

Application and Practice of Mathematical Optimization Method in Structural Mechanics Design

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Abstract: This paper discusses the application of mathematical optimization method in structural mechanics design, and emphasizes its importance in improving the efficiency and accuracy of structural design such as buildings and bridges. Traditional design methods are limited by empirical formulas and trial-and-error methods, and they are unable to solve complex structural problems, while mathematical optimization methods can better find the global optimal solution. This paper introduces several optimization algorithms, including scatter-based, gradient-based and heuristic algorithms. Taking a truss structure as an example, it shows how to use genetic algorithm (GA) to optimize the design. The results show that the optimized truss structure has achieved significant weight reduction under the constraint conditions. At the same time, compared with the gradient-based method, the advantages of GA in global optimization are highlighted. Finally, the paper discusses the challenges and solutions encountered in practice, such as improving the algorithm, introducing parallel computing and synthesizing various optimization methods. These studies provide valuable reference for engineers and promote the development of this field.

Keywords: Mathematical optimization; structural mechanics design; Genetic algorithm.

1. Introduction

In the field of modern engineering, structural mechanics design is the key link to ensure the safety, stability and economy of buildings, bridges, aerospace vehicles and other structures. With the continuous progress of science and technology, the complexity and accuracy requirements of structural design are also increasing. Traditional structural design methods often rely on empirical formulas and trial-and-error methods, which may be inadequate in dealing with complex structures, unable to find the global optimal solution or inefficient [1-2]. Therefore, exploring more efficient and accurate structural design methods has become the focus of engineers and researchers.

As a powerful tool, mathematical optimization method can find the optimal solution under given constraints, which provides strong support for solving complex structural design problems. These methods transform practical problems into solvable mathematical problems through mathematical modeling, and then use algorithms to find the optimal design parameters [3]. Mathematical optimization method can not only improve the design efficiency, but also ensure the superiority of design results to some extent. This paper shows in detail how the mathematical optimization method plays a role in structural mechanics design through specific cases, in order to provide useful reference and enlightenment for engineers and researchers.

2. Overview of Mathematical Optimization Methods

Mathematical optimization method is a subject to study and solve optimization problems, aiming at finding the best scheme from many possible schemes. In structural mechanics design, the application of mathematical optimization method can significantly improve the design efficiency and quality. Including scatter-based optimization algorithm, gradient-based optimization algorithm and heuristic optimization algorithm.

The optimization algorithm based on scattering points is a method to search the optimal solution by randomly scattering points in the design space [4-5]. This method is simple, intuitive and easy to implement, especially suitable for situations with large design space and high complexity. However, the shortcomings of the scattering method are also obvious, that is, the search efficiency is low and it is easy to fall into the local optimal solution. Therefore, in practical application, it is necessary to set up a reasonable scattering strategy to improve the search efficiency and global optimization ability.

Gradient-based optimization algorithm is a method to guide the search direction by using the gradient information of the objective function about the design variables [6]. This kind of algorithm usually has high convergence speed and accuracy, and is especially suitable for the case of objective function continuously differentiable. Common optimization algorithms based on gradient include gradient descent method and Newton method [7-8]. However, the gradient method may fall into local optimum when dealing with non-convex problems or problems with multiple local optimum solutions, so it needs to be combined with other strategies to improve the global optimization ability.

Heuristic optimization algorithm is a kind of algorithm based on intuitive or empirical construction, which can give a feasible solution to the problem within acceptable time and space complexity, but it may not guarantee the optimality of the obtained solution. This kind of algorithm usually has strong global optimization ability and good adaptability, especially suitable for dealing with complex nonlinear, multi-peak and other optimization problems. Common heuristic optimization algorithms include simulated annealing algorithm, genetic algorithm (GA) and ant colony algorithm [9-11]. These algorithms search for the optimal solution by simulating natural processes or biological behaviors, which have high practical value and broad application prospects.

3. Case Analysis of Application of Mathematical Optimization Method in Structural Mechanics Design

3.1. Problem description and model building

In the design of structural mechanics, a kind of problem often encountered is how to minimize the weight of the structure under the constraints of structural strength, stiffness and stability. This kind of problems usually have complex characteristics such as high nonlinearity and multi-peaks, and it is often difficult for traditional optimization methods to find the global optimal solution. Therefore, the heuristic algorithm is introduced to optimize the structure design.

Taking the truss structure as an example, it is assumed that the truss is composed of n members, the cross-sectional area A_i of each member is the design variable, and the total weight W of the structure is the objective function, namely:

$$W = \sum_{i=1}^n \rho_i A_i L_i \quad (1)$$

Where ρ_i, L_i is the density and length of the i bar, respectively. In addition, the displacement constraint and stress constraint of the structure should be considered.

Where the displacement and stress constraints can be expressed as:

$$\begin{cases} u_j \leq u_{j,\max}, & (j = 1, 2, \dots, m_u) \\ \sigma_i \leq \sigma_{i,\max}, & (i = 1, 2, \dots, n) \end{cases} \quad (2)$$

Where u_j is the displacement of the j degree of freedom and $u_{j,\max}$ is the maximum allowable displacement.

σ_i is the stress of the i bar, and $\sigma_{i,\max}$ is the maximum allowable stress.

3.2. The application of heuristic algorithm

Aiming at the above optimization problems, GA is chosen as a heuristic optimization algorithm to solve them. GA can find the global optimal solution in a complex search space by simulating natural selection and genetic mechanism.

Using real coding method, the cross-sectional area A_i of each bar is directly taken as the gene value to form a gene string with a length of n .

Since the objective function is a minimization problem, the fitness function can be designed as:

$$F = \frac{1}{1+W} \quad (3)$$

At the same time, in order to meet the displacement and stress constraints, a penalty term is introduced into the fitness function:

$$F = \frac{1}{1+W + P_u + P_\sigma} \quad (4)$$

Where P_u, P_σ is the penalty term of displacement and stress respectively, which can be defined as:

$$\begin{aligned} P_u &= \sum_{j=1}^{m_u} \max(0, u_j - u_{j,\max})^2 \\ P_\sigma &= \sum_{i=1}^n \max(0, \sigma_i - \sigma_{i,\max})^2 \end{aligned} \quad (5)$$

Roulette wheel selection strategy is adopted to select the parent individuals, and offspring individuals are generated through crossover and mutation operations. The crossover operation adopts two-point crossover, and the mutation operation adopts random disturbance. After each generation of evolution, individuals are sorted according to the fitness function value, and excellent individuals are selected to enter the next generation of evolution process. At the same time, in order to maintain the diversity of the population, the elite retention strategy is adopted to keep the best individuals in each generation directly to the next generation.

3.3. Analysis and discussion of optimization results

Through the optimization of GA, the truss structure design scheme which satisfies the displacement and stress constraints and minimizes the total weight is successfully found. In the optimization process, the algorithm gradually converges to the global optimal solution and stabilizes after a certain number of iterations. The final optimization result not only achieves the expected goal numerically, but also has good feasibility and stability in practical application.

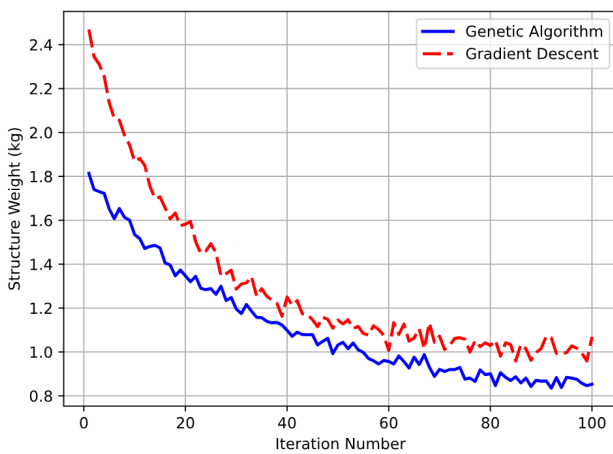
The optimized truss structure maintains sufficient strength and stiffness, and realizes the reasonable distribution and efficient utilization of materials. Compared with the original design scheme, the weight of the optimized structure has been significantly reduced, which is of great significance to improve the economy and environmental friendliness of the structure. The specific optimization results are shown in Table 1.

In order to further verify the advantages of GA in truss structure optimization design, it is compared with gradient-based optimization algorithm. Under the same constraints and initial design, the truss structure is optimized by GA and gradient descent method respectively.

The comparison results show that GA has obvious advantages in global optimization ability. Because GA does not depend on gradient information, it can better deal with complex problems such as nonlinearity and multi-peak and avoid falling into local optimal solution. In contrast, the gradient descent method is easily influenced by the initial value selection when dealing with this kind of problems, and may fall into local optimum and cannot jump out. Figure 1 shows the convergence comparison between GA and gradient descent method.

Table 1. Comparison of key parameters before and after optimization

parameter	Initial design scheme	Optimized design scheme
Sectional area of member 1 (m ²)	0.015	0.01147
Sectional area of rod 2 (m ²)	0.020	0.01632
Sectional area of rod 3 (m ²)	0.018	0.01389
...
Sectional area of rod n (m ²)	0.016	0.01256
Rod 1 length (m)	5.0	5.0
Length of rod 2 (m)	6.5	6.5
Rod 3 length (m)	4.8	4.8
...
Length of rod n (m)	7.2	7.2
Rod density (kg/m ³)	7850.0	7850.0
Total weight of the structure (kg)	2350.0	1874.331
Maximum displacement (mm)	15.0	14.579
Maximum stress (MPa)	180.0	175.403

**Figure 1.** Comparison of convergence between GA and gradient descent method

GA performs well in global optimization, and its convergence curve drops rapidly in the initial stage, which indicates that the algorithm can quickly approach the global optimal solution region. With the progress of iteration, GA gradually converges steadily, and maintains good search ability in the whole optimization process, effectively avoiding falling into local optimum. In contrast, the convergence curve of gradient descent method decreases slowly in the initial stage and is easily influenced by the selection of initial value. Although the gradient descent method gradually approaches the optimal solution with the deepening of iteration, in some cases, it may fall into local optimum and cannot jump out further. In addition, the convergence process of gradient descent method is relatively more fluctuating, which shows its strong dependence on local gradient. GA has obvious advantages over gradient descent method in global optimization and convergence stability, especially in dealing with complex problems such as nonlinearity and multi-peaks.

4. Practical Challenges and Solutions of Mathematical Optimization Method in Structural Mechanics Design

When using mathematical optimization method in structural mechanics design, there are some practical challenges, including computational efficiency, because a large number of calculations needed to deal with complex

structures may prolong the design iteration cycle and slow down the project progress; In addition, there are convergence problems, and some optimization algorithms may show slow convergence or no convergence at all, thus affecting the quality of the solution; Furthermore, the local optimal solution is easy to appear in the optimization process, which makes it difficult for the algorithm to break through and find the global optimal solution. This problem is particularly critical, because the final design performance may be affected.

In order to meet the optimization challenge in structural mechanics design, many strategies can be adopted, including improving existing algorithms, such as adjusting parameters or adopting more advanced algorithm variants to enhance convergence and global search ability; It is also critical to introduce parallel computing technology, which can greatly improve the computational efficiency by decomposing the optimization problem and executing it on multiple processors, which is especially important for large-scale structural optimization; In addition, the combination of different optimization methods can also give full play to their respective advantages, such as using GA for global exploration first, and then using gradient descent method to achieve local fine tuning, so as to locate the global optimal solution more efficiently and promote rapid convergence.

5. Conclusion

The application and practice of mathematical optimization methods in structural mechanics design show that these methods can effectively improve the design efficiency and quality. By transforming practical problems into solvable mathematical models, and applying different algorithms such as scatter-based optimization algorithm, gradient-based optimization algorithm and heuristic optimization algorithm to find the optimal solution, engineers can meet the constraints of structural strength, stiffness and stability, and at the same time realize the minimization of structural weight. In particular, heuristic algorithms such as GA show strong global optimization ability and adaptability when dealing with complex optimization problems such as nonlinearity and multi-peaks. In addition, although there are challenges such as computational efficiency, convergence and local optimal solution in practical application, these challenges can be effectively met by improving existing algorithms, introducing parallel computing technology and combining different optimization methods, thus ensuring the superiority and feasibility of the design scheme. In a word, mathematical

optimization method provides an efficient and accurate solution for modern structural mechanics design, which is helpful to promote the progress and development of engineering field.

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