

Three-dimensional Vortex-induced Vibration Nonlinear Dynamics of A Flexible Viscoelastic Fluid-conveying Riser

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Abstract: In this paper, the geometric nonlinearity and hydrodynamic characteristics of a long flexible viscoelastic riser under the interaction of internal and cross-flow velocity are presented. The system governing equation is obtained by using the extended Hamilton principle. The drag and lifting coefficients of coupled and distributed van der Pol wake oscillators are established to represent hydrodynamic of ocean currents. The effects of internal flow velocity, cross-flow velocity and viscoelastic coefficient on the stresses, displacements, natural frequency and mode variation of the pipeline are theoretically investigated. The results show that the cross-flow velocity, internal velocity and viscoelastic coefficient have important impacts on the vibration amplitude and critical internal fluid velocity of the riser. Additionally, the cross-flow velocity, internal fluid velocity and viscoelastic coefficient have vital effects on the riser mode order. Therefore, choosing the appropriate viscoelastic coefficient can reduce the vibration amplitudes of the riser. These findings have important reference value for the design and control of ocean riser.

Keywords: Viscoelastic fluid-conveying pipe; Fluid-structure interaction; Vortex-induced vibration; Nonlinear dynamic.

1. Introduction

Marine riser is a critical component connecting offshore floating platform and subsea oil and gas production system [1]. Fluid-Structure Interaction (FSI) refers to the complex interaction between structural forces and the forces exerted by external and internal fluid flow. This interaction has significant implications and can potentially lead to fatigue failure. Among them, the fatigue failure behavior of marine risers during drilling operations due to the vortex shedding frequency close to the natural frequency is the most common and unsafe. Moreover, the actual riser for steel material, the increasing weight and length of the marine risers cannot meet the requirements of deep-water also exacerbate pipeline caused by vortex induced vibration. Therefore, many researchers are working on the design of ideal systems to control the vibration of the riser and extend the service life of structures, such as considering the optimization of viscoelastic material.

At first, a lot of research has been done on the characteristics of long flexible risers made of common materials under VIV (vortex-induced vibrations) [2]. During the VIV experiment on a long flexible circular cylinder, it was first discovered the two branches of the hysteresis loop in the steady state amplitude are accompanied by a vortex shedding mode by Brika and Laneville [4]. The researchers then applied external forces to control the vortex-induced vibration. He et al. [5] presented a theoretical study of a slender and elastic pipe conveying fluid with a top-end excitation under the action of uniform cross flows. They indicated that an effective way of suppressing vortex-induced vibration is to balance the top excitation and the internal fluid velocity. Then taking into account the difference in phase of the support at the two ends of the pipe [6]. The dynamic response of vortex-induced vibrations of a fluid-conveying riser subjected to the axial harmonic tension was studied by Zhang et al. [7] They revealed the cross-flow velocities and harmonic tension

amplitudes have significant effects on the dominated resonance region.

To improve the accuracy of predicting experimental results of flexible pipe, Xie et al. [8] had made outstanding contributions in this respect. A classic van der Pol wake oscillator was employed to simulate the lift force induced by vortex shedding to detect the external vibration. However, the practical problems are complicated and changeable, so the VIV of the three-dimensional flexible fluid-conveying riser are investigated. With the aim of studying the VIV of flexible fluid-conveying pipes, Wang et al. [9] constructed a new model that is three-dimensional nonlinear. They observed that the joint impacts of the inward induced buckling and transverse induced resonance on the first mode of longitudinal vibration amplitude and transverse vibration amplitude of the pipeline in-plane may be significant. According to this model, Yang et al. [10] found that the internal flow speeds have prominent effects on the riser maximum stresses and displacements, and the critical internal fluid speeds will lead to the growth in mode order at various cross-flow speeds.

However, these pipes are not involved in the analysis of geometric and hydrodynamic nonlinear three-dimensional models of viscoelastic pipe under the interaction of internal and external flow. Under the action of alternating loads, viscoelastic materials produce viscous effects due to different phases of stress and strain to attenuate structural vibration of risers [11]. Taking advantage of this characteristic, viscoelastic pipes are widely used in industrial fields and so on, and have also received more and more attention from researchers [12].

At the same time, based on the characteristics of viscoelasticity, the interaction of internal and external flow on VIV of the pipes were also investigated. Ozhan and Pakdemirli [14] considered linear models of viscoelastic fluid-conveying pipes and viscoelastic beams in axial motion and discussed the effects of viscoelasticity on the natural frequency of different pipes and beams. They concluded

viscoelasticity can reduce the natural frequency of the pipes. Owoseni et al. [16] researched the nonlinear dynamics and the stability of slightly curved viscoelastic fluid-conveying pipes on two types of foundation, but lack of viscoelastic materials on the response of risers. Therefore, Yang et al. [17] researched the nonlinear vibrations of fluid-conveying pipes on simple supports via the method of multiple scales. Numerical results are presented to show the steady-state response of the pipe on the amplitude-frequency plane. Yang et al. [18] showed that viscoelastic coefficients have notable effects on the stresses, displacements and natural frequency of a long flexible pipe in three-dimensional nonlinear dynamics. Experiments and numerical simulations have also been carried out to study the influence of internal fluid on the vibrational response of risers [19]. For deep-water riser, a coupling vibration model was established, and they demonstrated that the complex environment and internal fluid will greatly alter the riser vibration characteristics [22]. However, there is a lack of research in the literature on how the vibration response of pipelines in completely coupled cross flow and internal flow is affected by the interaction between material viscoelasticity and internal flow.

Nowadays, Yazicioglu et al. [23] demonstrated that viscoelastic materials can reduce the difference of vibration response between empirical models and linear models. Shahali et al. [24] studied the nonlinear dynamics of viscoelastic fluid-conveying pipes subjected to the uniform cross flow. They found the suitable viscoelastic coefficients to alleviate the riser dynamic response. Furthermore, they showed how the pipe vibration amplitude changes with cross flow reduced velocity, internal flow velocity and viscoelastic damping. However, there have been relatively few studies on the vibration of slender cylinder under the joint impacts of

external and internal flow by employing the finite element method [25].

In this paper, we consider the response of flexible viscoelastic risers to three-dimensional vortex-induced vibration subjected to internal and external flow by finite element methods. Therefore, the content is conceived as follows: In the second part, considering the Hamilton's principle, the Euler-Bernoulli beam prediction model is applied to establish the three-dimensional vibration of a flexible viscoelastic riser for various internal fluid velocity. Hydrodynamics and lift-drag coefficients are introduced in the third part. Next, through the numerical verification and experimental comparison of the model, the influences of viscoelastic coefficient and internal fluid speed on the riser natural frequency and dynamic response under different cross fluid speed were studied. The last section details the conclusions.

2. Mathematical Vibration Model of A Flexible Viscoelastic Riser

A simply-supported marine riser at both ends is placed in a constant cross flow, $U(z)$, oriented along the z direction. Due to the action of effective gravity, the cylinder is completely upright in vertical static equilibrium, as shown in Fig. 1(a). According to previous experience, the extended Hamilton principle was utilized to establish a model of the pipeline, and the Kelvin-Voigt model was considered in the pipeline. Finally, according to the Euler-Bernoulli beam theory, a flexible 3-D viscoelastic marine riser under internal fluid was established.

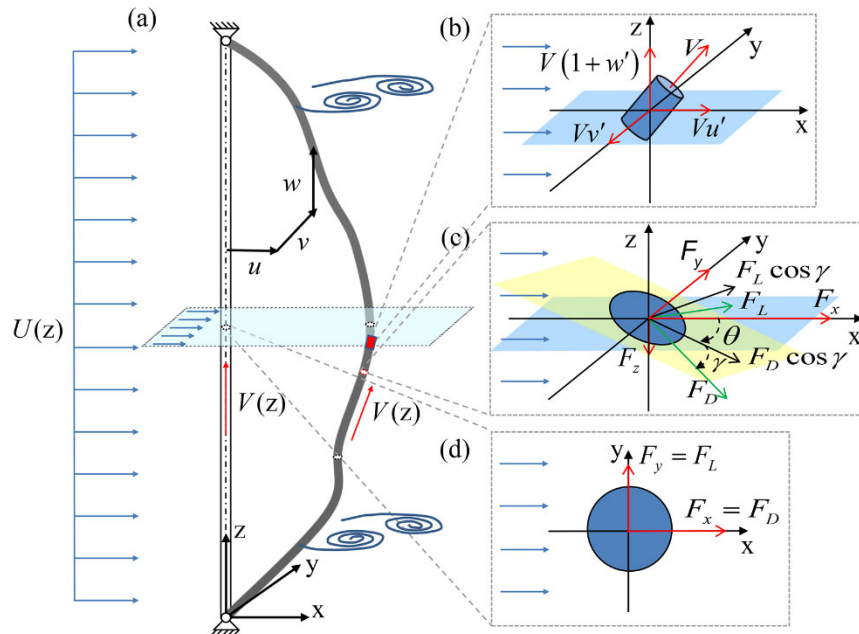


Figure 1. (a) A schematic three-dimensional model of flexible cylinder subjected to VIV, (b) speed of internal fluid relative to the riser, (c) sectional hydrodynamic force for three-dimensional oscillating cylinder, and (d) sectional hydrodynamic force for stationary cylinder [10].

Therefore, considering the Kelvin-Voigt viscoelasticity, the stress-strain relationship in the marine riser is represented as:

$$\sigma^* = \left(1 + \frac{E^*}{E} \frac{\partial}{\partial t}\right) E \varepsilon^* \quad (1)$$

in which, σ^* represents the stress of the pipeline, ε^*

represents the strain of the pipeline, the internal dissipation coefficient can be expressed by E^* , and t represents the time. $\frac{E^*}{E}$ can be regarded as a entirety, replaced by the value

\mathcal{A} , then the final expression can be obtained as follows:

$$\sigma^* = (1 + a \frac{\partial}{\partial t}) E \varepsilon^* \quad (2)$$

Take any point $(0, 0, z)$ on the vertical flexible tube, and its displacement vector is expressed as follows:

$$\vec{r} = u\vec{i} + v\vec{j} + (z + w)\vec{k} \quad (3)$$

In cartesian coordinates, the three orthogonal basis vectors are represented separately.

The velocity of the riser element in x, y and z directions is expressed as a function of the unit vector, as shown below:

$$\vec{V}_p = \frac{\partial \vec{r}}{\partial t} = \frac{\partial u}{\partial t} \vec{i} + \frac{\partial v}{\partial t} \vec{j} + \frac{\partial w}{\partial t} \vec{k} \quad (4)$$

As a result of the riser deformation, the fluid relative speed vector at the centre of the riser segment can be expressed by the following formula:

$$\vec{V}_r = V \left[u'\vec{i} + v'\vec{j} + (1 + w')\vec{k} \right] \quad (5)$$

in which, \vec{V}_r represents internal fluid speed relative to the riser, V represents internal fluid speed.

In the Cartesian coordinate system, considering the movement of the riser and combining (4) and (5), the absolute internal flow speed can be represented as the following formula:

$$\vec{V}_f = \vec{V}_p + \vec{V}_r = (\dot{u} + V u')\vec{i} + (\dot{v} + V v')\vec{j} + [\dot{w} + V(1 + w')]\vec{k} \quad (6)$$

where, \vec{V}_r represents the relative speed vector, \vec{V}_p represents the speed of the riser component, and \vec{V}_f represents the absolute internal flow speed.

Meanwhile, ignoring the effects of secondary flow and rotary inertia, the variation result of kinetic energy of the internal fluid at constant internal flow velocity is expressed by the following formula:

$$\begin{aligned} \int_{t_1}^{t_2} \delta T_f dt &= \frac{1}{2} m_i \int_{t_1}^{t_2} \int_0^L \delta \left\{ (\dot{u} + V u')^2 + (\dot{v} + V v')^2 + [\dot{w} + V(1 + w')]^2 \right\} dz dt \\ &= - \int_{t_1}^{t_2} \int_0^L m_i (\ddot{u} + 2V \dot{u}' + V^2 u'') \delta u dz dt - \int_{t_1}^{t_2} \int_0^L m_i (\ddot{v} + 2V \dot{v}' + V^2 v'') \delta v dz dt \\ &\quad - \int_{t_1}^{t_2} \int_0^L m_i (\ddot{w} + 2V \dot{w}' + V^2 w'') \delta w dz dt \end{aligned} \quad (7)$$

The fluid mass per unit length is represented by m_i which is positively correlated with the internal fluid density (ρ_i

represents the internal fluid density and $m_i = \rho_i \pi d^2 / 4$), the prime number represents the differential related to time t .

Without taking into account the internal fluid, the studies of Ricciardi and Saitta [29] and Srinil et al. [30] describe the variation results of a long cylindrical flexible riser under the action of stable and uniform cross flow. According to Zanganeh and Srinil [28], taking into the account internal dissipation, the equation describing the 3-D coupled motion of a viscoelastic pipeline in CF, IL and AX, is a nonlinear partial differential equation and can be represented as:

$$\begin{aligned} (m + m_i + m_a) \ddot{u} + c \dot{u} + E l u^{(IV)} + 2 m_i V \dot{u}' + m_i V^2 u'' + a E l \dot{u}^{(IV)} - (T u)' \\ = (1 + a \frac{\partial}{\partial t}) E A_r (w'' u' + w' u'') + \end{aligned} \quad (8)$$

$$\frac{1}{2} (1 + a \frac{\partial}{\partial t}) E A_r (3 u'' u'^2 + u'' w'^2 + 2 w'' w' u' + u'' v'^2 + 2 v'' v' u') + F_x$$

$$\begin{aligned} (m + m_i + m_a) \ddot{v} + c \dot{v} + E l v^{(IV)} + 2 m_i V \dot{v}' + m_i V^2 v'' + a E l \dot{v}^{(IV)} - (T v)' \\ = (1 + a \frac{\partial}{\partial t}) E A_r (v' w'' + v'' w') + \end{aligned} \quad (9)$$

$$\frac{1}{2} (1 + a \frac{\partial}{\partial t}) E A_r (v'' u'^2 + 2 u'' u' v' + u'' w'^2 + v'' w'^2 + 2 w'' w' v' + 3 v'' v'^2) + F_y$$

$$\begin{aligned} (m + m_i + m_a) \ddot{w} + c \dot{w} + E l w^{(IV)} + 2 m_i V \dot{w}' + m_i V^2 w'' + a E l \dot{w}^{(IV)} - (T w)' \\ = (1 + a \frac{\partial}{\partial t}) E A_r w'' + 2 (1 + a \frac{\partial}{\partial t}) E A_r w' w' + (1 + a \frac{\partial}{\partial t}) E A_r (u'' u' + v'' v' + w'' w') \end{aligned} \quad (10)$$

$$+ \frac{1}{2} (1 + a \frac{\partial}{\partial t}) E A_r (w'' u'^2 + 2 u'' u' w' + 3 w'' w'^2 + w'' v'^2 + 2 v'' v' w') + F_z$$

where, the displacements of IL, CF and AX are represented by u , v and w , separately. The relevant hydrodynamic forces along the three directions are represented by F_x , F_y and F_z respectively. The riser mass per length is represented by m , m_i represents the fluid mass per unit length, and m_a represents the additional fluid mass per unit length (cross flow). The inner diameter (d), outer diameter (D), bending stiffness ($E I$), moment of inertia (I), young's modulus (E), damping coefficient (c), axial stiffness ($E A_r$) and cross-sectional area (A_r) of the riser are all constant.

Taking into account the buoyancy impact [31], the static effective tension T in the absence of the influence of internal fluid can be varied in space, and the tension of the pipeline can be obtained as:

$$T = T_t - g(m + m_i - m_a)(L - z) \quad (11)$$

in which, the maximum preload is represented by T_t and the gravity is represented by g . For vertical or horizontal cylinders with neutral buoyancy, T represents a constant tension, and $T = T_t$ [32].

The boundary conditions for the fluid-conveying viscoelastic riser can be derived by the boundary conditions of simply-supported risers, as shown below:

$$u(0, t) = 0, v(0, t) = 0, w(0, t) = 0 \quad (12)$$

$$u''(0, t) = 0, v''(0, t) = 0, w''(0, t) = 0 \quad (13)$$

$$u(L,t) = 0, v(L,t) = 0, w(L,t) = 0 \quad (14)$$

$$u''(L,t) = 0, v''(L,t) = 0, w''(L,t) = 0 \quad (15)$$

3. Hydrodynamic Forces

Taking into account the riser's unique arrangement as shown in Fig.1(d) and combined with the riser's own movement and the relative velocity of the cross-flow, the lift force direction is defined as CF, and the drag force direction is defined as IL. Assuming that the riser segment is subjected to lift force and drag force respectively, on the x-y plane, θ is the horizontal angle clockwise, and on the x-z plane, γ is the horizontal angle clockwise.

$$F_x = F_L \cos \gamma \sin \theta + F_D \cos \gamma \cos \theta \quad (16)$$

$$F_y = F_L \cos \gamma \cos \theta - F_D \cos \gamma \sin \theta \quad (17)$$

$$F_z = F_L \sin \gamma + F_D \sin \gamma \quad (18)$$

in which lift and drag are expressed as:

$$F_D = 1/2 \rho D U_{rel}^2 (\bar{C}_d + C_D) \quad (19)$$

$$F_L = 1/2 \rho D U_{rel}^2 C_L \quad (20)$$

where \bar{C}_d represents the average drag coefficient, and it is usually presumed to be 1.2. Time-varying drag coefficient is represented as C_D and time-varying lift coefficient is represented as C_L . The overall speed with respect to the riser is calculated from the equation below:

$$U_{rel} = U \sqrt{\left(1 - \frac{\dot{u}}{U}\right)^2 + \left(\frac{\dot{v}}{U}\right)^2 + \left(\frac{\dot{w}}{U}\right)^2} \quad (21)$$

By incorporating wake variables $p = 2C_D / C_{d0}$ [31] (assuming $C_{d0} = 0.2$) and $q = 2C_L / C_{l0}$ [33] (assuming $C_{l0} = 0.2$), the changes of p and q are expressed in the following way:

$$\ddot{p} + 2\varepsilon_u \Omega_f (p^2 - 1) \dot{p} + 4\Omega_f^2 p = \frac{\Lambda_u}{D} \ddot{u} \quad (22)$$

$$\ddot{q} + \varepsilon_v \Omega_f (q^2 - 1) \dot{q} + \Omega_f^2 q = \frac{\Lambda_v}{D} \ddot{v} \quad (23)$$

In this paper, vortex shedding frequency is $\Omega_f = 2\pi S_t U / D$, and the excitation terms, which simulate

the influence of pipeline movement on the nearby wake, are on the right side of the equation. Λ_u and Λ_v represent the coupling empirical coefficients, ε_u represents wake empirical coefficients, usually $\Lambda_u = \Lambda_v = 12$, $\varepsilon_u = 0.3$, which are calibrated by Yang et al [18].

Finally, substituting equations (19-21) and (22-23) into equations (16-18), the hydrodynamic forces in three directions can be obtained as follows [28]:

$$F_x = \frac{1}{4} \rho D U_{rel} C_{l0} q \dot{v} + \frac{1}{4} \rho D U_{rel} C_{d0} p (V - \dot{u}) + \frac{1}{2} \rho D U_{rel} \bar{C}_d (V - \dot{u}) \quad (24)$$

$$F_y = \frac{1}{4} \rho D U_{rel} C_{l0} q (V - \dot{u}) - \frac{1}{4} \rho D U_{rel} C_{d0} p \dot{v} - \frac{1}{2} \rho D U_{rel} \bar{C}_d \dot{v} \quad (25)$$

$$F_z = \frac{1}{4} \rho D U_{rel} C_{l0} q v \dot{w} + \frac{1}{4} \rho D U_{rel} C_{d0} p \dot{w} + \frac{1}{2} \rho D U_{rel} \bar{C}_d \dot{w} \quad (26)$$

where C_{d0} and C_{l0} are the relative lift and drag coefficients of a stationary cylinder.

4. Numerical Studies and Discussion

The numerical simulation can be performed using the long flexible straight riser model established by Song et al. [32] and Yang et al. [18] in this section, and Table 1 lists the system parameters that are relevant for the numerical simulations. Highly nonlinear partial differential equations (8)-(10) together with (22)-(23) were solved by COMSOL. The static equilibrium position of the riser is specified by the initial conditions and satisfies $u = v = w = \dot{u} = \dot{v} = \dot{w} = 0$, the wake change satisfies $p = 2$ ($\dot{p} = 0$) and $q = 2$ ($\dot{q} = 0$). The flexible riser was discretized by Lagrange quadratic finite element method, $\Delta t = 0.001$ s and $\Delta z = 0.1$ m were chosen as discrete variables by means of a set of steady simulations of viscoelastic coefficient and flow velocity. In this study, Yang et al. [18] carried out numerical simulation verification with finite difference method and experimental model, and the effect of internal fluid speed and viscoelastic coefficient on the natural frequency and nonlinear dynamic response of the riser were studied for different cross-flow velocities.

4.1. Model validation

Firstly, the frequency comparisons are performed to verify the numerical model. The frequency of the riser under ocean currents can be described as follows:

$$f = \frac{\lambda_i^2}{2\pi L^2} \sqrt{1 + \frac{T_i L^2}{(i\pi)^2 EI}} \sqrt{\frac{EI}{m + C_A m}} (\lambda_i = i\pi) \quad (27)$$

The calculation results of the first 9 order natural frequency of the riser placed in water under the action of top tension are listed in Table 2. The results obtained by equation (27), finite element method and experiment are as follows:

Table 1. Physical parameters for the numerical analyses.

Quantities	Parameters	Value	Units
Length	L	28.04	m
Outer diameter	D	0.016	m
Inner diameter	d	0.015	m
Aspect ratio	L/D	1750	1
Cross-flow density	ρ_o	1000	kg/m ³
Internal fluid density	ρ_i	870	kg/m ³
Pipe mass per unit length	m	0.20106	kg/m
Additional fluid mass per unit length	m_a	0.20106	kg/m
Internal fluid mass per unit length	m_i	0.12064	kg/m
Cross-flow velocity	U	0.1-0.3	m/s
Internal fluid velocity	V	0-60	m/s
Bending stiffness	EI	153.71	N m ²
Axial stiffness	EA_t	5.11e6	N
Top tension	T_t	700	N
Viscoelastic coefficient	α	0-0.11	1

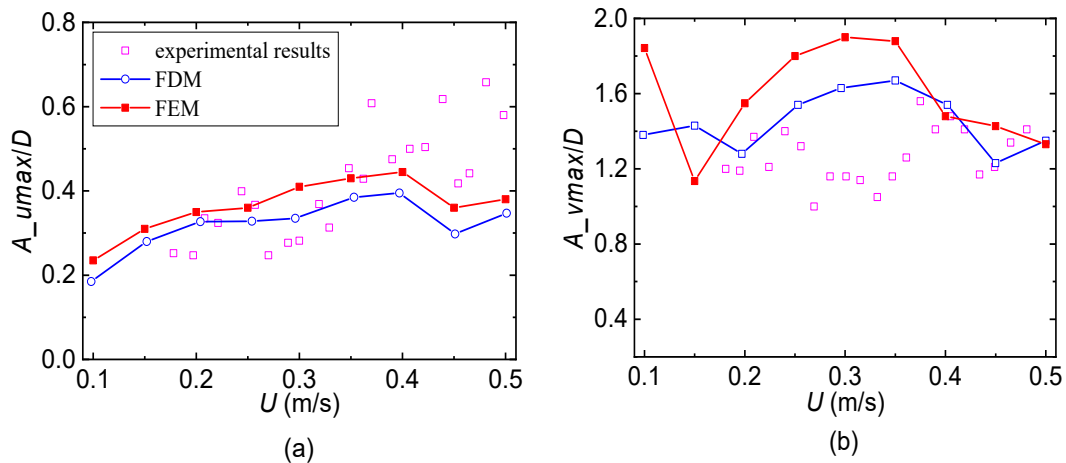
Table 2. Model order and frequency of top tension risers in water

Model order	1	2	3	4	5	6	7	8	9
Theoretical results f (Hz)	0.754	1.496	2.259	3.041	3.846	4.680	5.548	6.456	7.401
FEM f (Hz)	0.751	1.496	2.260	3.041	3.846	4.68	5.548	6.456	7.406

As can be seen in Table 2, the first 9 order frequency between theoretical results and finite element results agree very well. The riser natural frequency increases with increasing modal order.

In addition, comparison of the numerical simulation results and experimental results also validates the model. Fig. 2 depicts the corresponding maximum displacement amplitude

of flexible riser based on finite difference method and finite element calculation at various cross-flow velocities. The findings show that the experimental displacement amplitude coincides with the numerical simulation displacement amplitude when the cross-flow speed is within the prescribed range of 0.3 m/s. Therefore, the numerical result can predict VIV in two directions effectively.

**Figure 2.** Comparison of experimental results and numerical prediction with maximum displacement amplitudes of (a) IL and (b) CF VIV [10].

4.2. Natural frequency

According to the study of Yang et al. [10], the low order natural frequency of risers is little affected by different

viscoelastic coefficients as the internal flow velocity rises, and almost coincide. Therefore, Fig. 3 displays the variation of the flexible riser natural frequency at mode 2(a), mode 5(b), mode 6(c) and mode 9(d) as the internal fluid speed grows

while considering various viscoelastic coefficients. When the riser mode is higher than the 5th order and the critical speed does not reached, the riser natural frequency will reduce as the internal fluid speed and viscoelastic coefficient increase, while the riser vibration will decrease. However, the riser natural frequency will decrease more slowly as the viscoelastic coefficient increases when the internal fluid speed reaches the critical speed.

It also shows that the pipe natural frequency gradually

reduces as the internal fluid velocity grows. In addition, the riser natural frequency with various viscoelastic coefficients will deviate as the internal fluid speed develops for higher-order modes, in particular when the internal fluid speed is between 20m/s and 40m/s, the viscoelastic coefficient will grow as the riser natural frequency grows. In addition, the critical internal speed grows as the viscoelastic coefficients grow for the same high mode order.

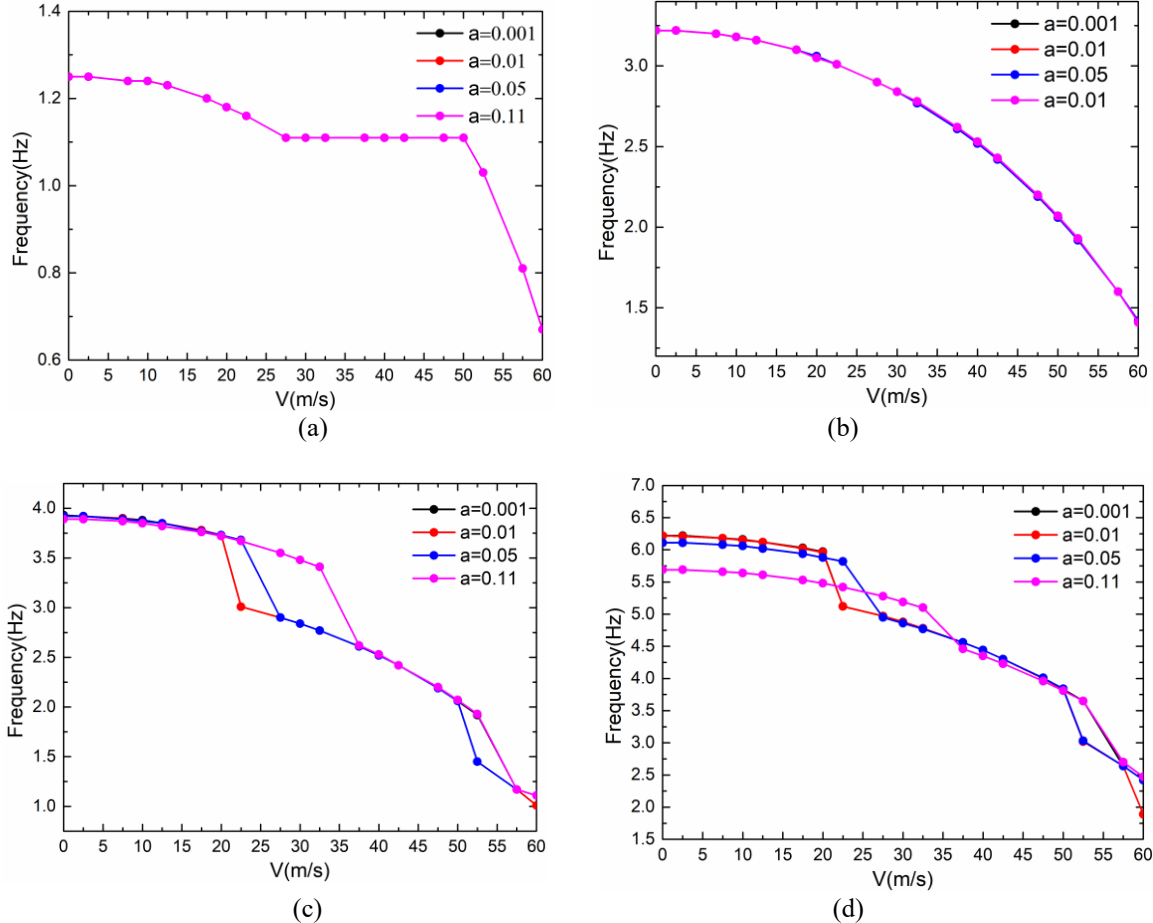


Figure 3. Natural frequency of a flexible pipe at mode 2 (a), mode 5 (b), mode 6 (c) and mode 9 (d) as the internal fluid velocity increases for various viscoelastic coefficients.

4.3. Three-dimensional VIV responses

The effect of viscoelastic coefficients on the displacement of a flexible fluid-conveying pipe under various cross-flow speed (0.1 m/s, 0.2 m/s, 0.3 m/s) was investigated as internal flow speed grows for inline(IL) and cross flow(CF) directions and are depicted in Fig. 4. According to Fig.4(a), the riser IL displacement grows with the growth of V , but reduces as the viscoelastic coefficient increases. However, Fig.4(b)-(f) show that the displacement amplitude decreases as the viscoelastic coefficient of the riser increases at the critical point. In addition, when the maximum displacement amplitude appears in CF direction, the displacement increases after a rapid decline. In addition, various viscoelastic coefficient and cross-flow velocity correspond to different critical velocity. The corresponding critical internal fluid speed is 45m/s both in IL and CF directions when the cross-

flow speed with various viscoelastic coefficients is 0.1 m/s. When cross-flow velocity reaches at 0.2m/s, the critical internal fluid velocity of the riser is 25m/s for $a = 0.001$, $a = 0.01$ and $a = 0.05$, but the critical internal flow velocity is 30m/s for $a = 0.11$. When cross-flow velocity is 0.3m/s, $a = 0.001$ and $a = 0.01$ corresponding to critical velocity is 30m/s, and $a = 0.05$ and $a = 0.11$ correspond to 35m/s. Most distinctively, when the cross-flow velocity is 0.2m/s and 0.3m/s, the displacement amplitude corresponding to the low viscoelastic coefficient is higher than the high viscoelastic coefficient, because at the high cross-flow velocity, the natural frequency reduces slowly as the internal flow speed grows. At the same time, the viscoelastic coefficient weakens the displacement amplitude in the same high mode order.

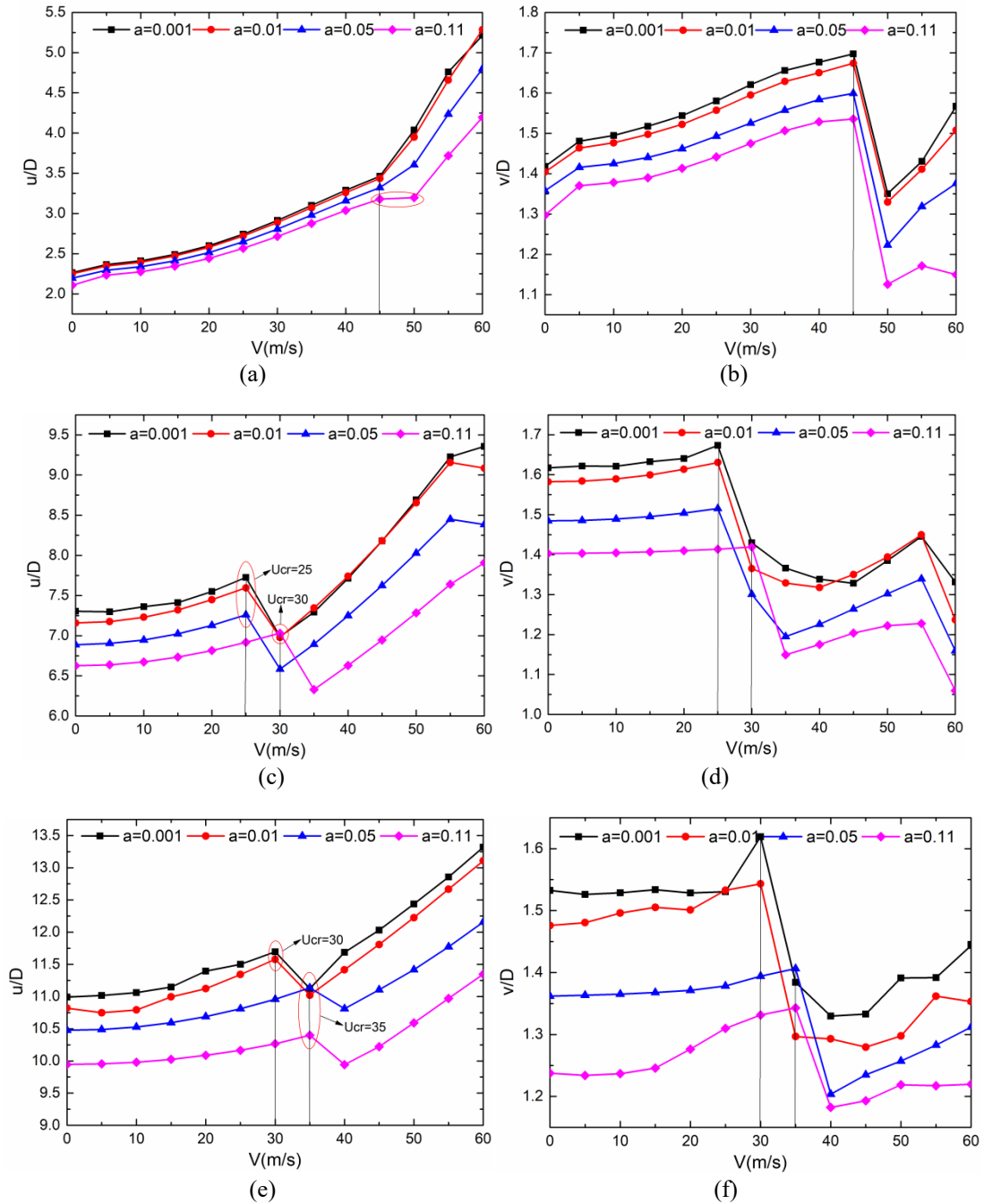


Figure 4. IL (a,c,e) and CF (b,d,f) displacements of the flexible riser at $U=0.1\text{m/s}$ (a,b), $U=0.2\text{m/s}$ (c,d) and $U=0.3\text{m/s}$ (e,f) with increasing internal flow velocity for various viscoelastic coefficients.

When the cross-flow velocity is 0.1m/s , 0.2m/s and 0.3m/s respectively, Fig. 5 shows the variation of bending stress amplitudes in IL and CF directions. And Fig. 5(a) and 5(b) show that the stress amplitudes decrease as the viscoelasticity grows and increase as the internal fluid speed grows. The variation of IL stress in Fig. 5(c) is complicated. Although the stress amplitude increases first, then reduces and grows again as the internal flow velocity grows. Meanwhile, the smaller the viscoelastic coefficient is, the more drastic the change of stress amplitude is. The Fig. 5(e) and 5(f) shows that there is

a negative correlation between the viscoelastic coefficient and the stress amplitude. However, as the internal fluid speed rises, the CF stress amplitude changes steadily before the critical point and then increases rapidly. The results indicate that the viscoelastic coefficient can simultaneously reduce the stress amplitudes and displacements of the fluid-conveying riser. In other words, the viscoelastic coefficients have an obvious restraining role in the vibration response of three-dimensional flexible riser.

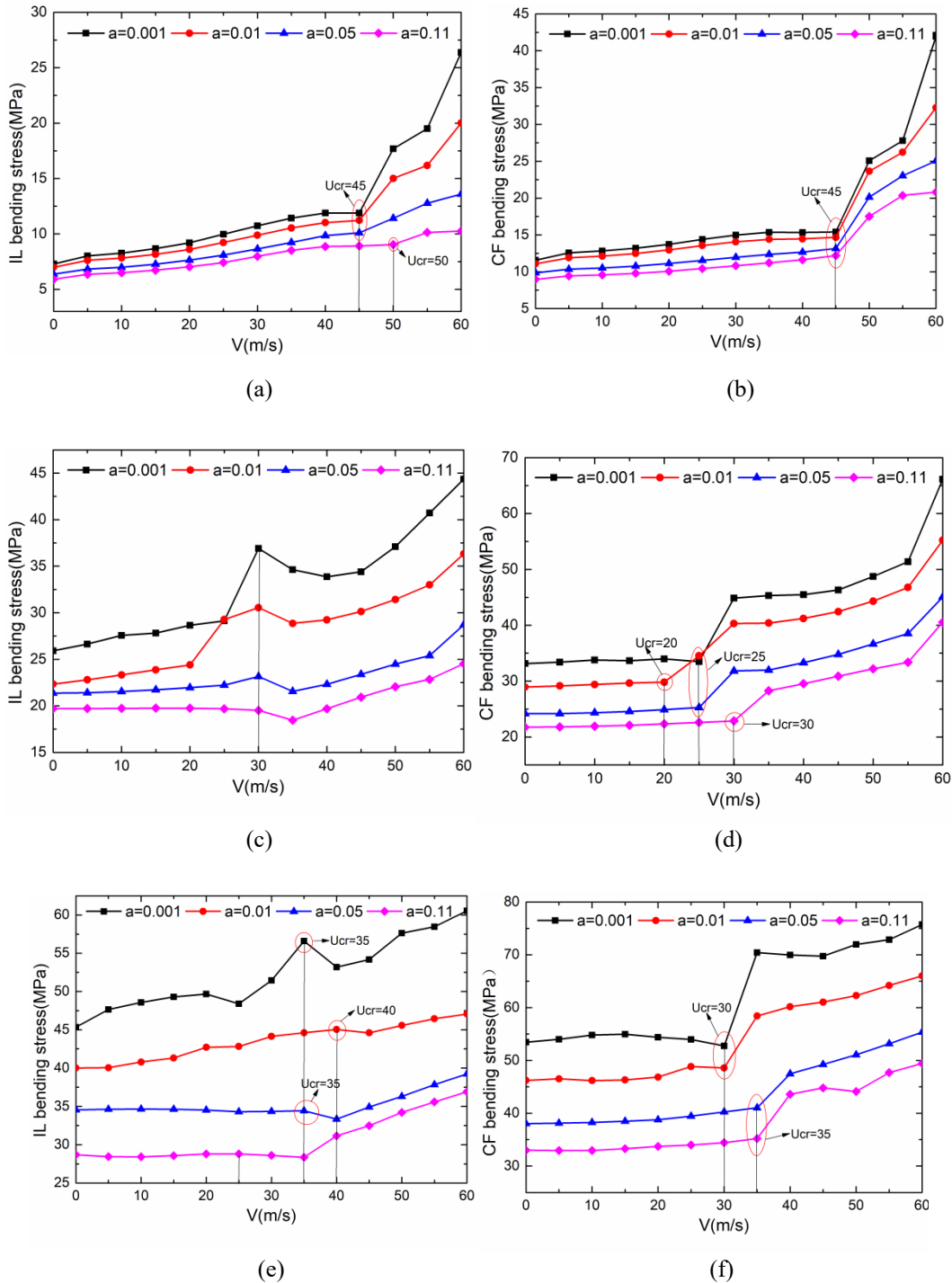


Figure 5. IL (a,c,e) and CF (b,d,f) stresses of flexible pipe at (a,b) $U=0.1$ m/s, (c,d) $U=0.2$ m/s and (e,f) $U=0.3$ m/s as internal flow velocity increases at various viscoelastic coefficients.

The aforementioned results show that the stress and displacement amplitude of risers with different viscoelastic coefficients will present "jump phenomena" around the critical internal flow speed. To further verifying the influence of internal flow speed and viscoelastic coefficient on the riser stress and displacement, the maximum cross-flow speed of 0.3m/s is taken as an example. The steady-state dynamic responses of the corresponding spatial time-varying IL(Fig.6(a)(c)(e)(g)) and CF(Fig.6(b)(d)(f)(h)) shifts when

$a = 0.001$ and $a = 0.11$ are shown in detail below. These responses were stripped of average values at $V = 30$ m/s, $V = 35$ m/s, and $V = 40$ m/s. Fig.6 shows that the overall riser responses corresponding to low viscoelastic coefficients and internal flow velocities (Fig. 6(a)(b)(e)(f)) are mainly controlled by standing wave, while the response of the riser (Fig. 6(c)(d)(g)(h)) is affected by both standing wave and traveling wave.

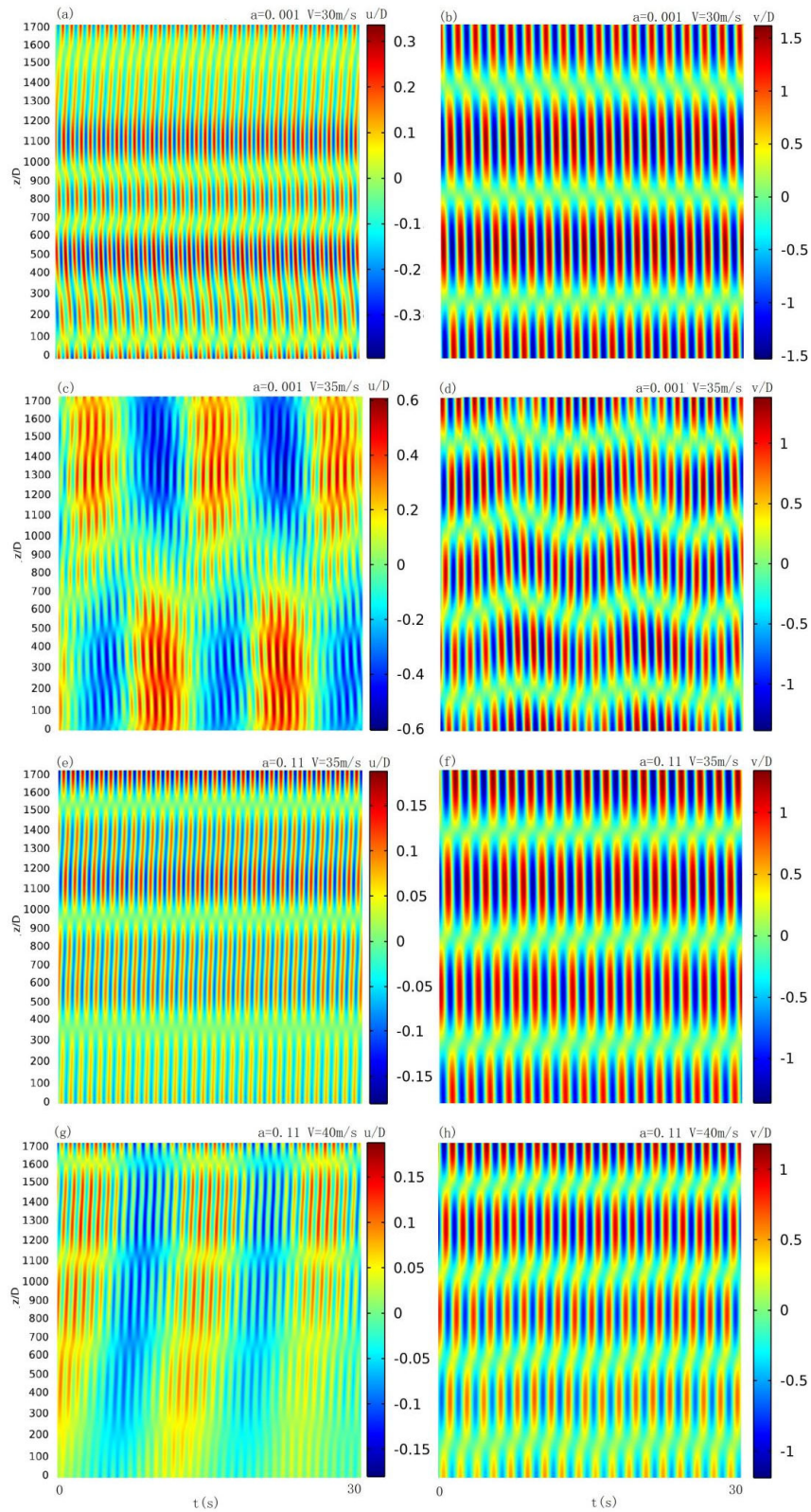


Figure 6. Space-time responses of IL (a, c, e, g) and CF (b, d, f, h) displacements with removed mean values at $V=30\text{m/s}$ (a, b) with $a=0.001$, $V=35\text{m/s}$ (c, d, e, f) with $a=0.11$ and $a=0.001$, and $V=40\text{m/s}$ (g, h) with $a=0.11$.

For IL displacement, the mode order response increases as the internal fluid velocity increases with higher viscoelastic coefficient. When $a = 0.001$ and $V = 30\text{ m/s}$, the IL displacement of the riser vibrates in 6 order mode. The riser IL mode varies from order 6 to 7 as the internal fluid speed grows. At this time, the riser CF displacement response presents complex characteristics of the superposition of 3 order mode and 4 order mode. The riser CF displacement

vibrates stably in the mode of 4. When the internal flow speed and the viscoelastic coefficient are at the maximum. It is clear that IL response mode is always higher than CF response mode, and IL vibration frequency is 1.5 times of CF vibration frequency, which is caused by the riser large vortex vibration frequency in the IL direction.

With the aim of analyzing the stress space-time responses, equation (28)-(29) can represent the stresses in IL and CF

directions, where the bending and compressive stresses in both directions are represented by the negative and positive signs, separately.

$$\sigma_u(z,t) \approx \pm \frac{D}{2} Eu''(z,t) \quad (28)$$

$$\sigma_v(z,t) \approx \pm \frac{D}{2} Ev''(z,t) \quad (29)$$

Based on the characteristics of the stress amplitudes depicted in figure 5, figure 7 shows the distribution of stress amplitudes at high cross flow velocity, the stress responses of the critical values ($V = 30$ m/s and $V = 30$ m/s) and the critical values ($V = 35$ m/s and $V = 40$ m/s) in the direction of IL and CF when the viscoelastic coefficient is 0.11. In general, the responses of risers in both directions are affected by the waveform characteristics, and the magnitude of the bending stresses is the same, but the amplitude of the CF is significantly higher.

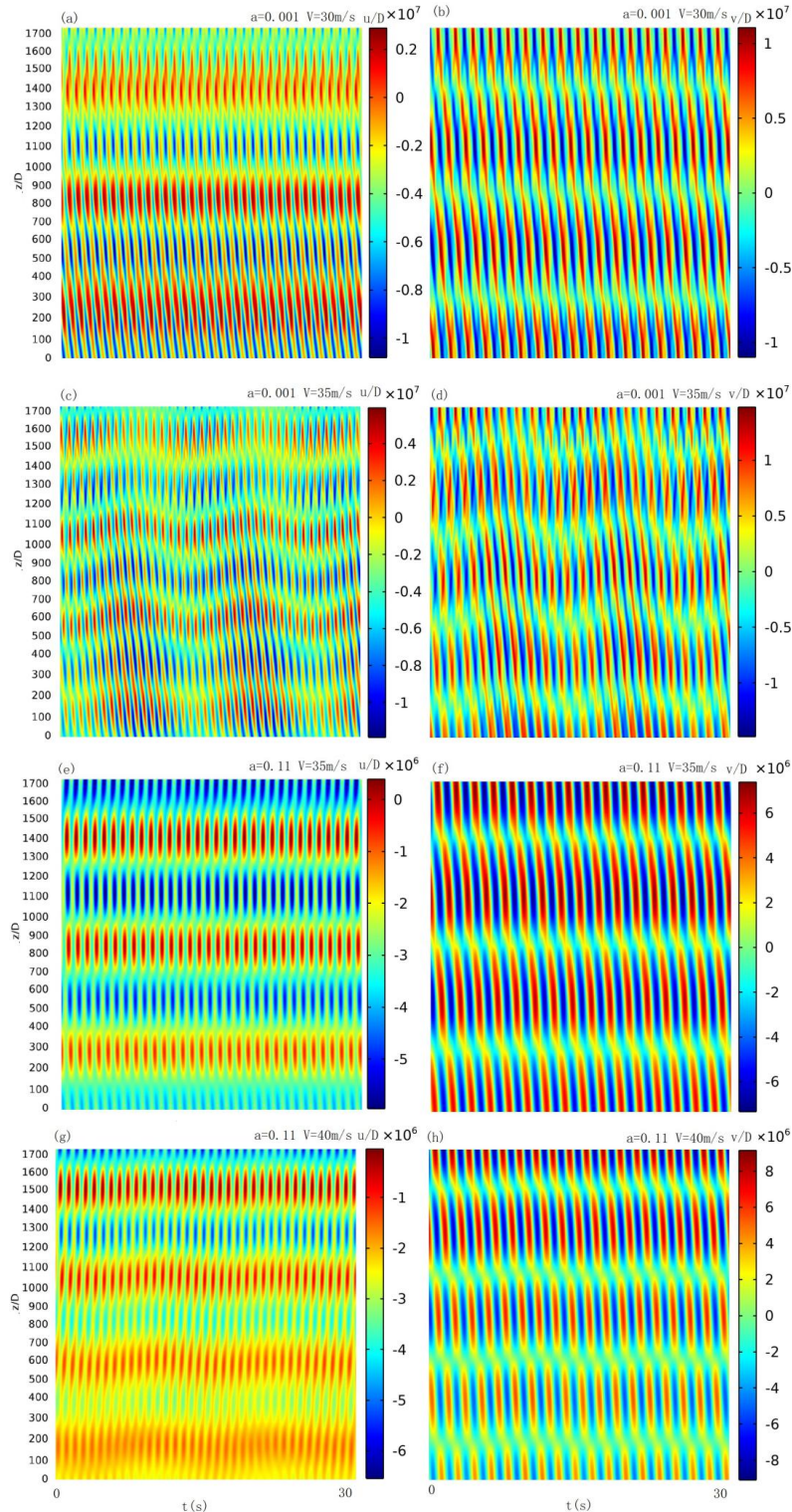


Figure 7. Space-time varying responses of IL (a, c, e, g) and CF (b, d, f, h) stresses (Pa) when $V=30$ m/s (a, b), $V=35$ m/s (c, d, e, f) and $V=40$ m/s (g, h).

As can be seen from Fig. 7 that the mode order will increase as the internal fluid speed and the viscoelastic coefficient grow for the stresses in both directions. The growth of viscoelastic coefficient will cause the stress in the CF direction to decrease. It can be interpreted as the impact of viscoelastic coefficient on the natural frequency of the flexible pipeline corresponds to higher modes. Therefore, the viscoelastic coefficient and the internal flow speed have notable impacts on the riser response.

5. Conclusions

In this paper, considering hydraulic and geometric nonlinearity, a three-dimensional vibration prediction model based on Euler-Bernoulli beam for a long flexible straight viscoelastic riser conveying fluid is established. The fluid-structure interaction (FSI) analysis of a flexible fluid-conveying riser for different viscoelastic coefficients is performed by utilizing finite element method (FEM). The viscoelastic coefficients on the nonlinearity dynamic and natural frequency of fluid-conveying riser for different CF velocities are elaborated and some significant conclusions are drawn.

As for the frequency analyses, in the same high mode order, when the internal speed does not reach the critical speed range, the natural frequency decreases obviously as the viscoelastic coefficient and the internal speed grow. At the same time, different viscoelastic coefficients will change the critical flow velocity in the riser. In other words, the critical fluid speed increases with increasing viscoelastic coefficient.

For the stress response and displacement response, the stress amplitude and displacement amplitude will "jump" at the position of critical fluid speed. When the internal fluid speed does not reach the critical point, the displacement and stress amplitudes reduce as the viscoelastic coefficient grows, but grow as the internal fluid speed grows. However, the various viscoelastic coefficient corresponding to critical point become complicated at high cross-flow speed of the riser displacement amplitude. It shows that the critical internal flow velocity corresponding to high viscoelastic coefficient is also higher. Simultaneously, the viscoelastic coefficient weakens the riser frequency in the same high mode order and leads to the displacement amplitude corresponding to the low viscoelastic coefficient is higher than the high viscoelastic coefficient. At the same time, the mode order grows as the internal fluid speed or viscoelastic coefficient grows. The results show that the amplitude can be suppressed effectively by proper internal cross flow velocity and viscoelastic coefficient.

Therefore, the response of a flexible pipe conveying fluid can be effectively suppressed by choosing appropriate viscoelastic coefficient and internal fluid velocity to avoid the fatigue failure caused by vibration. The establishment of the model and the good choice of parameters have certain guiding effect on the suppression of riser vibration.

6. Conflict of Interest

All the authors have no conflict of interest.

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8. Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

9. Nomenclature

x, y, z	Space coordinates, m
u, v, w	Displacement components, m
L	Length of pipe, m
D	Outer diameter, m
d	Inner diameter, m
ρ_o	Outer fluid density, kg/m ³
ρ_i	Internal fluid density, kg/m ³
m	Pipe mass per unit length, kg/m
m_a	Additional fluid mass per unit length, kg/m
m_i	Internal fluid mass per unit length, kg/m
U	Internal flow velocity, m/s
U_r	Reduced cross-flow velocity, m/s
V	Cross-flow velocity, m/s
E	Elasticity modulus, GPa
E^*	Coefficient of internal dissipation
c	Damping coefficient, N/s
I	Moment of inertia, m ⁴
A_r	Section area of pipe, m ²
T_t	Top pre-tension, N
T	Static effective tension, N
F_x	x-direction hydrodynamic force, N
F_y	y-direction hydrodynamic force, N
F_z	z-direction hydrodynamic force, N
σ^*	Stress, MPa
ε^*	Strain
a	Viscoelastic coefficient

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