Academic Journal of Science and Technology
ISSN: 2771-3032 | Vol. 11, No. 3, 2024

Research on Ice Load Optimization based on Monte Carlo Algorithm and Finite Element Analysis

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Abstract: This paper mainly focuses on the ice bearing capacity and ice vehicle layout optimization problems for in-depth research. By modeling the ice thickness and analyzing the ice stresses, the load-bearing capacity and safety margins of the ice were assessed. Firstly, the research considers the freezing time, environmental conditions, load factor, strength, temperature and salt influence coefficient, and obtains the exact value of the bearing capacity of the ice under different conditions and the safe distance of the ice truck. Secondly, Monte Carlo algorithm and finite element analysis are used to simulate the random distribution of the crowd and the influence of the ice truck on the ice surface stress, which provides an important reference for the safety management and planning of the ice surface activities. The research results show that under certain conditions, the ice surface can withstand a certain number of people, and the layout of the ice vehicle needs to consider the safe distance from the ice hole to ensure that the ice surface does not break and does not exceed the maximum allowable stress. These research results are of great significance for ensuring the safety of ice activities and improving the efficiency of ice utilization.

Keywords: Finite Element Analysis; Monte Carlo Algorithm; Load-bearing Capacity.

1. Introduction

In winter, ice and snow activities become the first choice for leisure and entertainment for many citizens, and the natural ice rink attracts many tourists with its unique charm. However, the carrying capacity of the ice surface is the key factor to ensure the safety of these activities. With the change of climate and the diversification of people's activities, the research on the carrying capacity of ice surface becomes particularly important.

In this paper, based on the perspective of spatial distance optimization, finite element analysis, Monte Carlo algorithm and genetic algorithm are used to deeply study the carrying capacity of ice rink. Firstly, we set up the ice thickness model and the maximum weight model to scientifically quantify the bearing capacity of the ice surface. On this basis, through the research of the ice bearing capacity model under unit area, the random distribution of the population on the ice surface and its influence on the stress on the ice surface are simulated. The research results of this paper will provide scientific basis for the security of ice and snow activities, and provide a new idea and method for the research of ice bearing capacity.

2. Ice Rink Carrying Capacity Model

2.1. Ice Thickness Model

According to the freeze degree-day method, the thickness of ice is related to freezing time and environmental conditions. Use the following model to calculate the thickness of ice [1]:

$$h = a\sqrt{I} \tag{1}$$

Where, h is ice thickness and I is freezing degree-day, can be calculated by formula (2). a is a typical coefficient, usually related to ice sheet conditions, such as whether there is snow cover, whether there is wind blowing, etc. [2].

$$I = \sum_{i=1}^{n} (T_b - T_i) \tag{2}$$

Where, T_b is the reference temperature, in this paper, the freezing point is chosen 0°C as the reference temperature; $T_i, i = 1, \cdots, t$ is the average temperature of the i th measurement day.

Figure 1 is a plot of ice surface thickness over time.

2.2. Model of Maximum Load Bearing Weight

It is assumed that the bearing capacity of natural ice can be regarded as the theory of thin plate on elastic foundation, whose thickness to width ratio is very small, floating on the water body, while the bearing capacity of ice and the thickness of ice h square relationship to deduce the maximum bearing capacity of ice p, apply the following formula [3]:

$$p = 9.81\frac{B}{N}h^2KS \tag{3}$$

Which, B represents the load factor, N represents the strength and considers the ice crack coefficient ($N = 1.75$ for no cracks, $N = 2.0$ for cracks) take temperature influence coefficient K, S represents the salt influence coefficient.

The load bearing curve of the ice surface varies with the date, as shown in Figure 2.

The yellow line in Figure 2 shows $\alpha = 1.7205$ the load-bearing capacity of the ice, the blue line shows $\alpha = 2.70$ the load-bearing capacity of the ice, and the green line shows the force of 5,000 people on the ice. During this period, the yellow and blue lines are above the green line, indicating that the ice can support 5,000 people during this time.

2.3. Model of Ice Bearing Capacity Per Unit Area

2.3.1. Model Proposed

Consider the load-bearing capacity of the local ice surface, i.e. the maximum number of people that can be accommodated per unit area of the ice surface. After the water surface is frozen, the ice sheet floats on the water surface, and the ice sheet is an elastic, homogeneous and isotropic material plate, which acts on the elastic basis [4]. Under the action of load, the ice has deflection deformation and internal tensile and compressive stress. However, because the ice is a brittle material, under the action of allowable load P. The ice strain is small, which conforms to Hooke's Law. Moreover, since the compressive strength of the ice is obviously greater than the tensile strength, the ice failure is determined by the ultimate tensile stress at the bottom of the layer. Application formula:

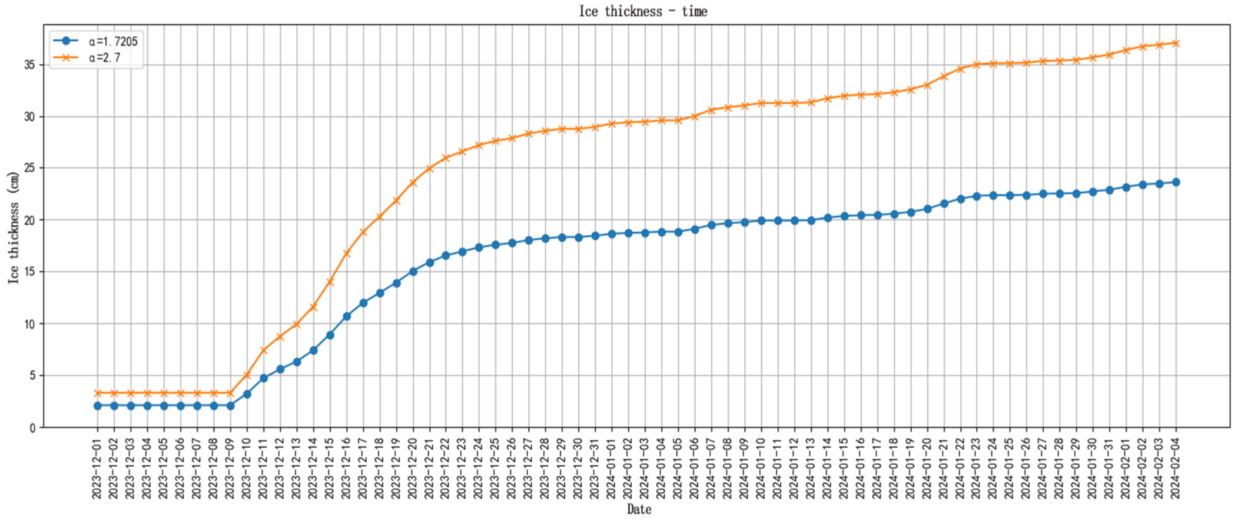

Figure 1. Curve of ice thickness with time

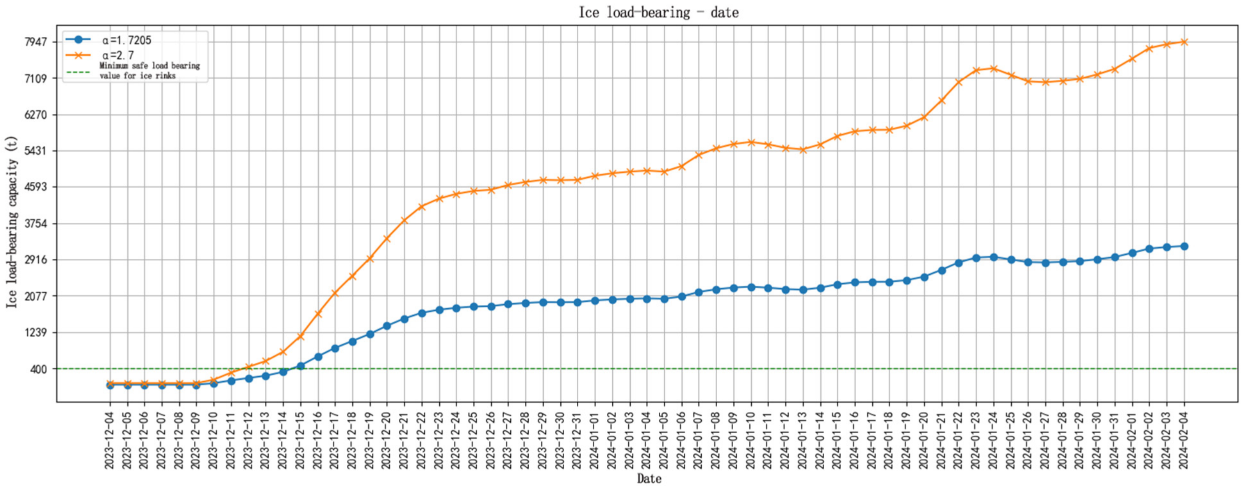

Figure 2. Curve of ice surface load bearing with date

$$\sigma_{\max} = 0.275(1+\mu)\frac{P}{h^2}\log\left(\frac{Eh^3}{kb^4}\right) \quad (4)$$

Where, the Young's modulus represents $E(\text{Pa})$, the foundation modulus k, $k=\rho_{\text{W}}\text{g}, a$ represents the area of the load area, when $a < 1.724\,h$, $b=\sqrt{1.6a^2+h^2}-0.675h$. when $a \geqslant 1.724\,h, b=a$.

The stress value of the distance from the center of the load area x where the uniformly distributed load is applied on the infinite ice sheet is:

$$\sigma_{\text{x}} = 0.249\frac{P}{h^2/6}\exp\left(\frac{-x}{0.691l}\right) \quad (5)$$

Where, l represents the characteristic length and the calculation method is $l=\left[\frac{Eh^3}{12(1-\mu^2)k}\right]^{1/4}$.

2.3.2. Algorithm Proposed

To realize the random distribution of the crowd, we will adopt Monte Carlo algorithm. The algorithm generates many random points in a given area through random sampling to simulate the distribution of the crowd. The steps of the algorithm are shown in Table 1:

Table 1. Steps of Monte Carlo algorithm

Procedure	Description
Step 1: Define the variables	Determine the number of simulations and the number of people required.
Step 2: Random sampling	Coordinate points are randomly selected on the unit area.
Step 3: Simulation process	For each random point, run the simulation process in the unit area, using the hook stock calculate the distance from that point to the center.
Step 4: Output the result	The output result is a list of distances from the center for each person in the simulated population distribution.

2.3.3. Model Solving

Formula (4) represents the maximum bearing capacity of the ice surface when tourists gather together σ_{x}, and formula (5) represents the bending stress of the ice surface when a tourist stands x meters away from the center of the area. When n tourists in a unit area are randomly distributed, the bending stress of the ice surfaceσ at that moment is compared with the maximum bearing capacity of the ice surface σ_{x} to determine how many people can be accommodated in the unit area. Compared with the actual maximum number of people that can be accommodated in the unit area, judge whether the ice will break if part of 5,000 people gathers together, as shown in Figure 3:

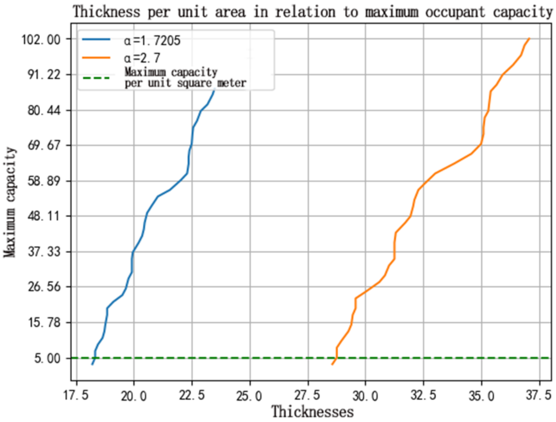

Figure 3. Relationship curve between thickness and maximum number of people per unit area

It can be seen from Figure 3 that the ice can withstand the crowd gathering per unit area during the Open Day.

3. Ice Surface Stress Optimization Model based on Finite Element Analysis

The layout of the skaters in the rink is optimized to minimize the proximity of the single and 10-person skaters to the ice hole, while ensuring that the ice does not break and does not exceed the maximum allowable stress of the ice.

3.1. Model Building

Stress distribution often becomes complicated when ice caves are present, especially around the edges and near the holes. A detailed description of the specific stress distribution around the hole has not been found in the available literature. Finite element analysis is an effective tool for dealing with such complex geometry. This method can accurately simulate the stress distribution around the hole by dividing the problem area into multiple small units and applying the appropriate approximation function to each unit.

(1) Optimization objectives:

Optimize the closest distance between the single ice truck and the hole, and optimize the closest distance between the 10-person team and the hole.

(2) Constraints:

First, consider the situation of an individual ice truck. The goal is to maximize the distance r_{only} between the vehicle and the ice hole while sliding around the area with the hole, ensuring that it does not cause the ice to break, so:

$$r_{\text{only}} \geq r_0 \geq r_{\text{ice\_hole}} \tag{6}$$

To move on to the next stage, we need to consider the situation of a convoy of ten vehicles coasting around the area with a hole. The total length of this convoy is $N \times L$, where,N is the number of vehicles in the convoy, L is the length of a single vehicle. Since the mass of the convoy is evenly distributed, and the motion of the entire convoy is regarded as the motion of the center of mass. Our goal is to ensure that the entire convoy does not slide into the ice hole, so the distance between the center of mass and the center of the ice hole r_{more} should be at least:

$$r_{\text{more}} \geq (N \times L)/2 + r_{\text{ice\_hole}} \tag{7}$$

At the same time, ensure that the entire team maximizes its distance from the ice hole when sliding around the area with a hole, so that the critical distance between the team and the ice hole is expressed r_0', so there are:

$$r_{\text{more}} \geq r_0' \tag{8}$$

Due to the existence of the load of the ice hole, the stress distribution on the ice surface must be within an acceptable range. The stress of the ice sheet when a single ice vehicle is sliding to a certain position σ_{only} and the stress of the ice sheet when the whole team is sliding to a certain σ_{more} position should meet:

$$\sigma_{\text{only}} < \sigma_{\max} \tag{9}$$

$$\sigma_{\text{more}} < \sigma_{\max}' \tag{10}$$

Which, $\sigma_{\max}$ represents the maximum bearing capacity of the ice to a single ice vehicle, $\sigma_{\max}$' represents the maximum bearing capacity of the ice to the ice vehicle fleet.

The large data prediction model for the user's electricity consumption is implemented in the Clementine software.

3.2. Model Solving

After finite element analysis, we can obtain the maximum stress of the ice surface under the pressure exerted by a single ice skater or a team of 10, as shown in Figure 4. According to the literature, the maximum allowable stress of ice is 0.98912 MPa [5]. Currently, however, it is still not possible to determine the optimal distance for a single skater or a team of 10. Therefore, we will use genetic optimization algorithm to optimize the distance between the single ice skater or the team of 10 people to the ice hole.

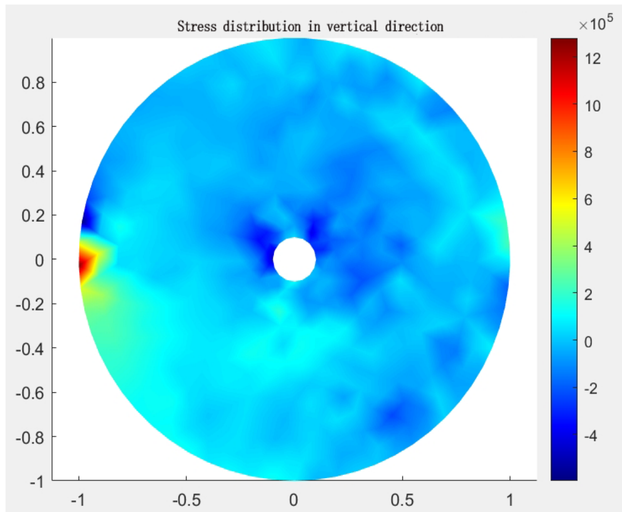

Figure 4. Ice surface stress distribution at 1 m from the ice hole by an ice truck

3.3. Result Analysis

Through matlab solution, we obtained the safety range of the single ice truck and the 10-person team, as shown in Figure 5 and Figure 6.

By setting the minimum moving radius of the ice vehicle as 0.1 meters and the maximum moving radius as 10 meters, we get that when the distance between the ice vehicle and the center of the hole is greater than or equal to 3.550705 meters, the single ice vehicle is in a safe state. It is shown in Figure 5. Considering the length and constraints of the convoy, and if the total length of the 10-person convoy is 7 meters, the minimum moving radius of the convoy is 3.6 meters and the maximum moving radius is 10 meters. We can get that when the distance between the convoy and the center of the hole is greater than or equal to 8.737758 meters, the convoy is in a safe state. It is shown in Figure 6.

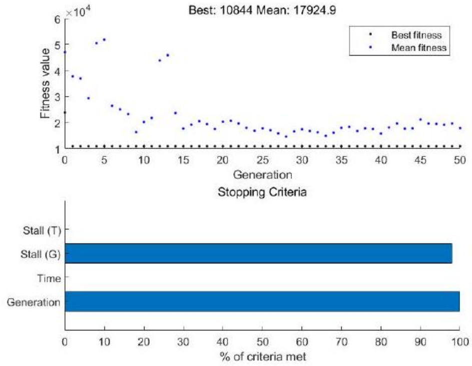

Figure 5. Simulation results of single ice vehicle

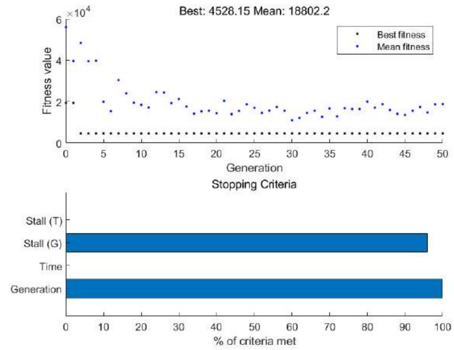

Figure 6. Simulation results of a 10-person team

3.4. Model Checking

After obtaining the safe range, use Solidworks software to build a solid model to simulate the distribution of people on the ice rink. The model was then meshed with the help of its built-in Simulation plug-in. Through the analysis of the grid model, the maximum stress value in the ice rink is obtained, and the rationality of the results is tested. The results obtained are shown in Figure 7 and Figure 8 below.

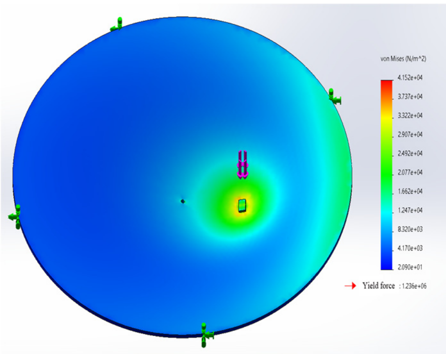
Figure 7. Diagram of static stress analysis for one person

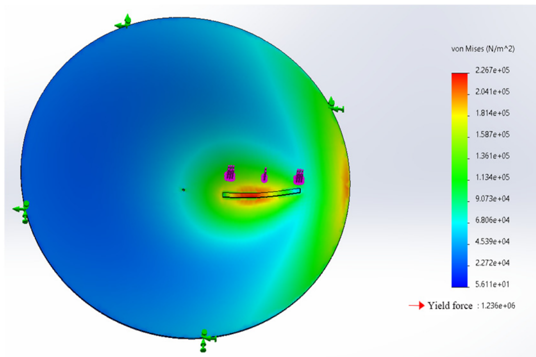

Figure 8. Static stress analysis diagram of vehicle fleet

According to the above analysis results, it is found that the maximum stress value of the ice rink remains below the allowable stress value within the minimum distance range obtained. 0.98912MPa. This conclusion not only validates the reasonableness of the safety range set by us, but also further proves the structural stability and safety of the ice rink under this condition.

4. Conclusion

This paper makes an in-depth study on the bearing capacity of ice surface, and reveals the spatio-temporal variation and influencing factors of the bearing capacity of ice surface by constructing the ice thickness model and the maximum bearing weight model. At the same time, a detailed ice surface model is established by finite element analysis (FEA), which can accurately simulate the ice surface pressure generated by the ice vehicle moving on the ice surface and calculate the maximum stress value in the area around the ice cave. With the distance between the ice truck and the ice cave as the objective function, the spatial distance of the ice truck is optimized by using genetic algorithm. Finally, it is concluded that when the distance between the single ice truck and the center of the hole is greater than or equal to 3.550705 meters, and the distance between the 10-person team and the center of the hole is greater than or equal to 8.737758 meters, it is in a safe state. It provides theoretical support for ice rink management and safety evaluation.

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