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Design and Simulation of Butterworth Low-pass Filter

Hao Feng, Linjie Sun

Taiyuan University of Technology, China

Abstract: Filters are required to filter the signal to improve the signal-to-noise ratio. It is known that the maximum frequency of the human ear is about 20 kHz, and filtering out the signal components above 20 kHz can greatly improve the audio signal quality. Therefore, the design requires the cut-off frequency of 20 kHz. In this design, the normalized transfer function of the fourth-order Butterworth filter is obtained by using the normalization process, and the denormalization process is carried out according to the cutoff frequency of 20kHz, the transfer function of the fourth-order Butterworth filter is obtained.The idea of circuit cascade is applied to derive the parameters of the components, so that a filter that meets the design requirements is obtained, and the filter is used in the Multisim and Matlab simulation software to verify the filtering effect.

Keywords: Filter, cut-off frequency, normalization process, Butterworth.

1. Introduction

The Butterworth filter, also known as the maximum flat filter, is characterized by a high degree of flatness of the waveform curve in the passband, small ripple, the waveform curve can be kept monotonically decreasing, and the amplitude has the same trend as the diagonal frequency. Compared with Bessel and Chebyshev filters, Butterworth filters have the advantage of balanced characteristics in the three aspects of linear phase, decay slope and loading characteristics.In practice, Butterworth filters have been listed as the first choice, which is widely used in the field of communications, motor testing, radar signal processing field, high-quality audio and so on.

In this paper, the specific parameters of the fourth-order Butterworth filter are calculated by normalization, according to the design requirement of the cutoff frequency of 20kHz, and simulated and verified by Multisim, and the fourth-order Butterworth filter that can be applied to audio devices is designed.

2. The Butterworth Low-pass Filter Design Principle

Amplitude squared function of the Butterworth low-pass filter $|H_a(j\omega)|^2$ can be expressed by the following equation:

$$|H_a(j\omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} \tag{1}$$

Where N represents the order of the filter, and Ω_c is the cutoff frequency of the filter, the frequency when the amplitude decayed by $3dB$.Around $\Omega = \Omega_c$, the amplitude decays rapidly as the Ω increases. The speed of the amplitude decrease is related to the order N. The larger N, the flatter the pass band, the narrower the transition band, the faster the amplitude decrease of the transition band and the resistance band, and the smaller the error between the total frequency sound characteristics and the ideal low-pass filter.

With s instead of $j\Omega$,write the amplitude square function $|H_a(j\Omega)|^2$ as a function about s[1] :

$$H_a(s)H_a(-s) = \frac{1}{1+\left(\frac{s}{j\Omega_c}\right)^{2N}} \tag{2}$$

The amplitude square function has $2N$ poles,and the poles s_k is represented by the following equation:

$$s_k = \Omega_c e^{j\pi\left(\frac{1}{2}+\frac{2k+1}{2N}\right)} \tag{3}$$

In the equation $k = 0,1,2,\cdots,2N-1$。 The distribution of 2N poles is equally spaced on a circle of radius Ω_c.In order to ensure that the filter is causally stable, the N poles in the left half plane of the s in $2N$ poles are taken to form $H_a(s)$, and the remaining N poles in the right half plane of the s form $H_a(-s)$. The expression of $H_a(s)$ is:

$$H_a(s) = \frac{{\Omega_c}^N}{\prod_{k=0}^{k=N-1}(s-s_k)} \tag{4}$$

Due to the different boundary frequency and filter amplitude frequency characteristics corresponding to different technical indicators, the frequency is normalized to unify the design formula and chart. The Butterworth filter is normalized by the $3dB$ cutoff frequency, and the normalized system function is:

$$G_a\left(\frac{s}{\Omega_c}\right) = \frac{1}{\prod_{k=0}^{k=N-1}\left(\frac{s}{\Omega_c}-\frac{s_k}{\Omega_c}\right)} \tag{5}$$

Order $p = \frac{s}{\Omega_c}$, $\lambda = \frac{\Omega}{\Omega_c}$, λ is called the normalized frequency, p is called the normalized complex variable, and now we get the system function of the normalized Butterworth low-pass filter:

$$G_a(p) = \frac{1}{\prod_{k=0}^{k=N-1}(p-p_k)} \tag{6}$$

Write in a polynomial form as:

$$G_a(p) = \frac{1}{p^N+b_{N-1}p^{N-1}+b_{N-2}p^{N-2}+\cdots+b_1p+b_0} \tag{7}$$

In this funtion $p_k = \frac{s_k}{\Omega_c}$ is the pole after normalization:

$$p_k = e^{j\pi\left(\frac{1}{2}+\frac{2k+1}{2N}\right)} \tag{8}$$

Here, the polynomial coefficient table and the polynomial factorization table corresponding to the Butterworth filter order N are given as shown below[2]:

Table 1. Table of polynomial coefficients

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
1	1.0000								
2	1.0000	1.4142							
3	1.0000	2.0000	2.0000						
4	1.0000	2.6131	3.4142	2.6131					
5	1.0000	3.2361	5.2361	5.2361	3.2361				
6	1.0000	3.8637	7.4641	9.1416	7.4641	3.8637			
7	1.0000	4.4940	10.0978	14.5918	10.0978	4.4940			
8	1.0000	5.1258	13.1371	21.8462	25.6884	21.8642	13.1371	5.1258	
9	1.0000	5.7588	16.5817	31.1634	41.9864	41.9864	31.1634	16.5817	5.7588

Table 2. Table of polynomial factorization

N	polynomial form
1	(p^2+1)
2	$(p^2+1.414p+1)$
3	$(p^2+p+1)(p+1)$
4	$(p^2+0.7654p+1)(p^2+1.8478p+1)$
5	$(p^2+0.6180p+1)(p^2+1.6180p+1)(p+1)$
6	$(p^2+0.5176p+1)(p^2+1.4142p+1)(p^2+1.9319p+1)$
7	$(p^2+0.4450p+1)(p^2+1.2470p+1)(p^2+1.8019p+1)(p+1)$
8	$(p^2+0.3902p+1)(p^2+1.1111p+1)(p^2+1.6629p+1)(p^2+1.9616p+1)$
9	$(p^2+0.3473p+1)(p^2+p+1)(p^2+1.5321p+1)(p^2+1.8974p+1)(p+1)$

In this way, check the system function of the Butterworth low-pass filter normalized in Table 2 according to the filter order N, and then according to:

$$s_k = \Omega_c p_k \tag{9}$$

Realize the normalization, get $H_a(s)$,and finally calculate each component parameters according to the circuit structure.

3. Design Process of Butterworth Low-pass Filter

3.1. The process of deriving the expression for parameter calculation

Figure 1 shows the circuit structure of a typical second-order Butterworth filter, and its transfer function is:

$$H(s) = \frac{KB{\Omega_c}^2}{s^2+A\Omega_c s+B{\Omega_c}^2} \tag{10}$$

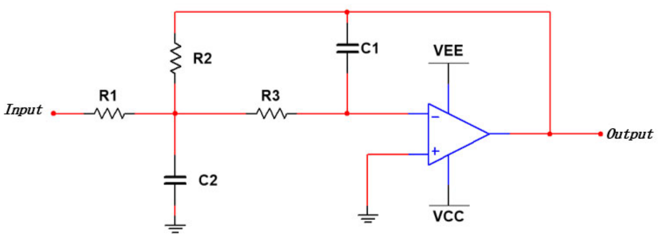

Figure 1. Circuit diagram of a second-order Butterworth filter

According to the derivation:

$$A{\Omega_c}^2 = \frac{1}{R_2R_3C_1C_2} \tag{11}$$

$$B\Omega_c = \frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) \tag{12}$$

$$K = \frac{R_2}{R_1} \tag{13}$$

Sorted out:

$$R_1 = \frac{R_2}{K} \tag{14}$$

$$R_2 = \frac{2(K+1)}{\left[AC_2+\sqrt{A^2C_2^2-4BC_1(K+1)}\right]\Omega_c} \tag{15}$$

$$R_3 = \frac{1}{BR_2{\Omega_c}^2C_1C_2} \tag{16}$$

C_1 and $C_2$are arbitrary values. Each resistor is measured in Ω,The unit of each capacitor is F, therefore, when K,A,B and Ω_c are given, First, arbitrarily select the values of the C_1

and C_2, The values of each resistance of the circuit are then calculated. There is a rule of thumb for the capacitance selection of capacitors: The C_2 selected is an approximate $\frac{10}{f_c}$ μF nominal value, which C_1 satisfied:

$$C_1 \leq \frac{A^2 C_2}{4B(K+1)} \tag{17}$$

3.2. Filter order selection

Figure 2 shows the amplitude-frequency characteristics of different order filters:

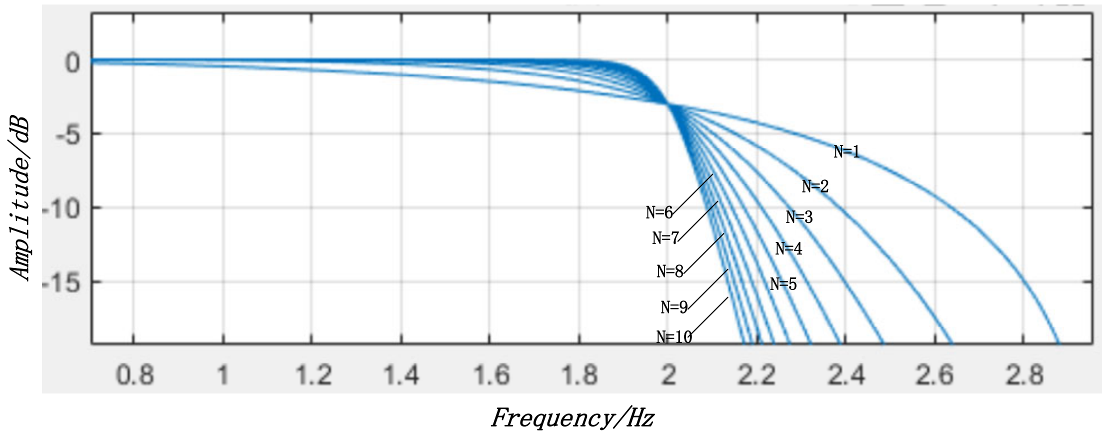

Figure 2. Amplitude-frequency characteristics of filters of different orders

When the order of the filter is greater than the fourth order, the stopband descent rate does not change obviously, and the larger the order, the more complex the design[3], All things considered, the design adopts a fourth-order filter.

3.3. Filter parameter calculations

Looking up the Table 2, the normalized transfer function is:

$$G_a(p) = \frac{1}{(p^2+0.7654p+1)(p^2+1.8478p+1)} \tag{18}$$

Its four poles are released as shown in Figure 3:

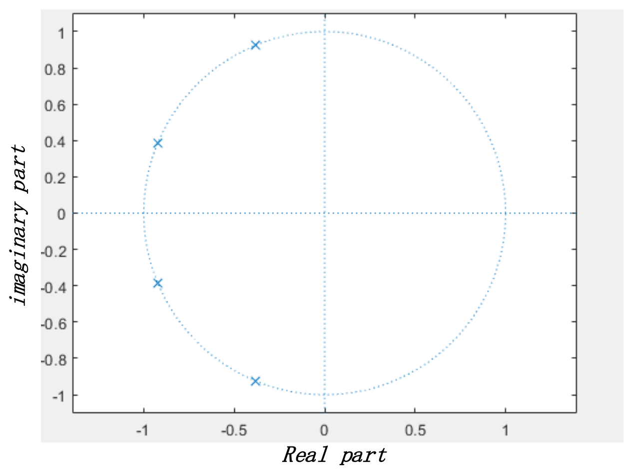

Figure 3. Zero pole plot

The four poles of the transfer function are all in the left half plane of the imaginary axis, and the system is stable[4].

$G_a(p)$ is the cascade of Eq. (19) and (20):

$$G_{a_1}(p) = \frac{1}{(p^2+0.7654p+1)} \tag{19}$$

$$G_{a_2}(p) = \frac{1}{(p^2+1.8478p+1)} \tag{20}$$

According to Eq. (9), the $G_{a_1}(p)$ and $G_{a_2}(p)$ are denormalized to obtain:

$$H_{a_1}(s) = \frac{1.58\times10^{10}}{s^2+9.62\times10^4 s+1.58\times10^{10}} \tag{21}$$

$$H_{a_2}(s) = \frac{1.58\times10^{10}}{s^2+2.32\times10^5 s+1.58\times10^{10}} \tag{22}$$

The first-stage filter parameters are calculated according to Eq. (10) and Eq. (21).: $K = 1, A = 0.77, C = 1$,select $C_2 = \frac{10}{20\times10^3}\mu F = 500pF$, $C_1 = 50pF$, $R_2 = 83k\Omega$, $R_1 = 83k\Omega$, $R_3 = 41k\Omega$.

Calculate the second-stage filter parameters: $K = 1$, $A = 1.85$, $B = 1$,select $C_2 = \frac{10}{20\times10^3}\mu F = 500pF$, $C_1 = 110pF$, $R_2 = 34.4k\Omega$, $R_1 = 34.4k\Omega$, $R_3 = 17.2k\Omega$.

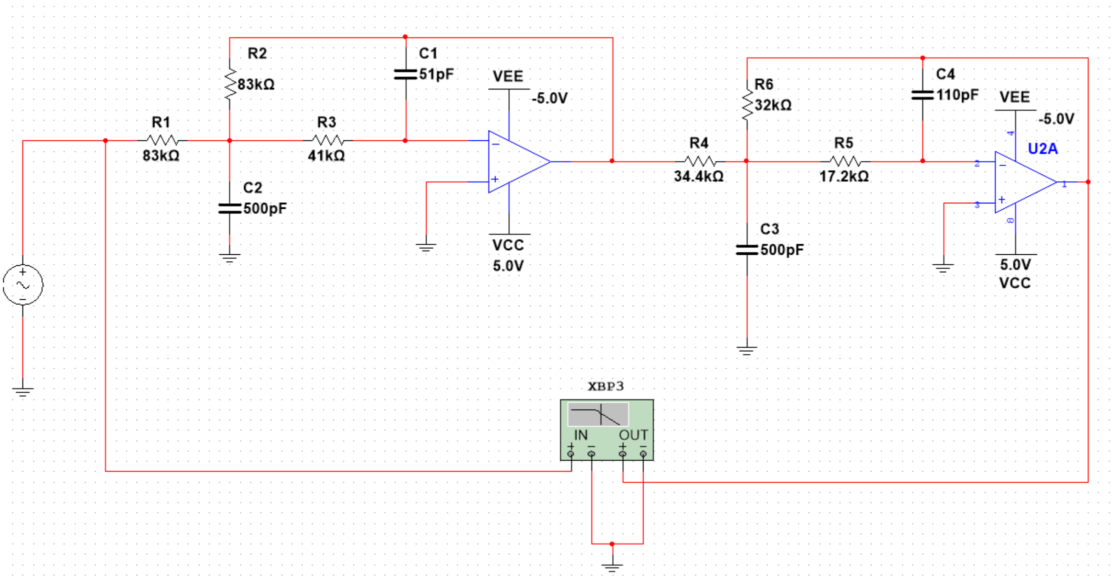

Figure 4. Multisim Simulation circuit diagram

4. Simulation and Analysis of Fourth-order Butterworth filter

The amplitude-frequency characteristics of the filter obtained by Multisim simulation are as follows:

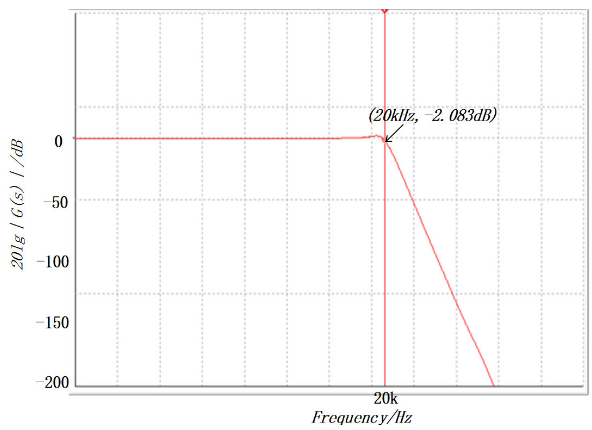

Figure 5. Multisim simulation amplitude-frequency characteristic plot

From the amplitude and frequency characteristics obtained by the simulation, it can be seen that the attenuation is -2.083dB at the cut-off frequency of 20kHz, which meets the design requirements.

Five voltage sources with different frequencies are used to simulate sound signals with different frequencies in series, as shown in Figure 6, the filtering experiment is carried out, and the frequencies of the five voltage sources are 0.5kHz, 1kHz, 3kHz, 25kHz, and 30kHz respectively.

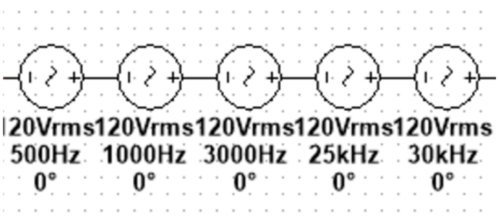

Figure 6. Multisim sound signals of different frequencies

The prefiltered signal waveform is shown in Figure 7:

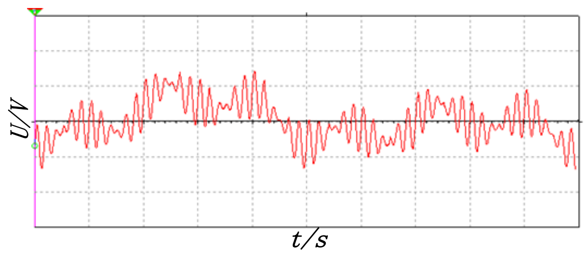

Figure 7. Waveform of the signal before Multisim filtering

The filtered signal waveform is shown in Figure 8:

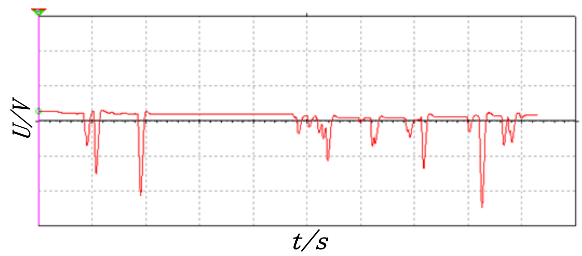

Figure 8. Waveform of the signal after Multisim filtering

In order to better observe the changes of different frequency components before and after filtering, MATLAB is used to decompose the output signal to obtain its frequency domain characteristics[5]:

The filter front is shown in Figure 9:

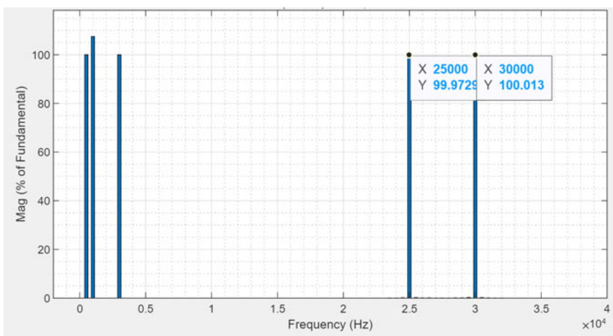

Figure 9. Frequency domain characteristic diagram of MATLAB filter front

There are components at both 25kHz and 30kHz before filtering.

After filtering, it is shown in Figure 10:

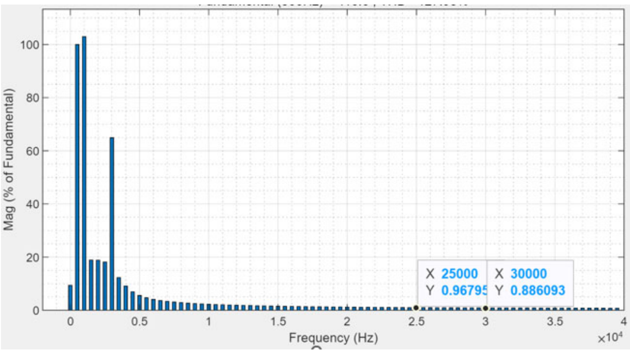

Figure 10. Frequency domain characteristic diagram after MATLAB filtering

The components at 25kHz and 30kHz are close to 0 after filtering, which proves that the filter has good filtering characteristics.

5. Conclusion

Through the study of the design principle of Butterworth low-pass filter, the system function of the normalized Butterworth low-pass filter is obtained, the zero pole is obtained in Matlab to verify the stability, and the amplitude-frequency characteristics of Butterworth filter of different orders are compared, and the fourth-order Butterworth low-pass filter is determined to be used in this design, and the resistance values and capacitance values in the circuit model are deduced by checking the polynomial table of the corresponding order, and the circuit model is built in mutisim, and the amplitude-frequency characteristics meet the design requirements. Five sound signals with different frequencies are used to simulate the sound signal in reality, and the signal before and after filtering is Fourier decomposed, and the performance of the filter is analyzed at the frequency.

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