect other species in the local ecosystem. In some cases, imbalanced sex ratios in lamprey populations may lead to a decrease in the stability of local ecosystem resistance.

3. Food Web Model

The development and gender ratio changes of lamprey populations are simulate under different ecosystem conditions. The formula is as follows:

$$P(n) = P(n - 1) + Lf \quad (18)$$

$$Lf = \max(-0.1P(n), \min(0.1P(n), \varphi)) \quad (18)$$

$$P(n) = \max(0, \min(P(n))) \quad (19)$$

The above equation indicates that the change in the male sex ratio of each generation of lampreys depends on the male sex ratio of the previous generation.

To limit fluctuations, the male sex ratio of the lamprey population can be controlled within 10% of the initial value. The following formula can be derived:

$$Rs(n) = \text{Reproductive success rate} \cdot |1 - P(n)| \quad (20)$$

The success rate of reproduction is influenced by the male sex ratio of the lamprey population.

$$Pt(n) = \tau \cdot Pt(n - 1) \cdot (Rs(n) + 1) \quad (21)$$

In the above equation, τ is the survival rate of lampreys.

$$Fr(n) = \min(1, \max(0, Resource\ quantity - Pt(n))) \quad (22)$$

$$Ci = w_{ra} \cdot P(n) + w_{re} \cdot Rs(n) + w_{pr} \cdot Pt(n) + w_{fd} \cdot Fr(n) \quad (23)$$

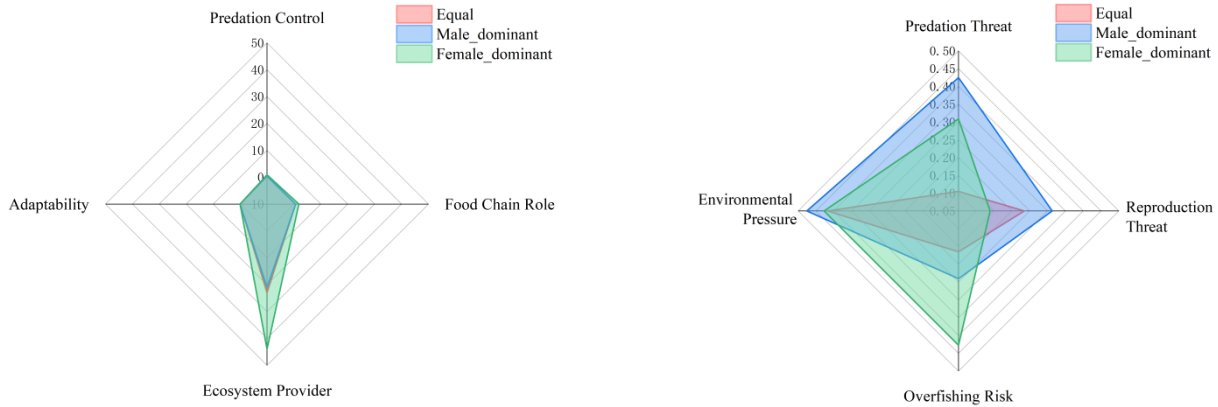


Figure 7. Radar chart of lamprey’s sex ratio under different ecosystem conditions

Figure 7 indicates that ecosystem provider is the main advantage of lampreys. At the same time, the advantage of female dominance is greater than that of male dominance and gender equality, while the latter two are basically the same in their evaluation advantages. It shows that they provide a variety of ecological services and contributions to the entire ecosystem. These services and contributions are of great significance for maintaining ecological balance, stability and biodiversity. On the whole, in the three cases, the dominant disadvantage is environmental pressure, and the following disadvantages need to be discussed in three cases. The first is the average gender ratio, followed by reproduction threat. When the male is dominant, Predation Threat is a secondary disadvantage, and finally the female is dominant, Overfishing Risk is a secondary disadvantage.

4. Conclusion

The aim of this paper is to investigate the environmental impacts of changes in the sex ratio of the lampreys, the assessment of the strengths and weaknesses of sex adaptations in the lampreys, and their impacts on ecosystem stability. First, Verhulst and logistic models of sex ratios in lampreys’ populations were developed to predict and analyze the effects of variation in lampreys sex ratios on larger ecosystems under different scenarios. It was found that while variability in the sex ratio of lampreys enhanced their adaptability, it also increased the uncertainty of the ecosystem as a whole. Second, the relative strengths and weaknesses of lampreys were assessed by modeling and comparatively analyzing food webs in their habitats. It was found that the main advantage of the

In the above equation, $Fr(n)$ is the limited food resource. " w_{ra} ", " w_{pr} ", " w_{re} ", and " w_{fd} " represent the weights of various variables such as the male sex ratio, reproductive success rate, predator number, food, and resources of the lamprey population, respectively. This formula represents the calculation of comprehensive indicators, which can be used to evaluate the state of the entire ecosystem. The weight can be adjusted according to the actual situation to reflect the relative importance of different factors.

By simulating the male dominant ratio, female dominant ratio, and male to female equal ratio in the population of lampreys, a radar chart is obtained, as shown in Figure 7.

lampreys was its role as an ecosystem provider, while the main disadvantage was environmental stress, and secondary advantages and disadvantages also varied by sex ratio. This paper investigates the importance of the balanced sex ratio of the lamprey’s population for the maintenance of ecological balance, stability and biodiversity.

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