

Research on Light Pollution Based on AHP and Fuzzy Comprehensive Evaluation Methods

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Abstract: In the current context of increasing global light pollution, we urgently need an effective control system. In this study, we constructed an extensive and objective evaluation model through the AHP method to classify light pollution into standard and index layers, and successfully established an evaluation model that accurately predicts the level of light pollution. Through practical tests, we found that light pollution is higher in cities, followed by suburbs, and lower in rural areas and protected areas. Based on the established evaluation model, we propose three most effective intervention strategies: regulating the use of LED billboards, reducing the use of reflective building materials, and controlling nighttime construction lighting. In response to the possible negative impacts, we propose specific solutions and verify the effectiveness of the intervention programs through a multi-objective planning model, achieving significant results in reducing light pollution levels. This study provides a useful reference and lesson for light pollution management.

Keywords: AHP, Fuzzy Comprehensive Evaluation, Multi-Objective Planning Modeling.

1. Introduction

In today's society, the problem of light pollution has gradually become a focus of attention as the country's level of development improves and increasing attention is paid to the ecological environment [1]. Globally, light pollution seriously affects the clarity of the starry sky, and the situation is worsening every year. The problem has attracted extensive attention from German scientists, further emphasizing its urgency. The complexity and diversity of light pollution makes the establishment of a broad, universal and objective evaluation model an important challenge [2]. At the same time, many factors, such as the impact on corporate profits, delays in project implementation, and public satisfaction with the policy, need to be taken into account when formulating an intervention plan in order to minimize the negative impacts while reducing light pollution.

The aim of this paper is to explore how to effectively solve the problem of light pollution, we use the AHP method to establish an assessment system, divide light pollution into standard and index layers, determine the weights through the judgment matrix filled in by experts, and establish a model to accurately predict the level of light pollution. Practical validation shows that there are obvious differences in light pollution levels in different areas, with the highest in cities and the lowest in rural areas and protected areas. Based on the evaluation model, three effective intervention strategies were proposed, including regulating the use of LED billboards, reducing the use of reflective building materials, and controlling nighttime construction lighting. We verified the effectiveness of these interventions through a multi-objective planning model to reduce light pollution levels while minimizing its negative impacts. This study aims to provide a

useful reference for light pollution management and promote a balance between ecological protection and sustainable development.

2. Light Pollution Risk Level Index System

2.1. Construction of evaluation index system

Analytic Hierarchy Process (AHP) is a multi-objective decision analysis methodology proposed by professor T.L. Stty of the University of Pittsburgh in the 1970s [3]. The principle is to decompose the factors related to decision into several levels, such as target layer, criterion layer and scheme layer. Through the calculation and comparison of each factor, the weight of different factors can be obtained to provide reference basis for decision makers to choose the optimal scheme.

According to reference [4], the degree of light pollution is classified into artificial day, color light pollution and white light pollution, as shown in the Figure 1 below:

Matrix distribution model solution, Construct $C - D$ matrix: compare the four elements D_1, D_2, D_3 and D_4 in pairs to get the comparison matrix. The general form is shown in the following Table. 1.

Table 1. Compare matrix.

C	D_1	D_2	D_3
D_1	1	1/2	2
D_2	2	1	3
D_3	1/2	1/3	1

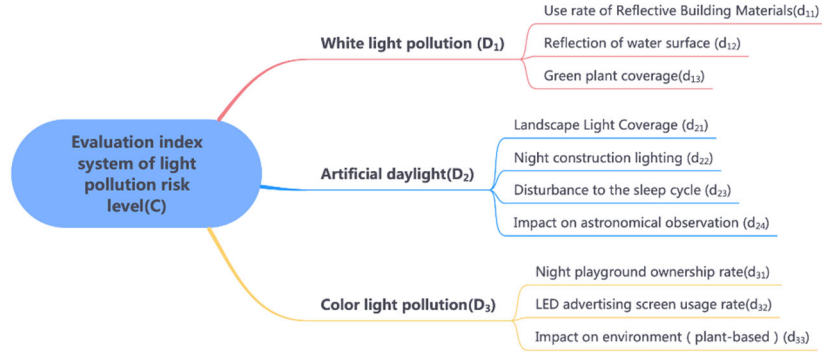


Figure 1. Light pollution index

Any of these matrices should satisfy:

$$d_{ij} = \frac{1}{d_{ji}} (i, j = 1, 2, \dots, n) \quad (1)$$

d_{ij} in the judgment matrix generally adopts the nine-point scale method, in which the value is determined according to the comprehensive balance of government website, government work report, statistical data and expert opinions.

Judgment matrix and consistency test results. The judgment matrix $d_{1i} - D_1$, $d_{2i} - D_2$ and $d_{3i} - D_3$ were constructed, and the consistency ratio was determined by Matlab, and then the weight was obtained. The judgment matrices are shown in Table. 2, Table. 3 and Table. 4.

Table 2. Judgment matrix $d_{1i} - D_1$

D_1	d_{11}	d_{12}	d_{13}	ω_1	ω_2	ω_3	$\bar{\omega}$
d_{12}	1	3	6	0.6667	0.6667	0.6667	0.6667
d_{22}	1/3	1	2	0.2222	0.2222	0.2222	0.2222
d_{32}	1/6	1/2	1	0.2222	0.1111	0.1111	0.1111

$CR = 8.35 \times 10^{-4} < 0.1$, passe the consistency test

Table 3. Judgment matrix $d_{2i} - D_2$

D_2	d_{21}	d_{22}	d_{23}	d_{24}	ω_1	ω_2	ω_3
d_{21}	1	1/2	1/4	1/7	0.0701	0.0699	0.0700
d_{22}	2	1	1/2	1/4	0.1354	0.1354	0.1354
d_{23}	4	2	1	1/2	0.2708	0.2708	0.2707

$CR = 0.0053 < 0.1$, passe the consistency test

Table 4. Judgment matrix $d_{3i} - D_3$

D_3	d_{31}	d_{32}	d_{33}	ω_1	ω_2	ω_3	$\bar{\omega}$
d_{31}	1	1/8	1/5	0.0701	0.0701	0.7013	0.0702
d_{32}	8	1	2	0.6044	0.6044	0.6044	0.6042
d_{33}	5	1/2	1	0.3255	0.3255	0.3255	0.3257

$CR=0.0088<0.1$, passe the consistency test

2.2. Fuzzy comprehensive evaluation

The basic idea of fuzzy comprehensive evaluation model construction is that, based on fuzzy mathematics theory, the general standard of evaluation is decomposed into a fuzzy set composed of multiple indexes, which is used to deal with uncertain information [5]. The construction of fuzzy comprehensive evaluation method is generally carried out according to the following steps:

1. Determine the comprehensive evaluation factor set: after the evaluation index is selected, a set $U = [U_1, U_2, U_3, U_4]$ is established for the factors affecting the light pollution factors, and then the set $U_1 = [U_{11}, U_{12}, U_{13}, \dots]$ is the second-level index under U , and so on.

2. The establishment of comprehensive evaluation comments: give a grade to the importance of each indicator, and finally select the ideal result. The comments set is $V = [V_1, V_2, V_3, V_4]$, V_1 is high risk, V_2 is medium risk, V_3 is low risk, and V_4 is lower risk.

3. Determine each weight factor: $A = [0.2970, 0.5394, 0.1635]$.

4. The fuzzy comprehensive evaluation judgment matrix is determined the matrix R : for index U_i , the membership of each comment is a fuzzy subset on V . The evaluation of indicator U_i is denoted as.

$$R_i = [r_{i1}, r_{i2}, \dots, r_{im}] \quad (2)$$

The fuzzy comprehensive judgment matrix of each index is:

$$R = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} \quad (3)$$

5. Fuzzy comprehensive evaluation, perform matrix composition operations.

$$B = A \cdot R \quad (4)$$

Take the comments with the largest data as the comprehensive evaluation result.

3. Light Pollution Risk Assessment for Different Locations

3.1. 3.1 Urban community

Densely populated, the impact of artificial light is large, such as high density of lighting equipment, wide use of reflective materials in buildings, can cause color light pollution in many places. Following the steps of the fuzzy comprehensive evaluation method, we first calculate the risk of light pollution in urban areas.

Determine the comprehensive evaluation factor set.

$$\begin{aligned} U &= [U_1, U_2, U_3] \\ U_1 &= [U_{12}, U_{12}, U_{13}] \\ U_2 &= [U_{21}, U_{22}, U_{23}, U_{24}] \\ U_3 &= [U_{31}, U_{32}, U_{33}] \end{aligned} \quad (5)$$

The comments set is:

$$V = [V_1, V_2, V_3, V_4] \quad (6)$$

Each weight factor:

$$A = [0.2970, 0.5394, 0.1635] \quad (7)$$

The evaluation matrix of U_1, U_2, U_3 is obtained by using the formula $B = A \cdot R$ through the first-level fuzzy comprehensive evaluation.

$$\begin{aligned} B_1 &= [0.6623 \quad 0.1888 \quad 0.1111 \quad 0.0666] \\ B_2 &= [0.63581 \quad 0.32357 \quad 0.02708 \quad 0.01354] \quad (8) \\ B_3 &= [0.45578 \quad 0.27912 \quad 0.23255 \quad 0.03255] \end{aligned}$$

The fuzzy comprehensive judgment matrix of each index is:

$$R = \begin{bmatrix} 0.6623 & 0.1888 & 0.1111 & 0.0666 \\ 0.63581 & 0.32357 & 0.02708 & 0.01354 \\ 0.45578 & 0.27912 & 0.23255 & 0.03255 \end{bmatrix} \quad (9)$$

By the formula:

$$B = A \cdot R \quad (10)$$

Getting the weight factor:

$$A = [0.2970, 0.5394, 0.1635] \quad (11)$$

Then the second-level fuzzy evaluation was carried out to obtain the membership degree of light pollution risk level to the rating:

$$B = [0.6056 \quad 0.2763 \quad 0.0856 \quad 0.0324] \quad (12)$$

Take the comments with the largest data as the comprehensive evaluation result. The result is rated as "high risk".

3.2. Suburban community

The population density is medium, and the artificial light has a great influence, among which the high development prospect may lead to a large risk of light pollution caused by construction lighting at night.

With the same assessment method for urban light pollution risk level, the evaluation matrix of U_1, U_2, U_3 is obtained by using the formula $B = A \cdot R$ through the first-level fuzzy comprehensive evaluation.

$$\begin{aligned} B_1 &= [0.05555 \quad 0.41111 \quad 0.37778 \quad 0.2] \\ B_2 &= [0.21705 \quad 0.367768 \quad 0.40127 \quad 0.014] \quad (13) \\ B_3 &= [0.16044 \quad 0.41853 \quad 0.24657 \quad 0.17446] \end{aligned}$$

The fuzzy comprehensive judgment matrix of each index is:

$$R = \begin{bmatrix} 0.05555 & 0.41111 & 0.37778 & 0.2 \\ 0.21705 & 0.367768 & 0.40127 & 0.014 \\ 0.16044 & 0.41853 & 0.24657 & 0.17446 \end{bmatrix} \quad (14)$$

By the formula:

$$B = A \cdot R \quad (15)$$

Getting the weight factor:

$$A = [0.2970, 0.5394, 0.1635] \quad (16)$$

Then the second-level fuzzy evaluation was carried out to obtain the membership degree of light pollution risk level to the rating:

$$B = [0.1598071 \quad 0.3888559 \quad 0.35577612 \quad 0.0954758] \quad (17)$$

Take the comments with the largest data as the comprehensive evaluation result. The result is rated as "medium risk".

3.3. Rural community

Densely populated, with little artificial light impact, light pollution caused by natural factors may exist.

With the same assessment method for urban light pollution risk level, the evaluation matrix of U_1, U_2, U_3 is obtained by using the formula $B = A \cdot R$ through the first-level fuzzy comprehensive evaluation.

$$\begin{aligned} B_1 &= [0.06666 \quad 0.12221 \quad 0.5889 \quad 0.22223] \\ B_2 &= [0 \quad 0.02708 \quad 0.40222 \quad 0.5707] \quad (18) \\ B_3 &= [0.03255 \quad 0.18598 \quad 0.45809 \quad 0.32338] \end{aligned}$$

The fuzzy comprehensive judgment matrix of each index is:

$$R = \begin{bmatrix} 0.06666 & 0.12221 & 0.5889 & 0.22223 \\ 0 & 0.02708 & 0.40222 & 0.5707 \\ 0.03255 & 0.18598 & 0.45809 & 0.32338 \end{bmatrix} \quad (19)$$

By the formula:

$$B = A \cdot R \quad (20)$$

Getting the weight factor:

$$A = [0.2970, 0.5394, 0.1635] \quad (21)$$

Then the second-level fuzzy evaluation was carried out to obtain the membership degree of light pollution risk level to the rating:

$$B = [0.0251199 \quad 0.0813111 \quad 0.4667585 \quad 0.4267105] \quad (22)$$

Take the comments with the largest data as the comprehensive evaluation result. The result is rated as "low risk".

3.4. Protected area community

Light pollution from natural causes is minimal and there is no artificial light.

With the same assessment method for urban light pollution risk level, the evaluation matrix of U_1, U_2, U_3 is obtained by using the formula $B = A \cdot R$ through the first-level fuzzy comprehensive evaluation.

$$\begin{aligned} B_1 &= [0.04444 \quad 0.08888 \quad 0.08888 \quad 0.7778] \\ B_2 &= [0 \quad 0 \quad 0.2707 \quad 0.7293] \quad (23) \\ B_3 &= [0 \quad 0.03255 \quad 0.09765 \quad 0.8698] \end{aligned}$$

The fuzzy comprehensive judgment matrix of each index

is:

$$R = \begin{bmatrix} 0.04444 & 0.08888 & 0.08888 & 0.77778 \\ 0 & 0 & 0.2707 & 0.7293 \\ 0 & 0.03255 & 0.09765 & 0.8698 \end{bmatrix} \quad (24)$$

By the formula:

$$B = A \cdot R \quad (25)$$

Getting the weight factor:

$$A = [0.2970, 0.5394, 0.1635] \quad (26)$$

Then the second-level fuzzy evaluation was carried out to obtain the membership degree of light pollution risk level to the rating:

$$B = [0.0131987 \quad 0.0317193 \quad 0.1883787 \quad 0.7666033] \quad (27)$$

Take the comments with the largest data as the comprehensive evaluation result. The result is rated as "lower risk".

4. Intervention Strategy and Validation

4.1. Intervention strategies

According to the weights of indicators obtained by AHP-fuzzy comprehensive evaluation method, corresponding intervention strategies are made.

1. Standardize the use of LED billboard: Make strict requirements on LED light brightness, color, flicker frequency, use time, use time and other factors, and check regularly.

2. Reduce the impact of reflective building materials: According to the different area population environment conditions for the use of reflective material construction planning layout, minimize its impact on everyday life. Or the degree of light pollution can be reduced from the source by matching and replacing the curtain wall with other materials or changing the Angle of the building.

3. Light control for nighttime construction: The lighting shall be placed in a scientific way in the construction area. Reduce light overflow, resolutely prevent direct exposure to residential buildings. Introduce advanced technology to improve construction efficiency during the day.

4.2. Validation

The above analyzed the risk level of light pollution in urban, suburban, rural and protected areas, and found that the risk level of light pollution was higher in urban areas. Therefore, we chose urban areas to implement the intervention policy we developed and to verify whether the intervention policy is effective.

The four levels of high risk, medium risk, low risk and lower risk are quantified, as shown in the Table. 5 below:

Table 5. Light pollution level

Level	High risk	Medium risk	Low risk	Lower risk
score	8	5	3	1

Under the influence of intervention policies, the membership degree of urban light pollution index changes.

Then, fuzzy comprehensive evaluation is carried out for the index, and the score is calculated using the light pollution level 18.

For the impact of intervention strategies on the risk level of light pollution, we concretized the mathematical programming model multi-objective programming model of light pollution is obtained [6].

f_1 as selected as the impact of intervention strategy on risk level, f_2 as the adverse impact of intervention strategy, x_1 as the regulation of LED billboards, x_2 as the reduction of building reflection, and x_3 as the control of nighttime construction light. Based on the assessment of the importance of the factors, we determined that the weights of f_1 and f_2 were 0.3 and 0.7.

The programming model is obtained as:

$$\min f_1 = a_1 x_1 + b_1 x_2 + c_1 x_3 \quad (28)$$

a_1 is the process of minimizing the product of x_1 weight and the relative change value for the specification LED billboard.

b_1 is the process of minimizing the product of x_2 weight and the value of the relative change to reduce the reflection of light in the building.

c_1 is the process of minimizing the product of x_3 weight and the value of the relative change in light control for night construction.

$$\min f_2 = a_2 x_1 + b_2 x_2 + c_2 x_3 \quad (29)$$

Through the study of literatures, we can get the harm coefficient $a_2 = 0.055, b_2 = 0.136, c_2 = 0.109$ of the implementation of intervention policies on cities.

The weighted combination function is obtained as follows:

$$f = ax_1 + bx_2 + cx_3 \quad (30)$$

We can get the coefficient value:

$$\begin{aligned} \min f_1 &= 0.419x_1 + 0.354 + 0.627x_3 \\ \min f_2 &= 0.055x_1 + 0.136x_2 + 0.109x_3 \\ f &= 0.474x_1 + 0.490x_2 + 0.736x_3 \end{aligned} \quad (31)$$

According to the coefficients before x_1, x_2 and x_3 we can judge the effectiveness of the intervention strategy for this site. The smaller the coefficient, the more effective it is for light pollution control.

By comparing $a < b < c$, we can get that regulating the use of LED advertising is the most effective intervention policy for cities.

5. Conclusions

In this study, we constructed an evaluation system that accurately predicts the level of light pollution by introducing the AHP method and proposed an effective intervention strategy. We divided light pollution into standard and index layers and established an objective evaluation model by determining the weights through a judgment matrix filled in by experts. Practical validation shows that there are obvious differences in light pollution levels in different areas, with the highest in cities and the lowest in rural areas and protected areas. Through intervention strategies such as regulating the use of LED billboards, reducing the use of reflective building

materials, and controlling lights for nighttime construction, we are committed to solving the problem of light pollution while minimizing its negative impacts. By testing the effectiveness of our intervention programs, we have successfully reduced light pollution levels and minimized negative impacts in both urban and suburban areas. This research provides an important reference for light pollution management, balancing ecological conservation and sustainable development. Our work provides practical methods and strategies for solving light pollution problems and will certainly have a positive impact on future environmental protection and urban planning.

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