

Construction of line Layout Model Based on Geometric Deduction and Constrained Heuristic Search

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Abstract: Based on problem B of the National Mathematical Contest in Modeling for College Students in 2023, this paper establishes a model through geometric deductive deduction, and establishes and solves an optimal model of survey line layout through constrained heuristic algorithm. In problem 1, a mathematical model of coverage width and overlap rate is established based on triangle relation and triangle sine theorem by finding the mathematical relationship between target quantity and known quantity. In problem 2, the angle between the known direction of the survey line and the projection of the normal direction of the submarine slope on the horizontal plane is β . We can imagine a surface perpendicular to the direction of the measurement line, and then find the slope of the imaginary surface according to the geometric relationship, and replace the slope Angle of problem 1 with the imaginary slope Angle to obtain the mathematical model of the coverage width. Based on this, the constraints of the Angle and overlap rate of the line are added, and the problem 3 is simplified into the optimization problem of multiple routes in problem 2. Finally, by changing the parameter setting of slope Angle in the model, the sensitivity analysis is carried out, and it is found that the model is stable and reliable. To sum up, the problems from one to four are gradually deepened and progressively advanced, and the development of Marine survey field is facilitated by establishing a model of depth survey line layout.

Keywords: Geometric relation, heuristic algorithm multi-objective programming, overlay width, overlap rate, surface integral.

1. Introduction

With the urgent need for high-precision seabed topographic survey, ocean exploration has developed from discrete, low-precision and low-efficiency to full coverage, high-precision and high-efficiency [1][2]. The traditional single beam sounding can not meet the requirement of high efficiency seabed exploration. On this basis, the multi-beam sounding system has been proposed and developed continuously, and has been widely used as a new technology of ocean exploration. The layout of line is a very important link in the technical design of sea area, but at present, the actual wiring design methods in China mostly rely on experience. Based on the current situation, the research of multi-beam line measurement is particularly important. There are two key problems in the layout of multi-beam line. The first is the survey line interval. Shallow water depth is used to design the survey line interval, and the problem of missing measurement may occur in the shallow place^[3]. Using deep water depth design line interval, there will be more overlapping areas, and the data redundancy is large, which affects the efficiency of the line. The other is the selection of the direction of the measurement line. Different directions of the measurement line will affect the difficulty of the measurement line^[4].

2. Problem Restatement

2.1. Solution to Problem 1

Considering the existence of a certain slope on the sea floor, the slope α is defined when the plane perpendicular to the direction of the survey line forms a diagonal line with an Angle of α from the horizontal plane. To find the expression of the coverage width W and the overlap rate η between adjacent strips of multi-beam sounding. According to the mathematical expression, the coverage width and overlap rate under the given parameters are solved by substitution^[5].

2.2. Solution to problem 2

In a rectangular sea area, when the Angle between the direction of the survey line and the projection of the normal direction of the submarine slope on the horizontal plane is arbitrarily determined (recorded as β), the mathematical expression of the coverage width is required, and the value of the coverage width is obtained with several groups of given parameters.

2.3. Solution to Problem 3

In a given rectangular sea area, a planning model is established, which can cover the entire sea area under the constraints required in the question, and the overlap rate between adjacent strips is between 10% and 20%. A set of optimal measuring lines is designed to make the total measuring line length shortest.

2.4. Solution to Problem 4

On the basis of some existing reference data, simulate the bottom of the sea area, and make appropriate simplified processing according to the bottom of the sea area. Under constrained conditions, cover the sea area to be measured as much as possible, keep the overlap rate of adjacent strips below 20%, minimize the total length of the side line as much as possible, search for the approximate optimal solution, and calculate relevant indicators according to the optimal solution: The total length of the measuring line, the proportion of the missing measuring area, the total length of the overlap of more than 20%.

3. Problem Analysis

3.1. Analysis of Problem 1

The first problem requires a mathematical model of coverage width and overlap rate. The key point of this

problem is to find the mathematical relationship between the target quantity and the known quantity in plane geometry problems. In this regard, according to the triangle relation and the sine theorem of triangle, the mathematical expressions for covering width W , overlap rate η , slope α , opening Angle of multi-beam converter θ and seawater depth D at the center of sea area are established respectively [6].

3.2. Analysis of problem 2

Problem 2 is extended to some extent on the basis of problem 1. In problem 2, the angle between the direction of the survey line and the projection of the normal direction of the submarine slope on the horizontal plane is β . In fact, problem 2 is reduced to the situation of problem one when $\beta = 90^\circ$ or $\beta = 270^\circ$. Problem 1 gives a mathematical model of coverage width and overlap rate on the surface perpendicular to the direction of the measurement line. In problem 2, we can imagine a surface perpendicular to the direction of the measurement line, and then find the slope of the imaginary surface according to the geometric relationship, and replace the slope Angle of problem 1 with the imaginary slope Angle to obtain the mathematical model of coverage width.

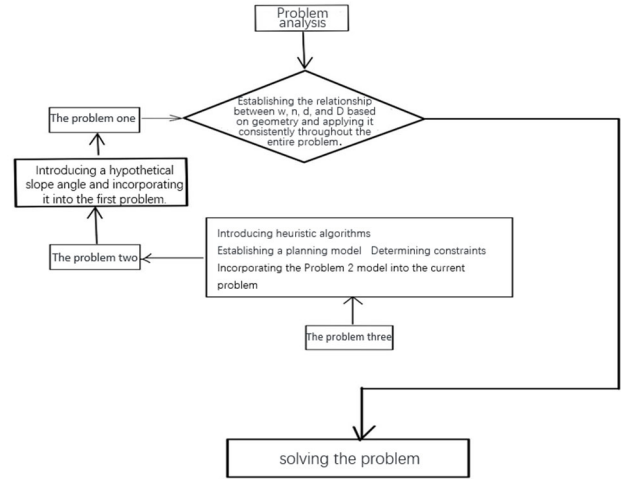
3.3. Analysis of problem 3

Problem 3 is a problem on optimization, which needs to design a set of routes with the shortest measurement length, and can completely cover the entire sea area and meet the requirement of 10%~20% overlap rate. According to problem 2, a route can be determined by the Angle β projected on the horizontal plane between the direction of the survey line and the normal direction of the submarine slope (hereinafter referred to as the direction of the route) and the interval d of the route. Therefore, the problem is simplified to the optimization problem of solving the two routes in multiple problems. (1) The total length of the survey lines is required to be the shortest. When the course direction is fixed, the survey line interval d is as large as possible, and the number of survey lines is as small as possible. Therefore, the overlap rate of survey lines at the shallowest part of the sea water is at least 10%, and the overlap rate at the deepest part of the sea water is not more than 20%; When there is overlap between each line, it will definitely meet the complete coverage of the sea area between the survey lines, we only need to consider the interval from the most marginal survey line to the edge of the sea area d . When the first survey line is determined, the second survey line will be determined solely by the first survey line according to the maximum interval, and so on until all survey lines have scanned the entire sea area [7].

3.4. Analysis of Problem 4

Firstly, the data given in the question is visualized. In MATLAB, a three-dimensional map is made. Considering that the data is a plurality of data points, in order to make that undersea topography more intuitive and smooth, fitting the given data, a fitted surface equation is obtained. By looking at the contour plot and gradient plot of the surface, it is found that that contour line on both sides along the diagonal line are nearly parallel and approximately straight, and the gradient direction is basically the same. Considering that the sea floor on either side of the diagonal along the rectangular sea area is approximately planar, the original surface is then decomposed into two planes, and the question is reduced to question three. According to the title, the mathematical model of multi-

objective programming is established and solved [8].



4. Model Assumption

1. According to the relevant literature in the field of Marine survey, it is generally possible to arrange parallel routes.
2. The measurement lines laid out are straight lines.
3. The distance between the surveying ship and the center point of the sea area is only considered as the distance between the surveying ship and the center point of the sea area in the vertical direction of the surveying line, which means, the distance between the surveying ship and the center point of the sea area along the direction of the surveying line is not considered.
4. Since the propagation speed of sound wave in water is much higher than the speed of ship, it is considered that there is no time delay effect during each measurement.

5. Symbol Description

Table 1. Symbol Description

symbol	Instructions	unit
W	Covering width	meter
θ	Multi-beam transducer opening Angle	Angle
α	submarine slope Angle	Angle
η	Overlap rate	—
D	Depth of sea water at the center point of sea area	meter
d	Distance between adjacent survey lines	meter
l_i	The i line	
$\tilde{\alpha}$	Total length of measuring line	Angle
L_i	Article i Measuring line length	meter
Z	Total length of measuring line	meter
x	Distance between survey line and center point of sea area	meter
D'	Sea depth	meter

Note: Some length units in the question are in nautical miles, and all length units in this paper are in the International standard unit meter. The specific conversion method is given as 1 nautical mile = 1852 meters according to the question.

6.2. Establishment and solution of problem 2 model

Problem 2 introduces a more complex scenario on the basis of problem 1. Since the beam surface is actually a plane perpendicular to the survey line, we are more concerned about the change of the vertical surface of the survey line in the following analysis. In problem 1, the vertical plane of the line is parallel to the horizontal component of the slope's normal direction, while in problem 2, there is a certain Angle between the vertical plane of the line and the horizontal component of the slope's normal direction. If the position of the vertical plane can be determined, we can imagine a slope formed by the vertical plane of the vertical line and the reference plane of the seabed, and then solve the slope Angle of the imaginary slope surface $\tilde{\alpha}$ by using geometric relations. Then problem 2 can be simplified to problem 1.

Therefore, the center point of the sea area (denoted C) is the vertical plane of the survey line and intersects it with the submarine reference plane. The intersection line is denoted b. Points O and B are at the intersection line b, connect C and O, so that CO is perpendicular to the submarine reference plane (denoted line segment CO is line h). Point B is at the intersection line between the submarine reference plane and the bottom slope of the real sea γ (denoted l). Cross the center point C of the sea area to the intersection line of the submarine reference plane and the submarine reference plane as A vertical line. The intersection point is denoted as point A, connecting OA (denoted as line a), BC.

The scenario diagram is shown in Figure 2 below

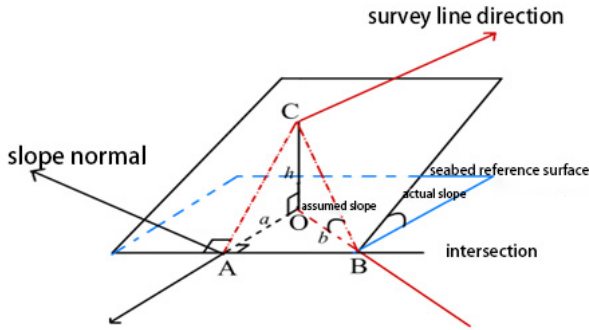


Figure 2.

It is easy to know from the figure that there is a trigonometric function relationship at this time:

$$\tan\alpha = \frac{|CO|}{|AO|}, \quad \tan\tilde{\alpha} = \frac{|CO|}{|BO|} \quad (15)$$

Since CO is the reference plane of the ocean floor, $CO \perp l$. And since $CA \perp l$, $CA \cap CO = C$. From the vertical theorem of line and plane, it can be concluded that $l \perp \text{Face OCA}$.

Since the line plane is perpendicular, l is perpendicular to any line in the face OCA, i.e. :

$$\forall \text{line } x \subset \text{face OCA}, \quad l \perp x \quad (16)$$

So the line OA is perpendicular to the intersection line l , i.e., $\angle OAB = 90^\circ$

In the triangle OAB, $\angle OAB = 90^\circ$, $\angle AOB = \angle\beta - 90^\circ$, so

$$\cos\angle AOB = \cos(\angle\beta - 90^\circ) = \frac{|AO|}{|BO|} \quad (17)$$

By combining equations (14) and (16), the mathematical relationship of imaginary slope Angle can be obtained:

$$\angle\tilde{\alpha} = \arctan(|\sin\angle\beta| \times \tan\alpha) \quad (18)$$

After the imaginary slope Angle is obtained, problem 2 is simplified to problem 1. $\angle\alpha$ in formula (9) and formula (14) is replaced by $\angle\tilde{\alpha}$, and D in formula (9) is replaced by D' in formula (14). The total formula for the coverage width of problem 2 is as follows:

$$W = \frac{2(D + x \tan\angle\tilde{\alpha}) \tan\frac{\theta}{2} \cos^2\frac{\theta}{2} \sin(90^\circ - \angle\tilde{\alpha})}{\sin[(90^\circ - \angle\tilde{\alpha}) - \frac{\theta}{2}] \sin[(90^\circ - \angle\tilde{\alpha}) + \frac{\theta}{2}]} \quad (19)$$

The formula (19) was programmed in MATLAB, and the parameters given in question 2 were substituted, and the results were obtained.

6.3. Establishment and solution of problem 3 model

This problem is an optimization problem, which needs to design a set of routes with the shortest measurement length. It can completely cover the entire sea area, and meet the requirement of overlap rate between 10% and 20%.

It can be seen from problem 2 that a route can be determined by the Angle β projected on the horizontal plane between the direction of the survey line and the normal direction of the submarine slope (hereinafter referred to as the route direction) and the interval d of the route. The slope is 1.5° and the opening Angle of the multi-beam transducer is 120° , which is the same as problem 2. Therefore, the problem is simplified to the solution optimization problem of routes in problem 2.

By searching the data, we learned that the survey lines need to be arranged in parallel in order to ensure that a set of survey lines can scan the entire sea area as fully as possible^[9].

(1) The total length of the survey lines is required to be the shortest. When the course direction is fixed, the survey line interval d is as large as possible, and the number of survey lines is as small as possible. Therefore, the overlap rate of survey lines at the shallowest part of the sea water is at least 10%, and the overlap rate at the deepest part of the sea water is not more than 20%; (2) When there is overlap between each line, it will definitely meet the complete coverage of the sea area between the survey lines, we only need to consider the interval from the most marginal survey line to the edge of the sea area d . When the first survey line is determined, the second survey line will be determined solely by the first survey line according to the maximum interval, and so on until all survey lines have scanned the entire sea area^[10].

Step1. Solve for the initial interval d_0

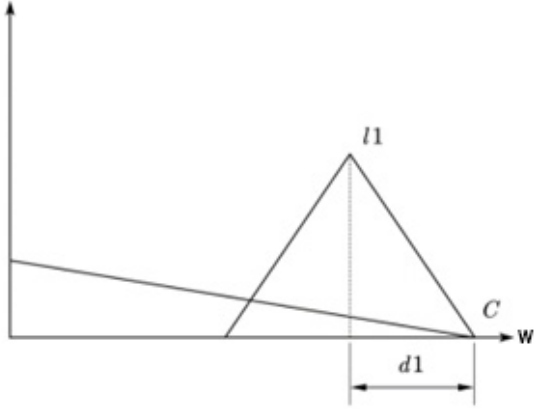


Figure 3.

Figure 3 shows the side view perpendicular to the direction of the survey line and passing the edge point C. As shown in Figure 3, when the scanning area of the edge survey line passes the most edge point C, the entire scanning of the edge area can be guaranteed. To maximize the spacing d_0 , we make the l_1 obscure right side scan width w_1 obscure exactly to the point C on the edge, where the deepest sea water is h_{max} , there is

$$d_1 = h_{max} \cdot \tan(\theta/2) \quad (20)$$

Step2. Establish a plane rectangular coordinate system for the top view of the entire sea area

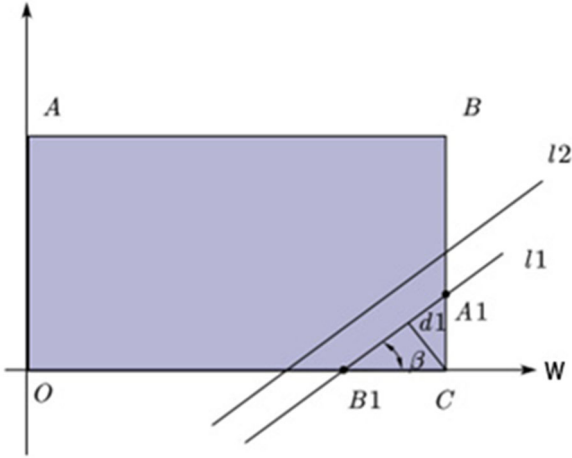


Figure 4

As shown in FIG. 4, the positive direction of the X-axis is set to be westward. According to the meaning of the question, in the rectangular sea area 2 nautical miles long from north to south and 4 nautical miles wide from east to west, the west is deep and the east is shallow. The Angle between the axis of the straight line l and x is the included Angle of the horizontal projection of the direction of the survey line and the normal direction of the submarine slope, β , and the Angle β is arbitrary.

Step3. Solve the upper intersection point A_1 and the lower intersection point A_2 of the survey line l_1 and the sea area to be measured

We set intersection $A_1(x_1, y_1)$, $B_1(x_2, y_2)$ when the initial interval grey is determined, then you can write in figure 4 l_1 dark in the coordinate system for the linear equation

$$Y_1 = \tan\beta \cdot (x - x_2) \quad (21)$$

Among them,

$$x_2 = x_0 - d_0/\sin\beta \quad (22)$$

The equation for line BC is

$$x = 4 \cdot 1852 \quad (23)$$

The equation of line AB is

$$y = 2 \cdot 1852 \quad (24)$$

If the equations (18) and (20) are combined, a point coordinate will be obtained, set as A'_1 ; if the equations (18) and (21) are simultaneously combined, a point coordinate will be obtained, set as A''_1 , which is obtained by comparing the horizontal coordinates of A'_1 and A''_1

$$\min\{x'_1, x''_1\} \quad (25)$$

$$L_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (26)$$

Then the upper intersection is the coordinate of the point with the smallest horizontal coordinate among the two points, and the upper intersection A_1 is obtained. Similarly, write the equation of a line AO and a line OC to obtain the coordinates of the lower intersection, then the length of the measurement line is the Euclidean distance between the upper and lower intersection points, that is, write the equation of a line AO and a line OC to obtain the coordinates of the lower intersection points, then the length of the measurement line is the Euclidean distance between the upper and lower intersection points, that is

Step4. According to the measurement line l_1 obscurity and constraints, find out the interval d_2 with the next measurement line

From the overlap rate formula (13) obtained in question 1, the relationship between interval d and overlap rate η is

$$d = \frac{2 \cdot (1 - \eta) \cdot D \cdot \sin(\theta/2) \cdot \cos\tilde{\alpha}}{\sin(90^\circ - \tilde{\alpha} + \theta/2)} \quad (27)$$

Here is the hypothetical slope angle $\tilde{\alpha}$ in Problem 2, and

$$\tan\tilde{\alpha} = \tan\alpha \cdot \sin\beta \quad (28)$$

As can be seen from the figure, the depth of the sea area at any point is only related to the abscissa of the point x , and the relation is satisfied

$$D = f(x) = h_{max} - h_{min} + \tan\alpha \cdot x \quad (29)$$

When the route is obscure at the shallowest sea depth $A_1, \eta = 10\%$ can ensure that the route spacing is as large as possible, and B_1 should be satisfied when the route is obscure at the deepest sea depth $\eta \leq 20\%$;

Step5. Determine that the last route covers the diagonal edge point A

Suppose there are n measuring lines to complete the full coverage, repeat Step1-4, and the expression of the i measuring line is.

$$Y_i = \frac{d_i}{\cos\beta} + Y_{i-1} \quad (30)$$

Find the final interval d_i on the basis of the last line l_i , if the last intersection A_i and the abscissa x_i are satisfied

$$x_i \leq \frac{d_i}{\sin(\beta)} \quad (31)$$

Then

$$n = i \quad (32)$$

On the contrary,

$$n = i + 1 \quad (33)$$

Finally, the total length Z of the whole group of measurement lines is

$$Z = \sum_{i=1}^n L_i \quad (34)$$

Again according to question 2, when $0^\circ \leq \beta < 180^\circ$ with $180^\circ \leq \beta < 360^\circ$ when routes, is just the starting point is different, so the subject need to consider $0^\circ \leq \beta < 180^\circ$ situation, above all, kinds of optimization objective function for the model

$$\min Z = \sum_{i=1}^n L_i \quad (35)$$

The constraint condition is

$$\text{s.t.} \begin{cases} 10\% \leq \eta \leq 20\% \\ 0 \leq x_i \leq 4 \times 1852, & (i = 1,2,3 \dots) \\ 0 \leq y_i \leq 2 \times 1852, & (i = 1,2,3 \dots) \\ 0^\circ \leq \beta < 180^\circ \end{cases} \quad (36)$$

The final result is that the number of measured lines is 38, and the total length is 14,0752m. Draw the group of lines with the shortest total distance as follows

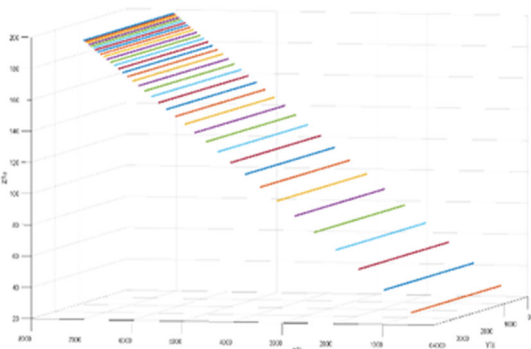


Figure 5a

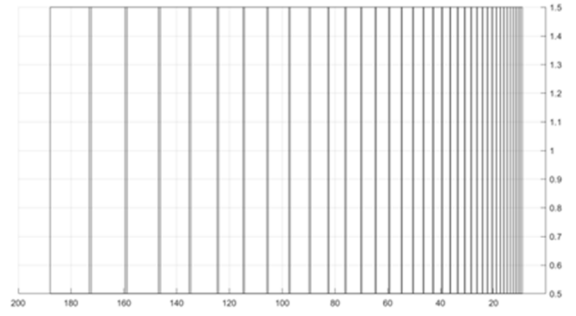


Figure 5b

6.4. Establishment and solution of problem 4 model

6.4.1. Data Visualization

We first visualized the data given in the question and made a three-dimensional diagram in MATLAB, as shown in Figure 5. Considering that the data are multiple data points, in order to make the submarine terrain more intuitive and smooth, cftool toolbox in MATLAB was used to fit the data given, and the surface diagram shown in Figure 6 can be obtained.

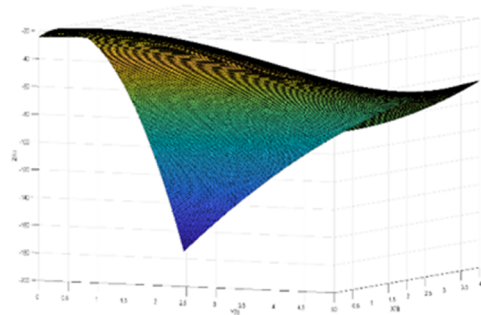


Figure 6

And the equation of submarine surface is

$$z = -3.173 - 17.6 \cdot x - 4.491 \cdot y - 6.4 \cdot x^2 + 9.6 \cdot x \cdot y - 3.2 \cdot y^2 \quad (37)$$

And the correlation coefficient $R^2 = 0.9648$ and the goodness of fit is good.

6.4.2. Model establishment

In the survey line group with the shortest total distance obtained from question 3, the direction of the survey line is parallel to the bottom edge of the rectangular sea area, that is, it runs along the contour line of the seabed. The feasibility of this operation is verified by searching data, and we specify the course direction of question 4 to run along the contour line of the coastal bottom^[11].

According to the equation of the submarine surface obtained in z , we make the contour map and gradient line of the submarine surface as shown in FIG. 7.

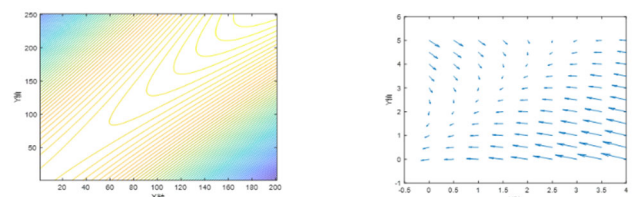


Figure 7

By analyzing the obtained contour map, we find that the

$$\text{s.t.} \begin{cases} \eta \leq 20\% \\ 0 \leq x_i \leq OE, \quad (i = 1,2,3 \dots) \\ 0 \leq y_i \leq d, \quad (i = 1,2,3 \dots) \\ \beta = 90^\circ \end{cases} \quad (47)$$

The procedure for S_2 is the same as above and will not be described here.

The result of question 4 is finally obtained, as shown in Table 3.

Table 3. The result of question 4

	Number of cycles/routes	Sailing distance (m)	Uncovered area (m ²)	Surface area/area ratio	Overlap length (m)
A1	33	442649.70	—	Surface area: 4280554491.0407 Uncovered area: 17895500.1888523 proportion: 0.41806500130551%	—
A2	4	45565.68	—		—
A	—	397084.02	—		—
B1	17	106675.11	18682510.86		—
B2	3	34503.62	787010.67		—
B	—	72171.49	17895500.19	—	
sum	50	469255.51	17895500.19	—	0.00

As can be seen from the table, the final total number of measured lines is 50, the optimal length is 469255.51m, the area of overlapping area is 0. The area of unmeasured area is 17895500.19m², accounting for 0.42% of the total surface area, and the result is good [13].

7. Model Analysis and Test

7.1. Model sensitivity analysis

Sensitivity analysis is one of the methods to evaluate the planning model, which is as follows^[14]:

(1) Set a bias for the required parameters of the model (take 3%, 6%, 9% as an example).

(2) When the slope Angle of the parameter \dagger is changed, the remaining parameters are kept unchanged to observe whether the difference between the final results is significant. The results are as follows

Table 4. The results

slope sensitivity analysis					
α	1.41	1.455	1.5	1.545	1.6377
Number of routes	41	39	38	37	35
total length of routes	151864.00	144456.00	140752.00	137048.00	129640.00

The relationship between the change of slope Angle and the number of lines and the total length of lines is shown in Figure 10.

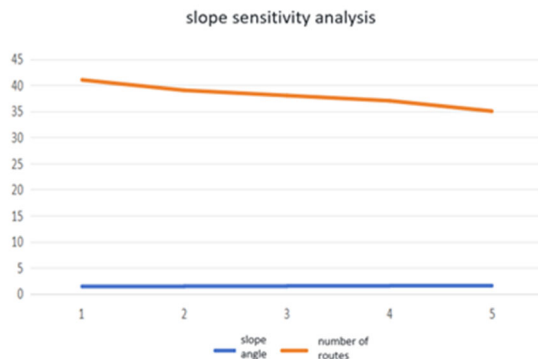


Figure 10a

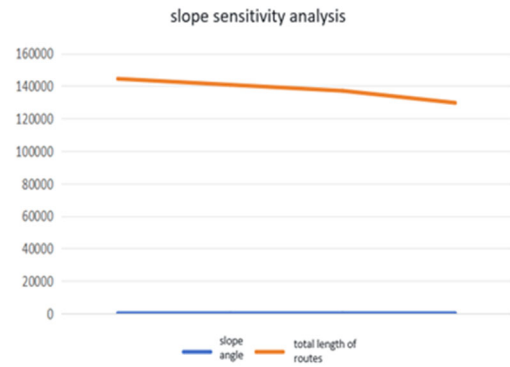


Figure 10b

The results show that when the parameters change, the model results will not change greatly, and the model stability is reliable.

7.2. Model error analysis

The model assumes that the delay caused by the beam propagation time does not affect the results, which was reviewed: The propagation speed of sound waves in seawater. The main factors of the propagation speed of sound waves are temperature, salinity and pressure (depth). The change of temperature has the greatest influence on sound velocity. Due to the uneven distribution of temperature and salinity in the seawater medium, the distribution of sound velocity will be uneven, thus forming a sound velocity gradient in the ocean. The speed of sound wave propagating in the seawater is three and a half times faster than that in the air (about 1531 m/s)^[15].

We estimate the speed of sound at 1 nautical mile per second, and the maximum time for the beam to sweep the rectangular sea area is not more than 5 seconds, so it is easy to know that this assumption will not introduce significant errors.

8. Evaluation, Improvement and Extension of The Model

8.1. Advantages of the model

1. The model is based on the integrity of hydrographic survey and has practical application prospects.
2. Some assumptions are made in the model to simplify the problem and reduce the amount of calculation.
3. The model is based on geometric ideas and simple to prove.
4. The model qualitatively analyzes the effects of acoustic wave propagation to simplify the problem.

8.2. Disadvantages of the model

1. Problem 4 model retains the integrity of measurement as much as possible, and the measurement line obtained is too long.
2. In the modeling of problem 4, we chose the fitting graph instead of the actual surface, which may result in the loss of some data points.

8.3. Model improvement

The line spacing in the model can be assumed to be equal spacing, which can simplify the problem and be easy to implement, but will sacrifice the accuracy of the model. At the same time, for problem 4, we can further refine the segmentation of the slope bottom and divide it into more planes with different gradients for solving, so as to improve

the accuracy of the model.

8.4. Model promotion

1. In this paper, the problem of line layout for multi-beam measurement of classical seabed depth is studied, which can be applied to more complex seabed sounding problems with improvement.

2. By changing some parameters, a new method can be provided for the mapping of various complex maps.

3. It can be applied to the design of 3D imaging technology to scan the physical path.

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