Compositional Analysis of Ancient Glassware Based on CRITIC Weighting Method and Superior Order Approximation Model

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Abstract: Glass is a witness of early trade exchanges on the Silk Road. In this paper, based on the classification information of glass artefacts and their corresponding proportions of major components, the chi-square test, the CRITIC weighting method and the superior order approximation model are used. The data were first pre-processed to exclude the sample data whose sum of the proportions of chemical components did not belong to 85%~105%. After classifying the other glass artifacts according to their attributes, the chi-square test was carried out by SPSS, and according to the significance p-value <0.05, it was concluded that only the glass type within the three had a significant effect on the degree of weathering, while the decoration and colour had little effect on the degree of weathering. All the sample data were classified according to the combination of type-degree of differentiation, and then the CRITIC weight method was applied to find out the objective weight of each chemical indicator in the combination respectively, and finally the distribution of the weights of each chemical element in all the combinations was compared to establish a superior order approximation model, and the individuals whose predictive attributes were known according to the prediction attributes were taken as the prediction reference objects to obtain the final results.

Keywords: CRITIC Weighting Method; Superior Order Approximation Model; Chi-square Test; Glassware.

1. Introduction

The Silk Road carries the common memory of thousands of years of material and cultural exchanges between the East and the West, and glass is a witness to the early trade exchanges. Although the appearance of ancient native glass and foreign glass is similar, the internal chemical composition is different. Under the influence of the burial environment, the glass is very easy to weathering, resulting in the proportion of internal elemental composition will be changed, affecting people's judgement on its category. In this paper, according to the classification information of cultural relics and the proportion of their corresponding main components (the sum of the proportions of chemical components belongs to 85%~105%, which is valid sample data), according to the different degrees of weathering, the glass attributes, decorations and colours of the glass, the analysis of the three attributes of the weathering degree of the impact of neglecting the other two attributes, in accordance with the classification of the type of glass, analyse the weathering degree of different Neglecting the other two attributes and classifying the glass according to its type, the proportion of each chemical component is analysed at different degrees of weathering, and then a statistical law is derived; the weathering point data are analysed, and a model is chosen to predict the proportion of each chemical component at these weathering points when they are unweathered.

2. Methodologies

2.1. Chi-square Test

Table 1. The chi-square table

n	p												
	0.995	0.99	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.025	0.01	0.005
1	...	...	...	...	0.02	0.1	0.45	1.32	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.02	0.1	0.21	0.58	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	0.07	0.11	0.22	0.35	0.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84
4	0.21	0.3	0.48	0.71	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.2	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.4	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.7	3.33	4.17	5.9	8.34	11.39	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19
11	2.6	3.05	3.82	4.57	5.58	7.58	10	13.7	17.2	19.68	21.9	24.72	26.76

The chi-square test [1] is to calculate the degree of deviation between the actual observed value and the theoretical inferred value of the statistical sample, the degree of deviation between the actual observed value and the theoretical inferred value will determine the size of the chi-square value, if the chi-square value is larger, the greater the degree of deviation of the two; on the contrary, the smaller the deviation of the two; if the two values are completely equal, the chi-square value will be 0, indicating that the theoretical value is completely consistent [2]. The chi-square table (as in Table 1) was used to obtain the p-value to whether the original hypothesis was accepted or not and to determine whether there was a significant difference.

2.2. The CRITIC Method

The CRITIC method [3] is an objective weighting method that integrates the objective weights of the indicators based on the comparative strength of the evaluation indicators and the conflict between the indicators. Contrast intensity refers to the size of the difference in values between the various evaluation programmes for the same indicator, expressed in the form of standard deviation. The larger the standard deviation, the greater the fluctuation, i.e., the larger the difference in values between programmes, the higher the weight will be; the conflict between indicators is expressed by the correlation coefficient, if there is a strong positive correlation between the two indicators, it means that the smaller the conflict between them, the lower the weight will be. When the standard deviation is certain, the smaller the conflict between indicators, the smaller the weight; the larger the conflict, the larger the weight.

For the analysis of statistical patterns, the CRITIC weighting method was used to combine the objective weights assigned to each chemical indicator based on the contrasting strengths of the chemical components and the conflicting nature of the components. After all sample data were classified according to the combination of type-differentiation degree (high potassium-weathering, high potassium-non - weathering, lead-barium-weathering, lead-barium-non-weathering), then SPSS was used to find out the objective weights of each chemical index in the combination respectively, and finally the distribution of weights of each chemical element of all the combinations was compared to get the statistical law, as shown in Table 2.

Table 2. High-potassium-weathering assemblage artefact number and corresponding relative chemical content

Artifact number	7	9	10	12	22	27
SiO_2	92.63	95.02	96.77	94.29	92.35	92.72
Na_2O	0	0	0	0	0	0
K_2O	0	0.59	0.92	1.01	0.74	0
CaO	1.07	0.62	0.21	0.72	1.66	0.94
MgO	0	0	0	0	0.64	0.54
Al_2O_3	1.98	1.32	0.81	1.46	3.5	2.51
Fe_2O_3	0.17	0.32	0.26	0.29	0.35	0.2
CuO	3.24	1.55	0.84	1.65	0.55	1.54
PbO	0	0	0	0	0	0
BaO	0	0	0	0	0	0
P_2O_5	0.61	0.35	0	0.15	0.21	0.36
SrO	0	0	0	0	0	0
SnO_2	0	0	0	0	0	0
SO_2	0	0	0	0	0	0

The glass artefact samples were classified into four categories, taking into account the type of glass and its weathered or unweathered status: weathered high-potassium glass, unweathered high-potassium glass, weathered lead-barium glass, and unweathered lead-barium glass. (The unweathered portion of weathered glass is defined as unweathered.) An attempt was made to calculate the content of each component for each of the four categories of glass using the CRITIC algorithm to calculate the weights.

Weathered high-potassium glass is categorised as having 6 samples to be tested and 14 evaluation criteria for each sample.

$$\mathrm{x} = \begin{pmatrix} x_{11} & \cdots & x_{1_{14}} \\ \vdots & \ddots & \vdots \\ x_{61} & \cdots & x_{6_{14}} \end{pmatrix} \tag{1}$$

Next, the variability and conflict of the glass indicators were analysed. The standard deviation of each indicator was calculated as S_j,

$$\begin{cases} \bar{x}_j = \frac{1}{n}\sum_{i=1}^{n} x_{ij} \\ S_j = \sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_j)^2}{n-1}} \end{cases} \tag{2}$$

Find the correlation coefficients and conflict between each sample and each indicator as $\mathrm{r_{ij}}$ and R_j.

$$R_j = \sum_{i=1}^{p}\left(1 - r_{ij}\right) \tag{3}$$

Using the S_j and R_j obtained from variability and conflict, the amount of information C_j was calculated.

$$\mathrm{C_j = S_j \sum_{i=1}^{p}\left(1 - r_{ij}\right) = S_j \times R_j} \tag{4}$$

and use C_j to obtain the final objective weights W_j of each component in each glass.

3. Modelling of the Superior Order Approximation

The superior order approximation model can use the basic attribute approximation situation to make predictions by analogy, which can solve the problem of using some of the individuals with known predictive attributes to predict the predictive attributes of all the other individuals under the premise of by having the same or approximate basic attributes [4]. Firstly, a decision attribute can be determined from the basic attributes, and then the priority of other basic attributes can be obtained through the chi-square test, the smaller the p-value, the greater the significance of the effect on the results, that is, the greater the error value of the results obtained by analogy when this attribute is different, the priority is given to controlling this attribute unchanged [5].

After determining the priority of the basic attributes, priority is given to find the group of each basic attribute that is exactly the same, if it cannot be exactly the same, then according to the priority of other basic attributes obtained through the chi-square test before, through the python for loop traversal search processing to find the approximate similarity, to establish the approximate identical group, the prediction results of the exact same group are more accurate than the prediction results of the approximate identical group [6]. Degree of weathering is used as a decision attribute, glass type, grain and colour as other basic attributes and chemical composition as a predictive attribute. Based on the previous chi-square test, the priority was obtained: glass type > decoration > colour, using the search for three glass artefacts with identical other basic attributes and grouping them by type until no identical glass artefacts could be found. Next, controlling for glass type and ornament as quantitative invariants, artefacts were searched for and grouped by type until all types were found. If there are still remaining artefacts, then the glass type as the only quantitative control remains unchanged, continue to find and group until all the glass artefacts are divided into groups, the above steps can be traversed through the python for loop to find processing.

After the end of grouping, at least one individual with known prediction attributes is found in each group, which will be used as the reference object for prediction; if there are more than one known individual, they can be used as the reference object for prediction, and the average of their prediction results will be taken as the final result.

4. Solution Results

The three statistics, Pearson's chi-square, continuously corrected Yates' chi-square and Fisher's chi-square, and the corresponding p-values were calculated separately for glass type, grain, and colour versus degree of weathering using SPSS (e.g., Tables 3-Tables 5).

Table 3. Type-weathering chi-square test statistic process values

Item	Name	Value
Type - Weathering (2*2)	Pearson’s chi-squared test	5.061(p =0.024*)
	Continuously corrected Yates chi-squared test	3.790(p =0.052)
	Fisher chi-squared test	4.971(p =0.026*)
	E>=5	4(100.00%)
	1<=E<5	0(0.00%)
	E<1	0(0.00%)
	n	56
	Degrees of freedom df value	1

Note: *p<0.05 **p<0.01

Table 4. Process values for the texture-weathering chi-square test statistic

Item	Name	Value
Tattoo - Weathering (3*2)	Pearson’s chi-squared test	4.941(p =0.085)
	Continuously corrected Yates chi-squared test	4.941(p =0.085)
	Fisher chi-squared test	-
	E>=5	4(66.67%)
	1<=E<5	2(33.33%)
	E<1	0(0.00%)
	n	56
	Degrees of freedom df value	2

Note: *p<0.05 **p<0.01

Table 5. Process values of chi-square test statistic

Item	Name	Value
Colour - Weathering (9*2)	Pearson’s chi-squared test	9.770(p =0.282)
	Continuously corrected Yates chi-squared test	9.770(p =0.282)
	Fisher chi-squared test	-
	E>=5	4(22.22%)
	1<=E<5	8(44.44%)
	E<1	6(33.33%)
	n	54
	Degrees of freedom df value	8

Notes: *p<0.05 **p<0.01

The final statistic and p-value were selected by combining the expected frequency information, R*C crossover type, etc., and Pearson's chi-square was chosen to produce the results. Different types (high potassium 1 lead barium 2) samples show significance (p<0.05) for weathering (weathering 1 unweathered 2) all together (p<0.05), which means that different types (high potassium 1 lead barium 2) samples show variability for weathering (weathering 1 unweathered 2) all together (p<0.05). Different types of samples show significant differences for weathering all of them. The different ornament (A1 B2 C3) samples will not show significance (p>0.05) for weathering (weathering 1 not weathered 2) total 1, implying that the different ornament (A1 B2 C3) samples show consistency and no difference for weathering (weathering 1 not weathered 2) total 1. The different ornament samples do not show significant differences for weathering all together. The different colour (blank 1 black 2 blue green 3 green 4 light blue 5 light green 6 dark blue 7 dark green 8 purple 9) samples do not show significance ($p>0.05$) for weathering (weathering 1 undifferentiated 2) for a total of 1 item, implying that the different colour (blank 1 black 2 blue green 3 green 4 light blue 5 light blue 6 dark blue 7 dark green 8 purple 9) samples show consistency and are not different for weathering (weathering 1 undifferentiated 2) for a total of 1 item. Then, none of the different colour samples show significant differences for weathering all. It can be concluded that only glass type causes a significant difference in the degree of weathering of the glass, with ornamentation and colour having less of an effect on the degree of weathering compared to the former.

For the analysis of statistical laws, all the sample data were firstly classified into combinations, and four combinations were obtained: high potassium-weathering, high potassium-no weathering, lead-barium-weathering, and lead-barium-no weathering. After analysed by CRITIC weighting method using SPSS respectively, the following table was obtained for the weighting values of each chemical component in the combination (Table 6-Table 9).

Table 6. Results of CRITIC weighting for high potassium-weathering combinations

Item	Indicator variability	Indicator conflict	Quantity of information	Weights
SiO_2	1.734	15.566	26.985	37.57%
Na_2O	0	13	0	0.00%
K_2O	0.445	14.007	6.236	8.68%
CaO	0.488	11.938	5.823	8.11%
MgO	0.306	12.552	3.845	5.35%
Al_2O_3	0.964	12.076	11.648	16.22%
Fe_2O_3	0.069	12.829	0.892	1.24%
CuO	0.935	14.491	13.546	18.86%
PbO	0	13	0	0.00%
BaO	0	13	0	0.00%
P_2O_5	0.21	13.557	2.846	3.96%
SrO	0	13	0	0.00%
SnO_2	0	13	0	0.00%
SO_2	0	13	0	0.00%

Table 7. Results of CRITIC weighting for high potassium-unweathered combination

Item	Indicator variability	Indicator conflict	Quantity of information	Weights
SiO_2	8.755	16.945	148.354	38.54%
Na_2O	1.287	14.377	18.502	4.81%
K_2O	3.92	13.326	52.243	13.57%
CaO	3.092	13.442	41.568	10.80%
MgO	0.676	11.524	7.792	2.02%
Al_2O_3	2.492	11.405	28.415	7.38%
Fe_2O_3	1.667	11.112	18.52	4.81%
CuO	1.66	12.197	20.246	5.26%
PbO	0.589	12.9	7.598	1.97%
BaO	0.982	11.861	11.649	3.03%
P_2O_5	1.434	11.79	16.906	4.39%
SrO	0.048	11.227	0.543	0.14%
SnO_2	0.681	14.774	10.065	2.61%
SO_2	0.186	13.915	2.581	0.67%

A best-order approximation model was developed using weathering degree as the determining attribute, glass type, grain and colour as the other basic attributes, and the proportion of chemical composition as the predicting attribute. According to the priority of the other basic attributes obtained based on the previous chi-square test ($0.024^*<0.085<0.282$): glass type<decoration<colour.

The predicted values of weathered artefacts before weathering were obtained, and the predicted values of the identical group were more accurate than those of the near-identical group, and the predicted results of the identical group are shown in Table 10.

5. Conclusion

In this paper, based on the classification information of glass artefacts and the proportion of their corresponding main components, the chi-square test, the CRITIC weighting method and the superior order approximation model are used.

After first pre-processing the data, excluding the sample data whose sum of the proportions of chemical components does not belong to 85%~105%, and classifying the other glass artifacts according to their attributes, the results are obtained through the chi-square test: within the three, only the glass type has a significant effect on the degree of weathering, while the decoration and the colour do not have a significant effect on the degree of weathering.

Table 8. Results of CRITIC weighting of lead-barium-weathering combinations

Item	Indicator variability	Indicator conflict	Quantity of information	Weights
SiO_2	9.038	12.983	117.34	18.97%
Na_2O	0.577	14.032	8.09	1.31%
K_2O	0.242	12.274	2.974	0.48%
CaO	1.726	12.222	21.092	3.41%
MgO	0.709	11.228	7.961	1.29%
Al_2O_3	2.689	11.106	29.862	4.83%
Fe_2O_3	0.747	11.618	8.674	1.40%
CuO	2.921	13.578	39.655	6.41%
PbO	12.19	16.065	195.825	31.65%
BaO	8.079	14.416	116.463	18.82%
P_2O_5	4.345	12.681	55.094	8.90%
SrO	0.272	13.449	3.653	0.59%
SnO_2	0.28	11.854	3.321	0.54%
SO_2	0.647	13.421	8.688	1.40%

Table 9. Results of CRITIC weighting for lead-barium-unweathered combinations

Item	Indicator variability	Indicator conflict	Quantity of information	Weights
SiO_2	11.829	16.303	192.838	34.67%
Na_2O	2.372	14.438	34.242	6.16%
K_2O	0.31	11.842	3.672	0.66%
CaO	1.285	11.526	14.808	2.66%
MgO	0.547	12.911	7.059	1.27%
Al_2O_3	3.262	12.924	42.164	7.58%
Fe_2O_3	1.155	12.561	14.505	2.61%
CuO	1.97	13.422	26.439	4.75%
PbO	8.215	12.858	105.634	18.99%
BaO	5.825	12.971	75.563	13.59%
P_2O_5	1.847	13.233	24.443	4.40%
SrO	0.243	12.12	2.951	0.53%
SnO_2	0.127	11.715	1.492	0.27%
SO_2	0.763	13.527	10.323	1.86%

Table 10. Predicted results for identical groups

Chemical composition	**2**	**28**	**29**	**42**	**44**	**53**
SiO_2	40.94	62.91	59.61	51.34	57.84	59.85
Na_2O	0.12	0.12	0.98	5.43	2.98	2.96
K_2O	1.05	0.26	0.3	0.35	0.2	0.11
CaO	2.31	1.33	2.94	0.01	2.12	0.77
MgO	1.17	0.99	1.47	1.15	0.02	1.13
Al_2O_3	5.70	4.70	14.06	5.62	12.46	6.02
Fe_2O_3	1.83	0.41	0.81	0.01	0.76	0.01
CuO	0.33	0.40	0.79	2.67	0.50	0.60
PbO	40.89	17.56	13.85	19.86	14.85	14.88
BaO	2.32	5.67	4.00	11.32	6.64	9.76
P_2O_5	3.42	1.07	0.49	0.11	0.11	0.11
SrO	0.19	0.12	0.25	0.00	0.26	0.27
SnO_2	0.00	0.23	0.00	0.00	0.00	0.00
SO_2	0.00	0.00	0.00	0.00	0.00	0.00

All the sample data were classified according to the combination of type-degree of differentiation, and then the CRITIC weight method was used to find out the objective weight of each chemical index in the combination, and finally the weight distribution of each chemical element in all the combinations was compared to obtain that the content of the main chemical components (magnesium oxide, aluminium oxide, and copper oxide) of high-potassium glass showed a decreasing trend after weathering, whereas the content of the main chemical components of lead-barium type of glass (lead oxide, barium oxide, and phosphorus pentoxide) content tends to increase after weathering; a superior order approximation model is established, and the final results are obtained based on the individuals whose prediction properties are known, and they are used as the reference objects for prediction.

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