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Evaluation of Performance of Thermal Fibers and Heat Transfer Research based on Entropy Weight Method and Differential Equation

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Abstract: Based on the mathematical modeling method, this study constructed a comprehensive evaluation index system to objectively evaluate the performance of different warming fibers. First, we assigned weights to the indexes by the entropy weighting method, and then used the TOPSIS method to rank nine common warming fibers for comprehensive evaluation. Further, we established a heat conduction model of thermal fibers based on differential equations and thermodynamic knowledge and explored the effects of fiber length and diameter on thermal performance. Finally, we evaluated the warmth-keeping ability of cotton and feather threads from both axial and radial perspectives and found that feather thread fibers had better warmth-keeping performance through comprehensive data analysis. These methods provide useful references and suggestions for the evaluation of thermal fiber properties and material development.

Keywords: TOPSIS; Entropy Weighting Method; Heat Transfer Modeling.

1. Introduction

In today's cold environment, the warmth-keeping performance of winter clothing is crucial [1]. This study aims to explore the performance of different warming fibers and construct a comprehensive evaluation index system through mathematical modeling methods to objectively assess their warming effect. We used the entropy weight method and TOPSIS method to rank and evaluate nine common warming fibers, and established a heat conduction model based on differential equations and thermodynamic knowledge to explore the effects of fiber length and diameter on the warming performance. By evaluating the warmth-keeping ability of cotton and feather threads in axial and radial directions, we found that feather thread fibers have superior warmth-keeping performance. These methods provide an important reference for the evaluation of warming fiber performance and material development and are expected to promote the development and application of winter clothing materials in the future.

2. Comprehensive Indicator System for Thermal Fiber

2.1. Construction of the Indicator System

We will take three different dimensions, i.e., physical indicators (including thermal conductivity, thermal resistance, and heat capacity), environmental adaptability indicators (including humidity adaptability H, wind speed adaptability W, and breathability P), and human comfort indicators (including moisture absorption M, and breathability P), to measure the warming ability of a certain thermal fiber.

2.2. Data Collection and Processing

After creating the above indicator system, we collected data related to the indicators using reliable sources such as CNKI and National Bureau of Statistics [2]. We eliminated the anomalies in the data, and then performed KMO test and Bartlett's sphericity test, aiming to assess the correlation between these secondary indicators under the first-level indicators, and then select the appropriate dimensionality reduction method. Here, we mainly apply principal component analysis to downscaling multidimensional indicators.

For the indicators that did not pass the KMO test and Bartlett's sphericity test, we used the t-SNE method to downgrade the multidimensional nonlinear indicators to a two-dimensional series in order to achieve the downscaling objective. Among them, KMO test and Bartlett's spherical test are important means to determine the covariance or correlation between indicators, which is important to ensure the effectiveness and accuracy of the downscaling method.

$$KMO = \frac{\sum\sum r_{ij}}{\sum\sum r_{ij} + \sum\sum a_{ij}} \tag{1}$$

Where, r_{ij} denotes the correlation coefficient of the two, and K denotes the partial correlation coefficient.

2.3. Ideal Solution Method based on Entropy Weight Method and TOPSIS

Ideal solution method based on entropy weight method is solved, 9 samples of the original data, three indicators, a_{ij} denotes, the value of the $j-th$ data of the $i-th$ indicator. Where $i=1,2,3$, $j=1,2,3...,9$.

The standardization of the data is done first:

$$a_{ij} = \frac{a_{ij} - \min\{a_{1j}...a_{ij}\}}{\max\{a_{1j}...a_{ij}\} - \min\{a_{1j}...a_{ij}\}} \tag{2}$$

After preliminary processing and analysis of the data, we used the TOPSIS method combined with the entropy weight method to establish a comprehensive evaluation model for warming fibers. The entropy weight method is used to automatically adjust the weights according to the variability of each index, while the TOPSIS method is used to integrate each index to evaluate the warmth-keeping ability of fibers.

For the evaluation system, we use the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method combined with the Entropy Weight Method (EWM) to build an evaluation model that includes indicators of physical and environmental adaptability [3], based on which we can develop a model for the thermal performance of synthetic fibers. Based on this model, we can provide a comprehensive and objective evaluation of the thermal performance of synthetic fibers. The entropy weighting method is used to determine the weights of the indicators and the TOPSIS method is used to calculate the distance of each fiber from the ideal solution in order to evaluate its performance [4].

By collecting and processing the evaluation index data of various warming fibers, and using the constructed comprehensive evaluation model, we are able to derive the comprehensive evaluation scores of different warming fibers, as shown in Table.1, so as to achieve a comprehensive assessment of their warming ability.

Table 1. Demonstration of the results of calculating some of the weights

Indicator	Information entropy value	Information utility value	Weights
Physical performance index_thermal resistance	0.954	0.046	10.0
Environmental adaptability index_Rating	0.945	0.055	11.937
Filling performance index_Score	0.923	0.077	16.796
CLO value adaptation_scoring	0.931	0.069	15.05
Comfort score	0.932	0.068	14.72
Persistence and durability_scoring	0.942	0.058	12.617
Physical performance indicator_Thermal conductivity	0.956	0.044	9.621
Physical performance index_thermal conductivity	0.958	0.042	9.219

According to the weight calculation results of entropy weighting method, the weight distribution of each thermal fiber performance index is as follows: the weight of thermal resistance value in physical performance index is 10.03%, the weight of environmental adaptability index scoring is 11.937%, the weight of filling performance index scoring occupies the largest proportion of 16.796%, the weight of CLO value adaptability adjusting scoring is 15.059%, the weight of comfort scoring is 14.721%, persistence and durability scoring weight is 12.617%, thermal conductivity weight of physical performance index is 9.621% and thermal conductivity weight of physical performance index is the smallest with 9.219%. The filling performance index score was ranked first with 16.796% weight while thermal conductivity in physical performance index was ranked last with 9.219% weight.

Table 2. Ranking results of TOPSIS scores for each fiber material

Index value	Positive ideal solution distance (D+)	Negative ideal solution distance (D-)	Combined score index	Ordering
Cotton	0.23727151	0.77250632	0.76502603	1
Wool	0.36587003	0.73575177	0.66788054	3
Nylon	0.65528963	0.35129135	0.34899463	8
Polyester	0.57194422	0.65056035	0.53215372	4
Acrylic	0.36430375	0.79144168	0.68478893	2
Cashmere	0.7201082	0.51683776	0.41783374	5
Silk	0.79160477	0.36344736	0.31465883	9
Camelhair	0.66828685	0.45579335	0.40548116	6
Castor	0.68380767	0.45563663	0.39987617	7

Table. 2 illustrates: the D+ and D- values, which represent the Euclidean distance between the evaluation object and the optimal solution (A+) and the worst solution (A-), respectively. The D+ value indicates the distance between the evaluation object and the optimal solution, while the D- value represents the distance between the evaluation object and the worst solution. In practical terms, the larger the values of these two values, the more distant the evaluation object is from the corresponding solution. Specifically, the larger the D+ value, the greater the difference between the evaluation object and the optimal solution; the larger the D- value, the greater the gap between the evaluation object and the worst solution. The ideal research object should be characterized by a smaller D+ value and a larger D- value.

3. Relationship between the Warming Capacity of Thermal Fibers and the Average Length and Diameter of Fibers

In order to establish the relationship between the warming capacity of thermal fibers and the average length and diameter of the fibers, we use polyester, which is inexpensive and easy to process, as the material of thermal fibers, and assume that the cross section of the fibers is circular, and that the average length of each fiber and the diameter of the fiber are known constants. We can develop a mathematical model based on heat transfer [5].

3.1. Heat Conduction Model

The propagation of the heat conduction model in an isotropic medium in three dimensions can be expressed as equation (3):

$$\frac{\partial u}{\partial t}=k\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=k\left(u_{xx}+u_{yy}+u_{zz}\right)\quad(3)$$

Where $u=u\left(t,x,y,z\right)$ denotes the temperature of the material as a function of the time variable t and the space variable $\left(x,y,z\right)$. $\frac{\partial u}{\partial t}$ denotes the rate of change of temperature at a point on the material with respect to time. u_{xx},u_{yy},u_{zz} is the second derivative of temperature with respect to the three spatial axes. k is the second derivative of temperature with respect to the three spatial axes. is the thermal diffusivity, which is determined by the thermal conductivity, density and heat capacity of the material.

The warming capacity is only related to the average fiber length as well as the fiber diameter, i.e., only the axial thermal conductivity equation is considered, so the model we developed can be described by Fourier's law [6], which is given by:

$$u(t,x) = \sum_{n=1}^{+\infty} D_n \left(\sin \frac{n\pi}{L} \right) e^{-\frac{n^2\pi^2 kt}{L^2}} \tag{4}$$

$$D_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \tag{5}$$

Where $u(t,x)$ is the function associated with t and x. t is the temperature. x is the length of the heat flow path, i.e., the length of the fiber L.

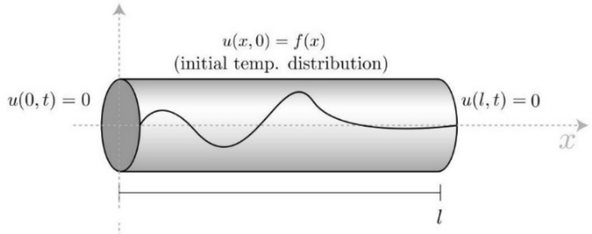

Figure 1. Thermal conductivity model for thermal fibers

Since the radial heat conduction is considered in the evaluation system as the fiber thickness, as shown in Figure 1, we only need to consider the heat conduction equation in the axial direction:

$$U(t,x) = ku_{xx} \tag{6}$$

Since the length and diameter of the warming fibers must be greater than or equal to 0, and since the warming fibers cannot cause the temperature to fall below 0 degrees Celsius, $\theta > 0$: $X(x)$ can be derived from equation (7):

$$\begin{aligned} X(x) &= Be^{\sqrt{-\theta x}} + Ce^{-\sqrt{-\theta x}} \\ X(x) &= B\sin\left(\sqrt{\theta x}\right) + C\cos\left(\sqrt{\theta x}\right) \\ \sqrt{\theta} &= n\frac{\pi}{L} \end{aligned} \tag{7}$$

Combining the above formulas yields, our proposed heat transfer model for the length of the diameter of warming fibers versus the warming capacity obtained through Fourier series.

$$u(t,x) = \sum_{n=1}^{+\infty} D_n \left(\sin \frac{n\pi x}{L} \right) e^{-\frac{n^2\pi^2 kt}{L^2}} \tag{8}$$

$$D_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \tag{9}$$

3.2. Surface Area and Volume of Fibers

The surface area of a polyester fiber A is related to its diameter and length and can be calculated by the following formula:

$$A = \pi dL \tag{10}$$

The volume of polyester fibers V can be calculated using the following formula:

$$V = \frac{1}{4}\pi d^2 L \tag{11}$$

3.3. Quantification of Warmth Retention Capacity

Warmth retention can be quantified by calculating the amount of heat that a fiber material blocks per unit of time. This can be achieved by calculating the thermal resistance of the fiber material $(R_{material})$, which is a measure of how well the material impedes the flow of heat. The thermal resistance can be calculated using the following equation:

$$R_{material} = \frac{\Delta x}{kA} = \frac{L}{k\pi d} \tag{12}$$

We know that the greater the thermal resistance of a material, the greater the ability to keep warm.

3.4. Model Solving

Combining the above equations, we bring the obtained values into equation (4). We consider the surface area to volume ratio of the polyester fiber as this will affect the heat transfer. Define a dimensionless surface area to volume ratio S as follows:

$$S = \frac{A}{V} = \frac{\pi dL}{\pi d^2 L/4} = \frac{4d}{d^2} \tag{13}$$

This ratio S is inversely proportional to the diameter of the fiber, meaning that the smaller the diameter, the greater the surface area relative to the volume, which may provide better warmth.

Finally, we can combine the thermal resistance of polyester materials $(R_{material})$ with S that obtained by equation (4):

$$u(t,x) = \sum_{n=1}^{+\infty} D_n \left(\sin \frac{n\pi x}{L} \right) e^{-\frac{n^2\pi^2 kt}{L^2}} = \frac{4td}{Lk} \tag{14}$$

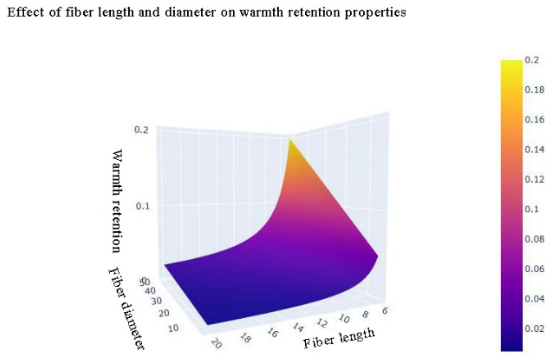

Figure 2. Effect of fiber length and diameter on warmth retention properties

Modeling has shown that the warming capacity of fibers is directly proportional to their length and inversely proportional to their diameter. In other words, the longer the fiber and the smaller its diameter, the greater its warmth-retaining capacity, as shown in Figure 2.

4. Evaluation of the Warming Capacity of Cotton and Down as Warming Materials

In this section, we use the heat conduction model in Section 3, combined with the index system, to find out which one of the cotton and down fibers has the better warmth-keeping ability. The length of cotton fibers is usually between 10 mm and 50 mm, and the diameter of cotton fibers is usually between 10 and 50 microns, while the length of down fibers is usually between 25 and 75 mm, and the diameter of down fibers is smaller, which may be in the range of 10 to 30 microns. The axial thermal conductivity of cotton fiber $\lambda_{cotton,axial} = 0.04$, radial thermal conductivity $\lambda_{cotton,radial} = 0.03$, and axial thermal conductivity $\lambda_{down,axial} = 0.015$. The axial thermal conductivity of down fiber $\lambda_{down,radial} = 0.01$. Here we have chosen the length of feather fibers to be 50 mm and the diameter to be 20 microns, and the length of cotton fibers to be 30 mm and the diameter to be 30 microns.

The material thermal resistance of the feather thread can be obtained from the equation of material thermal resistance value.

$$R_{material} = \frac{\Delta x}{kA} = \frac{L}{k\pi d} = 0.0013 \tag{15}$$

The material thermal resistance of down fibers can be calculated as.

$$R_{base,down} = \frac{R_{material}}{Thick_{material}} = 66.67\mathrm{m}^2\cdot\mathrm{K/W} \tag{16}$$

Calculating the CLO value of the plume line yields.

$$CLO = 6.451\cdot R_{base} = 430.06 \tag{17}$$

Assuming that the compressed fiber thickness is 0.004, the compression ratio can be expressed as follows.

$$\lambda = \frac{compressed\ thickness}{Thick_{material}} = \frac{0.004\ \mathrm{m}}{0.005\ \mathrm{m}} = 0.8 \tag{18}$$

According to the previous section its material properties adjustment weights are.

$$W_{com} = 1+\lambda\cdot(\alpha-1) = 1+0.01\cdot(0.7-1) = 1.03 \tag{19}$$

Environmentally adjusted weights for wind speed and humidity were taken into account.

$$W_{com} = 1+\lambda\cdot(\alpha-1) = 1+1) = 1.03 = 1-(-0.01)\cdot(0.4-1)-(-0.05)\cdot(2-1) = 1.056 \tag{20}$$

The integrated warmth evaluation index of the final down fiber can be calculated as.

$$TCI_{down} = CLO\cdot W_{act}\cdot W_{env}\cdot W_{com} = 430.06\cdot 1.03\cdot 1.056\cdot 1.2 = 561.32 \tag{21}$$

Similarly, we can find the material thermal resistance of cotton.

$$R_{material} = \frac{d}{k} = \frac{0.00003}{0.04} = 0.0075 \tag{22}$$

According to the obtained thermal resistance and index system of cotton materials, the evaluation indexes of cotton's warmth-keeping ability can be obtained as follows.

$$TCI_{cotton} = CLO\cdot W_{act}\cdot W_{env}\cdot W_{com} = 215.24 \tag{23}$$

Also, for axial data, according to the heat transfer model.

$$u(t,x) = \sum_{n=1}^{+\infty} D_n\left(\sin\frac{n\pi x}{L}\right)e^{-\frac{n^2\pi^2 kt}{L^2}} \tag{24}$$

$$D_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx \tag{25}$$

The axial heat fluxes can be calculated for cotton and down fibers, respectively. The radial heat flow $u_{cotton,axial} = -400\,W/m^2$ for cotton fiber, and $u_{down,axial} = -150\,W/m^2$ for down fiber.

Combining the above index system of warming capacity of warming fibers and the data of feather thread and cotton fibers calculated by the heat conduction model, from Figures 3-6, it can be learned that, for typical cotton fibers and feather thread fibers, the warming capacity of feather thread fibers will be better.

5. Conclusion

Through this study, we successfully established a comprehensive evaluation index system to effectively assess the performance of different thermal fibers. The entropy weight method and TOPSIS method were used to rank and evaluate the thermal fibers, which provided an objective reference basis for the performance evaluation of thermal fibers. In terms of heat transfer modeling, we established a model through differential equations and thermodynamic knowledge to reveal the influence law of fiber length and diameter on warmth retention performance. By axially and radially evaluating the warmth-keeping ability of typical cotton and feather threads, we found that feather thread fibers have better warmth-keeping effect. These methods and results provide important references for the development and selection of future winter clothing materials and help to promote further research and development in the field of warming fibers.

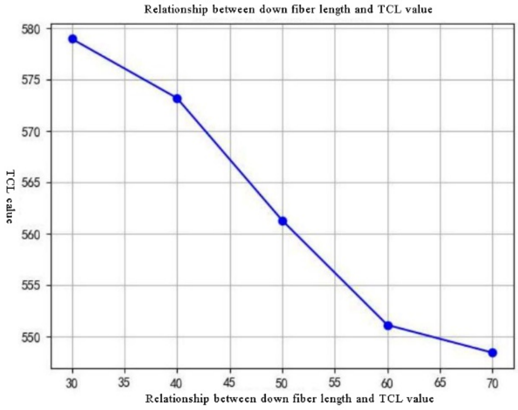

Figure 3. Relationship between feather fiber length and TCL value

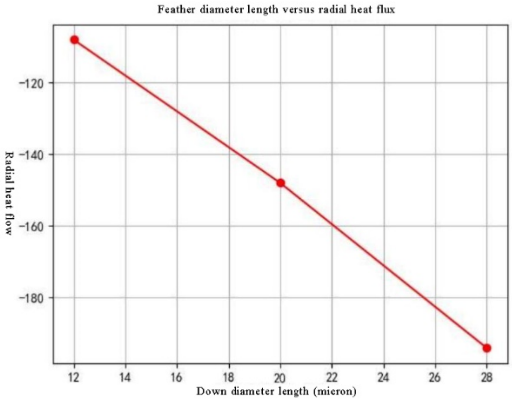

Figure 5. Plume diameter length versus radial heat flux

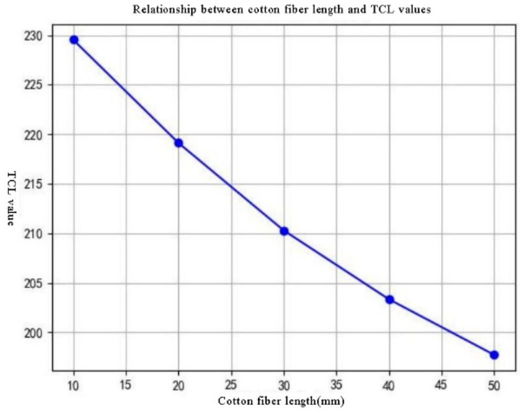

Figure 4. Relationship between cotton fiber length and TCL

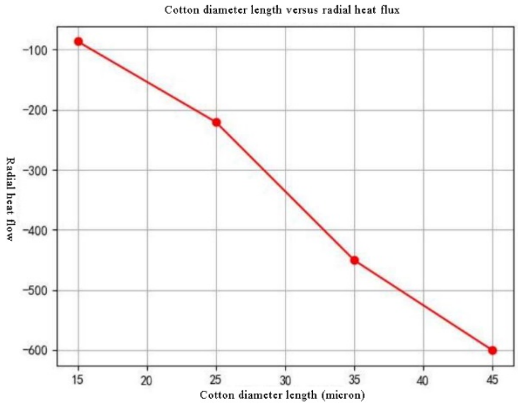

Figure 6. Cotton diameter length versus radial heat flux

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