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Finite-time Control Design of Flexible Risers with Input Saturation Constraints

Min Wan, Guangyang Zhou

School of Mechatronic Engineering, Southwest Petroleum University, Chengdu 610500, China

Abstract: To control the vibration of flexible riser systems affected by uncertain perturbation, this paper utilizes the transverse vibration control equations of marine flexible risers established based on Hamiton's principle and conducts a controlled study by combining the problems of input saturation and output limitation that exists in the practical engineering applications. A finite time control algorithm for flexible risers with input saturation constraints and output constraints is proposed. The barrier Lyapunov function is used to limit the end of the flexible riser to a given range, and the hyperbolic tangent function is introduced to deal with the saturation nonlinearity. The stability of the system is proved using the Lyapunov stability theory. Finally, the superiority of the control algorithm proposed in this paper is illustrated with simulation examples.

Keywords: Marine flexible risers; finite time control; output constraints; vibration control; input saturation constraints.

1. Introduction

Nowadays, people's demand for energy continues to grow, however, inland oil and gas field resources are increasingly depleted, the difficulty of exploration is gradually increasing, and the rational development of marine oil and gas resources has become a consensus of the world. Flexible riser is an important part of the marine oil and gas extraction and transportation system, due to the complexity of the marine environment, the marine flexible riser in the extraction process is very susceptible to the role of the disturbance load and thus mechanical vibration phenomenon. Moreover, flexible risers often need to bear the influence of self-weight, floating body movement, and ocean power environment factors for a long time during the operation process. (Ng et al., 2024), It will be subjected to relatively large tensile force and lateral cyclic reciprocating bending action. The fatigue failure of offshore flexible risers under this working environment will not only affect the efficiency and performance of the offshore oil and gas extraction system and reduce the production life but also may cause serious safety accidents. Therefore, it is necessary to research the vibration suppression of marine flexible risers. The control strategy of marine flexible risers mainly includes passive control and active control. Passive control is mainly achieved by improving the linear design of the riser, structural stiffness, internal fluid velocity, top tension, and structural damping, and improving the overall structure of the extraction and transportation system to achieve the purpose of vibration suppression. The research on passive control of flexible risers at home and abroad has achieved a lot of results. Literature (Feng et al., 2024) investigated the effect of top tension T and wave period on amplitude. Huang, C (Huang et al., 2024) investigated the mechanical behavior of DSRs under different buoyancy module configurations and different currents. Meenakumari, HNR(Meenakumari et al., 2024) studied the phenomenon of fluid-structure interaction in conveying flexible tubes. Experimental results of vortex-excited vibration of flexible risers filled with water and helium as typical liquids and light gases with different fluid media are presented in the literature (Zhu et al., 2024).Literature (Smith, 2008) demonstrates recent advances in the design of flexural reinforcement ribs. Yang, ZX; Wang, LF(Yang et al., 2022) Optimized design of flexible risers considering both local cross-section and global configuration parameters. Passive control requires no external energy in the control process and the structure is relatively simple and easy to implement, but the adaptive ability of passive control is relatively poor when there are changes in external disturbances. Active control, on the other hand, shows a good ability to deal with uncertainties, and at the same time can track and regulate the vibration condition well. Active control is mainly used to inhibit riser vibration by controlling the displacement of the top of the riser, and scholars at home and abroad have abundant results in the research of active control of flexible risers. Literature (Zhang et al., 2024) provides insights into the boundary control problem using the Dynamic Event Triggering Mechanism (DETM).Literature (Zhou et al., 2024) proposes an observer-based adaptive backpropagation boundary control method, the algorithm can make the controller adapt to the environment and improve the control accuracy by updating the control law online with real-time data. Zhang, BL; Sun, YT(Zhang and Sun et al., 2024) design finite-time discrete optimal backlash controllers with a feedforward compensation mechanism. Literature(Yu and Chen, 2019a) uses boundary control for active vibration control of flexible risers with variable distributed loads. Many research teams(Han et al., 2018; Seghour et al., 2016; Berkani et al., 2020; Seghour et al., 2018; Yu and Chen, 2019b; Song et al., 2021) have also modeled flexible oil risers considering the effects of time-varying internal flows and the design of related controllers based on Eulerian Bernoulli beam structures.

In actual engineering, due to the industrial production environment, the output drive device structure and system safety specifications and other internal and external factors, the output of the system is often strictly limited to the specified range or the output overshooting system must be within the specified range to ensure that the whole system is safe and reliable, the output constraints are difficult to avoid. If the output constraints are not taken into account in the controller design, the control performance will be impaired and even the system stability will be destroyed. Currently, the proposed solutions for the output constraint problem include the preset performance method(Song et al., 2023)、invariant set control, Barrier Lyapunov function (Maghenem and Sanfelice, 2019; Liu, 2022; Maghenem and Sanfelice, 2023; Kabzinski et al., 2016)、error transformation ,and model predictive control. Because of the systematic superiority of the Barrier Lyapunov Function (BLF, abbreviated as BLF) in the design process, the method of applying BLF to control the system to design it to satisfy the output constraints has been favored by researchers. Literature(Tee et al., 2009; Deng et al., 2017) shows that the barrier Lyapunov function ensures that the state constraints are in a given region.

Input saturation is prevalent in real systems, e.g., power transmission, and formation satellite attitude control(Zhang and Liang et al., 2024),and it should be considered when designing a control system to bind the system inputs within reasonable limits. Due to the limitations of the actuator's mechanical properties and manufacturing process, the input of the actuator is bounded. When the control input exceeds the limit, the actuator input will reach the upper limit, even if the control input continues to increase the input of the actuator will maintain the upper limit unchanged, i.e., entering the saturation state(Lin et al., 2012).Saturation of the actuator leads to the problem of not being able to achieve the required control input in the control objective, which has an impact on the stabilization effect. To address the input constraint problem, literature (He et al., 2016) considers an offshore riser model and proposes a boundary vibration control with input constraints. Unlike literature(He et al., 2016),literature (Ji and Liu, 2018) proposed a boundary vibration control under input constraints using a hyperbolic tangent function under a flexural satellite system, and literature(He et al., 2018) proposed a robust adaptive boundary vibration controller under input constraints for the Euler-Bernoulli beam system.

In previous control schemes for flexible risers, the system obtains asymptotic stability or consistent ultimately bounded stability, meaning that the error gradually converges to zero or a neighborhood of zero as time approaches infinity. (Fu et al., 2016) Finite-time stability is more concerned with the transient behavior of the system response than asymptotic or uniformly eventually bounded stability. (Zhao et al., 2015) Finite-time stability has more practical applications in the vibration control of marine flexible risers because of its fast convergence advantage.

In summary, a finite time control method is proposed in this paper for faster and more sensitive suppression of transverse vibration of flexible risers. An obstacle Lyapunov function is used to limit the top of the flexible riser to a given range to solve the time-varying constraint space problem. The input saturation constraint problem is handled using a hyperbolic tangent function and an adaptive term is added to compensate for the input saturation constraint error. Using Lyapunov stability theory, it is shown that all error signals satisfy globally consistent stability.

Comparing with the existing works, the contributions of this paper are summarized as follows.

1. Hyperbolic tangent functions and saturation functions are used to tackle the input constraint.

2. In order to eliminate the error due to input saturation and to cope with the uncertainty of the error parameter, an adaptive law is proposed.

3. Improved convergence of vibrations by introducing a finite-time stability criterion

The rest of the paper is organized into four sections. Section II presents the flexible riser system modeling and preparatory knowledge. In Section III, the main research results are provided. Section IV shows the simulation results of the flexible riser system and Section V draws the conclusions.

2. Problem Statement and Preliminaries

2.1. Flexible riser modeling

A typical marine flexible riser system is shown in Figure2-1. To establish the flexible riser model firstly, based on the kinetic analysis to obtain the kinetic energy of the flexible riser system E_k、the potential energy E_p、and the total virtual work done by the non-conservative force on the marine flexible riser W_0 ;furthermore, based on Hamilton's principle($\int_{t_1}^{t_2} \delta(E_k - E_p + W_0)\, dt = 0$)to obtain the control equations of the said flexible riser system as:

$$\begin{cases} \rho\ddot{y}(x,t) = -EIy''''(x,t) + Ty''(x,t) + f(x,t) - c\dot{y}(x,t), \\ y''(L,t) = y'(0,t) = y(0,t) = 0, \\ \dot{x}_1(t) = x_2(t) = \dot{y}(L,t), \\ \dot{x}_2(t) = \dfrac{1}{M_s}[EIy'''(L,t) - d_s x_2(t) - Ty'(L,t)] + \dfrac{1}{M_s}u(t) + \dfrac{1}{M_s}d(t) \end{cases}$$

The boundary conditions stated in Eq:

$$y''(L,t) = y'(0,t) = y(0,t) = 0;$$

$$-EIy'''(L,t) + Ty'(L,t) - d(t) + d_s\dot{y}(L,t) + M_s\ddot{y}(L,t) = u(t)\ ;$$

in the formula: t is the time variable、x is the spatial variable, L is the length of the riser, $y(x,t)$ is the actual offset of the transverse vibration, $f(x,t)$ is the distributed current load acting on the riser , $d(t)$ is the boundary disturbance, M_s is the mass of the ship, ρ is the mass per unit of the riser, EI is the bending stiffness of the riser, T is the tension of the riser , c is the structural damping coefficient, d_s is the damping coefficient of the ship, δ is the variational operator, $u(t)$ is the boundary control quantity acting on the tip of the riser; and, x_1、x_2 is the state of the system.

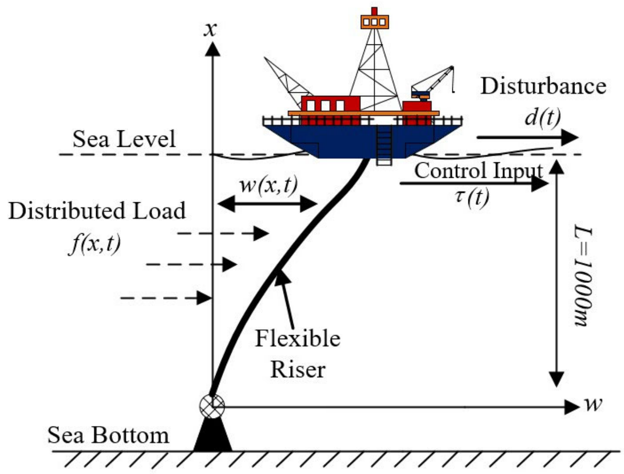

Figure 2-1. Typical Marine Flexible Riser System

2.2. Preparatory knowledge

Lemma 1(Zuo et al., 2020)

$x, y \in \mathbb{R}, \varepsilon > 0, a > 1, b > 1$ and satisfying $(a-1)(b-1) = 1$ the following equation exists

$$xy \le \frac{\varepsilon^a}{a}|x|^a + \frac{1}{b\varepsilon^b}|y|^b$$

Lemma 2 (Li et al., 2016)

For any real variablesω, υ and constants$\zeta > 0, \iota > 0, \gamma > 0$,there exists

$$|\omega|^{\zeta}|\upsilon|^{\iota} \le \frac{\zeta}{\zeta+\iota}\gamma|\omega|^{\zeta+\iota} + \frac{\iota}{\zeta+\iota}\gamma^{\frac{-\zeta}{\iota}}|\upsilon|^{\zeta+\iota}$$

Lemma 3(Wang and Li, 2023)

A nonlinear system is considered as$\dot{\Theta} = f(\Theta, F)$,for the Lyapunov candidate function $\dot{V}(\Theta)$,if there exist constants $\kappa > 0, C > 0$ and $0 < \ell < 1$,then

$\dot{V}(\Theta) \le -\kappa V^{\ell}(\Theta) + C, \forall t \ge 0,$

The solution of$\Theta(t)$ to $\dot{\Theta} = f(\Theta, F)$ is to satisfy Semi-globally uniformly stability (SGFS).

When$t > T^* \triangleq \frac{1}{(1-l)\upsilon\kappa}[V^{1-\ell}(\Theta_0) - (\frac{C}{(1-\ell)\kappa})^{\frac{1-\ell}{\ell}}]$ satisfies

$$V(\Theta) \leqslant \left(\frac{C}{(1-\upsilon)\kappa}\right)^{\frac{1}{\ell}}$$

3. Control Design

3.1. Control Design

In the design of the flexible riser boundary controller, the following nonlinear system is considered

$$\begin{cases} x_1(t) = w(L,t), \\ \dot{x}_1(t) = x_2(t) = \dot{w}(L,t), \\ \dot{x}_2(t) = \frac{1}{M_s}[EIw'''(L,t) - d_s x_2(t) - Tw'(L,t)] + \frac{1}{M_s}u(t) + \frac{1}{M_s}d(t) \end{cases} \quad (3.1)$$

Introduction of the cotangent function type obstacle Lyapunov function

$V = \cot\frac{\pi}{2}(1-x^2), -1 < x < 1$,for$\forall |x| < 1$,when $|x| \to 1, V \to +\infty$

Property1: when$|x| < 1$,there exists the inequality

$$x^2 \le \cot\frac{\pi}{2}(1-x^2) \le \pi x^2 \operatorname{cosec}\frac{\pi}{2}(1-x^2) \quad (3.2)$$

Tracking error is defined as:

$$z_1 = x_1 - y_d \quad (3.3)$$

$$z_2 = x_2 - \alpha \quad (3.4)$$

Where z_1 is the position tracking error z_2 is the velocity tracking error,α is the virtual control ,and y_d is the desired trajectory of x_1.

Assume that the error z_1 is within the constraint$-k_d < z_1 < k_c$ where $k_d \in \mathbb{R}^+$ and $k_c \in \mathbb{R}^+$ are known time-varying functions. Since $z_1 = x_1 - y_d$,there is $y_d - k_d < x_1 < y_d + k_c$.let $\overline{q}_c = y_d - k_d$ and $\overline{q}_c = y_d + k_c$.

For control design, the auxiliary variables $\xi_a, \xi_b, \xi_i, i = 1, \ldots, n$ are designed as

$$\begin{aligned} \xi_a &= \frac{z_1}{k_c} \\ \xi_b &= -\frac{z_1}{k_d} \\ \xi &= h\xi_a + (1-h)\xi_b \end{aligned} \quad (3.5)$$

Which $h = \begin{cases} 1, & z_1 > 0 \\ 0, & \text{others} \end{cases}$.

The asymmetric obstacle Lyapunov function is constructed as

$$V_1 = \frac{h}{2}cot\left(\frac{\pi}{2}(1-\xi_a^2)\right) + \frac{1-h}{2}cot\left(\frac{\pi}{2}(1-\xi_b^2)\right) \quad (3.6)$$

Derivation of the above equation gives

$$\dot{V}_1 = \left(\frac{h}{2}\pi\xi_a\dot{\xi}_a csc^2\left(\frac{\pi}{2}(1-\xi_a^2)\right) + \frac{1-h}{2}\pi\xi_b\dot{\xi}_b csc^2\left(\frac{\pi}{2}(1-\xi_b^2)\right)\right) \quad (3.7)$$

Substituting(3.5)with(3.7)yields

$$\dot{V}_1 = -(\frac{h\dot{k}_c}{2k_c}\pi\xi_a^2 csc^2(\frac{\pi}{2}(1-\xi_a^2)) + \frac{(1-h)\dot{k}_d}{2k_d}\pi\xi_b^2 csc^2(\frac{\pi}{2}(1-\xi_b^2))) + \frac{\pi}{2}\frac{\xi^2\dot{z}_1}{z_1}csc^2(\frac{\pi}{2}(1-\xi^2)) \quad (3.8)$$

According to(3.3),(3.4)we can get

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d = z_2 + \alpha - \dot{y}_d \quad (3.9)$$

Design the virtual control α as

$$\alpha = -k\,z_1(\pi\xi^2 csc^2(\frac{\pi}{2}(1-\xi^2)))^{\ell-1} + \dot{y}_d - k_1 z_1 \quad (3.10)$$

Where $\ell(0 < \ell < 1)$,k and k_1 are positive constants. Bringing(3.10)and(3.9)into(3.8)there are

$$\begin{aligned} \dot{V}_1 = &-\frac{1}{2}\pi\xi^2 csc^2(\frac{\pi}{2}(1-\xi^2))(k_1 + h\frac{\dot{k}_c}{k_c} + (1-h)\frac{\dot{k}_d}{k_d}) \\ &-\frac{1}{2}k\left(\pi\xi^2 csc^2\left(\frac{\pi}{2}(1-\xi^2)\right)\right)^{\ell} \\ &+\frac{\pi}{2z_1}\xi^2 csc^2(\frac{\pi}{2}(1-\xi^2))z_2 \end{aligned} \quad (3.11)$$

where the parameter k_1 satisfies$k_1 = \sqrt{\left(\frac{\dot{k}_c}{k_c}\right)^2 + \left(\frac{\dot{k}_d}{k_d}\right)^2}$

$\dot{V}_1$ can be concluded

$$\dot{V}_1 \le -\frac{1}{2}k\left(cot\left(\frac{\pi}{2}(1-\xi^2)\right)\right)^{\ell} + \frac{\pi}{2z_1}\xi^2 csc^2\left(\frac{\pi}{2}(1-\xi^2)\right)z_2 \quad (3.12)$$

Combining with (3.2), the derivative of x_2 can be rewritten as

$$\dot{x}_2(t) = \frac{1}{M_s}[EIw'''(L,t) - d_s\alpha - Tw'(L,t) + u(t) + d(t)] - \frac{d_s z_2(t)}{M_s} \quad (3.13)$$

Combining with(3.4),the derivative of z_2 can be written as

$$\dot{z}_2 = \frac{1}{M_s}[EIw'''(L,t) - d_s\alpha - Tw'(L,t) - \dot{\alpha}M_s + u(t) + d(t)] - \frac{d_s z_2(t)}{M_s} \quad (3.14)$$

To handle input saturation, this paper uses the following design:

Let $u(t) = \Delta U + s(\mu)$,and

$$s(\mu) = \left(\frac{2\bar{\mu}}{\pi}\right)\arctan\left(\frac{\pi\mu}{2\bar{\mu}}\right) \quad (3.15)$$

$\dot{z}_2$ can be rewritten as

$$\dot{z}_2 = \frac{1}{M_s}[EIw'''(L,t) - d_s\alpha - Tw'(L,t) - \dot{\alpha}M_s + \Delta U + s(\mu) + d(t)] - \frac{d_s z_2(t)}{M_s} \quad (3.16)$$

According to Lemma 1, the following equation holds

$$s(\mu) = \mu - \psi(\mu)\mu \quad (3.17)$$

Where $\psi(\mu) = [(\pi\eta\,\mu/(2\bar{\mu}))^2/(1+(\pi\eta\,\mu/(2\bar{\mu}))^2)] \in \mathbb{R}$, $0 < \eta < 1$ and there are

$$\begin{cases} \bar{\mu} = \mu_{max}, & \text{if } \mu_{max} \geq \mu_{min} \\ \bar{\mu} = \mu_{min}, & \text{if } \mu_{max} < \mu_{min} \end{cases}$$

With the symmetric saturation constraints on the inputs using the above method, the resulting error ΔU will be compensated by designing the adaptive parameters.

According to the definition and treatment of input saturation, it is known that ΔU is bounded, and let $P^* = \Delta U$, and this approximation error should be compensated in order to obtain a better control performance. To this end, the update law for $\hat{P}$ is designed as $\dot{\hat{P}} = z_2 - \sigma\hat{P}$, where $\hat{P} \in \mathbb{R}^n$ denotes the estimated value of P^*,σ is the number of normal, and $\tilde{P}$ denotes the estimation error,$\tilde{P} = \hat{P} - P^\star$.

The finite time controller μ is designed to

$$\mu = \mu_1 + \mu_2 \quad (3.18)$$

$$\mu_1 = -K_2 z_2^{2\ell-1} + d_s\alpha + \dot{\alpha}M_s - \frac{\pi}{2z_1}\xi^2 csc^2\left(\frac{\pi}{2}(1-\xi^2)\right) - EIw'''(L,t) + Tw'(L,t) + d_s z_2 - \frac{z_2}{2M_s} - z_2\hat{P}$$

$$\mu_2 = \begin{cases} 0, & \text{if } |z_2| = 0 \\ -\frac{|\mu_1| z_2}{|z_2|(1-\varsigma)}, & \text{if } |z_2| \neq 0 \end{cases}$$

Where $\psi(\mu) < \varsigma < 1$.

Construct the Lyapunov candidate function as

$$V_2 = V_1 + \frac{1}{2}z_2^2 M_s + \frac{1}{2}\tilde{P}^2 \quad (3.19)$$

Derivation of V_2 yields

$$\dot{V}_2 = \dot{V}_1 - d_s z_2^2 + \tilde{P}\dot{\hat{P}} + z_2\left(EIw'''(L,t) - d_s\alpha - Tw'(L,t) - \dot{\alpha}M_s + \mu - \psi(\mu)\mu + \Delta U + d(t)\right) \quad (3.20)$$

Substituting(3.12),(3.17)and(3.18)into(3.20)yields

$$\dot{V}_2 \leq -\frac{k}{2}\left(cot\left(\frac{\pi}{2}(1-\xi^2)\right)\right)^\ell - K_2 z_2^{2\ell} - z_2\hat{P} - \frac{1}{2}z_2^2 + \tilde{P}(z_2 - \sigma\hat{P}) + \Delta U z_2 - \psi(\mu)\mu z_2 + z_2\mu_2 \quad (3.21)$$

The following inequality can be derived from Lemma 1

$$\tilde{P}\sigma\hat{P} \leq \frac{\sigma}{2}|\tilde{P}|^2 - \frac{\sigma}{2}|P^*|^2 \quad (3.22)$$

since $P^* = \Delta U, -\psi(\mu)\mu z_2 + z_2\mu_2 \leq 0$.(3.21)can be rewritten as

$$\dot{V}_2 \leq -\frac{k}{2}\left(cot\left(\frac{\pi}{2}(1-\xi^2)\right)\right)^\ell - K_2 z_2^{2\ell} - \frac{\sigma}{2}|\tilde{P}|^2 + \frac{\sigma}{2}|P^*|^2 \quad (3.24)$$

let $\omega = 1$,$\upsilon = \frac{1}{2}|\tilde{P}|^2$,$\zeta = 1-\ell$,$\iota = \ell$,$\gamma = \ell^{\frac{\ell}{1-\ell}}$, and by Lemma 2 the following inequality can be derived

$$\left(\frac{\sigma}{2}|\tilde{P}|^2\right)^\ell \leq (1-\ell)\gamma + \frac{\sigma}{2}|\tilde{P}|^2 \quad (3.25)$$

Substituting(3.25)into(3.24)yields

$$\begin{aligned} \dot{V}_2 &\leq -\frac{k}{2}\left(cot\left(\frac{\pi}{2}(1-\xi^2)\right)\right)^\ell - K_2 z_2^{2\ell} - \left(\frac{\sigma}{2}|\tilde{P}|^2\right)^\ell + (1-\ell)\gamma + \frac{\sigma}{2}|P^*|^2 \\ &\leqslant -\kappa V_2^\ell + C \end{aligned} \quad (3.26)$$

where $\kappa = min\{k, \frac{(2)^\ell K_2}{(M_S)^\ell}, (\sigma)^\ell\}$, $C = (1-\ell)\gamma + \frac{\sigma}{2}|P^*|^2$.

3.2. Stability analysis

Define$T^* = \frac{1}{(1-\ell)\upsilon\kappa}[V_2^{1-\ell}(\Theta_0) - (\frac{C}{(1-\ell)\upsilon\kappa})^{(1-\ell)/\ell}]$,where $\Theta_0 = [z_1(0), z_2(0), \tilde{P}]^T$ then by Lemma 3, $\forall t > T^*$,$V_2^\ell \leq \frac{C}{(1-\ell)\upsilon\kappa}$, so the error signals $z_1, z_2, \tilde{P}$ satisfy the semi-global consistent stabilization

By the definition of V_2, $V_1 \leq V_2 \leq \left(\frac{C}{(1-\ell)\upsilon\kappa}\right)^{\frac{1}{\ell}}$, $\forall t > T^*$

Theorem 1: For the marine flexible riser controlled by(3.1)and the controller(3.17)、the adaptive law(3.18), it can be concluded that all the error signals satisfy semi-global consistent stabilization, and the system output x_1 converges in finite time to a small neighborhood of the desired trajectory y_d without violating the predefined constraint region. Specific convergence results are given below:

(1) In the initial value set within the constraint range $-k_{di}(0) < z_{1i}(0) < k_{ci}(0)$,the tracking error z_1 does not violate the specified constraint region,i.e., $\underline{q}_c < x_1 < \overline{q}_c$ where $\underline{q}_c = y_d - k_d$, $\overline{q}_c = y_d + k_c$, $i = 1, \ldots, n_2$

(2) The adaptive estimation error $\tilde{P}$ is semi-globally consistent and stable and satisfies

$$|\tilde{P}| \leq \sqrt{\mathcal{F}}, \text{for } t > T^*$$

Where $\mathcal{F} = 2(\frac{C}{(1-\ell)\upsilon\kappa})^{\frac{1}{\ell}}$.

(3)The tracking errors z_1 and z_2 are semi-globally consistent and stable and satisfy $\underline{C} < z_1 < \bar{C}$,where $\underline{C} = -k_d Q, \bar{C} = k_c Q, Q = min\{1, \sqrt{\mathcal{F}}\}$,for $\forall t > T^*$, $||z_2|| \leq \sqrt{\frac{\mathcal{F}}{M}}$.

Proof: (1) According to For $\forall t > T^*$,have $V_1 \leq V_2 \leq \left(\frac{C}{(1-\ell)\upsilon\kappa}\right)^{\frac{1}{\ell}}$ can derived that

$$\frac{h}{2}cot\left(\frac{\pi}{2}(1-\xi_a^2)\right) + \frac{1-h}{2}cot\left(\frac{\pi}{2}(1-\xi_b^2)\right) \leq (\frac{C}{(1-\ell)\upsilon\kappa})^{\frac{1}{\ell}}$$

$$\tilde{P}_i^2 \leq 2(\frac{C}{(1-\ell)\upsilon\kappa})^{\frac{1}{\ell}}$$

$$z_2^2 M \leq 2(\frac{C}{(1-\ell)\upsilon\kappa})^{\frac{1}{\ell}}$$

Case 1: If $z_1 > 0$,$h = 1$ and $\xi = \xi_a$,then we have

$\cot\left(\frac{\pi}{2}(1-\xi_a^2)\right) \le \left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}$,and applying Property 1 gives $\frac{1}{2}\xi_a^2 \le \left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}$,and hence there exists $0<\xi \le \sqrt{2(\frac{C}{(1-\ell)\nu\kappa})^{\frac{1}{\ell}}}$

Case 2: If $z_1 \le 0, h = 0$ and $\xi = \xi_b$,similarly to case 1, thus we have

$$0<\xi \le \sqrt{2\left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}}$$

Since $0<\xi<1$, there exists $0<\xi< \min\{1, \sqrt{2\left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}}\}$.

In summary, it can be shown that for $\forall t > T^*, \xi_i$ is semi−globally consistent and stable and bounded by $\min\{1, \sqrt{2\left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}}\}$.

(2) Recalling (3.6),V_1 is positive definite when $|\xi_{a,i}| < 1$ or $|\xi_{b,i}| < 1$,considering ξ_a and ξ_b:

$$-k_d < z_1 < k_c, \forall\, t > 0$$

Since $z_1 = x_1 - y_d$, there are

$$y_d - k_d < x_1 < y_d + k_c, \forall\, t > 0$$

$$0<\xi < \min\{1, \sqrt{2\left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}}\}.$$

By the definitions of ξ, ξ_a and ξ_b,for $\forall t > T^*$ there are

$$-k_d min\{1, \sqrt{2\left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}}\} < z_1 < k_c min\{1, \sqrt{2\left(\frac{C}{(1-\ell)\nu\kappa}\right)^{\frac{1}{\ell}}}\}$$

According to Theorem 1, when $t < T^*$,there is a rapid decrease and eventual convergence of z_1, z_2 , $\tilde{\theta}$ to their respective sets. when $t > T^*$,it is shown that the errors z_1, z_2 , $\tilde{P}$ are bounded. Since the initial error is bounded, $z_1, z_2, \tilde{P}$ is bounded when $t > 0$.Since y_d is bounded,x_1 is also bounded, and therefore the virtual control α is bounded. Since $x_2 = z_2 + \alpha$,x_2 is bounded. Notice that P^* is bounded, hence $\hat{P}$ is bounded. Thus it is learned that μ and α are bounded when $t > 0$.

4. Simulations

To verify the validity of the boundary control law in this paper, the vibration of the marine flexible riser system using the finite difference method under the joint action of distributed current loads and boundary disturbances in 400 s is simulated for simulation analysis, and the system parameters are shown in Table 1.

Table 1. Marine flexible riser parameters

Parameters	Parameter Description	value
L	Length of flexible riser	1000m
M_s	Weight of Ship	9.6×10^6kg
EI	Stiffness of flexible riser	1.5×10^7NM^2
D	external diameter of flexible riser	152.40mm
d_s	Ship Damping	1×10^3Ns/m
T	flexible riser strain	8.11×10^7N
ρ	flexible riser unit mass	500kg/m
ρ_s	density of sea water	1024.00kg/m^3
C_D	drag coefficient	1.361
S_t	Strouhal number	0.200
f_v	shedding frequency	2.625
θ	phase angle	0.00
c	Damping of flexible riser	2.00Ns/m

The distributed interference on the marine flexible riser is denoted as

$$f(x,t) = \frac{1}{2}\rho_s C_D U^2(x,t) D + A_D cos\left(4\pi f_v t + \theta\right)$$

Where ρ_s is the seawater density, C_D is the damping coefficient,A_D is the fraction of the oscillatory part of the damping, usually 20% of the first term of $f(x,t)$,and the dimensionless vortex fall-off frequency is denoted by:

$$f_v = \frac{S_t U(x,t)}{D}$$

Where S_t is the Strouhal number,D is external diameter of flexible riser.

The sea current can be expressed as:

$$U(x,t) = \frac{x}{1000} U(t)$$

The time-varying current $U(t)$ can be expressed as:

$$U(t) = \overline{U} + U' \sum_{i=1}^{4} sin(w_i t)$$

Where $w_i = (w_1, w_2, w_3, w_4) = (0.867, 1.827, 2.946, 4.282)$, $\overline{U} = 2m/s$ is the mean current speed and $U' = 0.2$ is the amplitude of the oscillatory flow.

The marine flexible riser boundary perturbation is:

$$d(t) = [3 + 0.8sin\,(0.7t) + 0.2sin\,(0.5t) + 0.2sin\,(0.9t)] \times 10^5$$

To verify the effectiveness of the control algorithm proposed in this paper, three dynamic response cases of the riser system are discussed and analyzed.

1. Free vibration of marine flexible risers

The free vibration results of the marine flexible riser are shown in Fig.4-1, The riser periodically oscillates due to the lack of external force control, and its vibration offset ranges from 0m to 7m, which has a serious impact on the performance of the marine flexible riser.

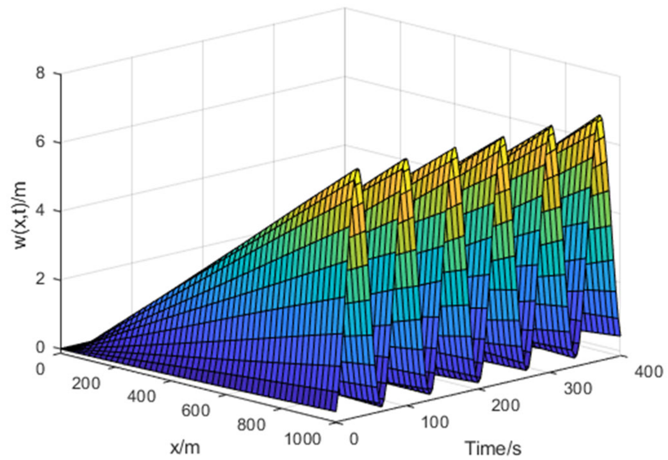

Figure 4-1. Lateral vibration excursion of the riser without control

2. PID control

The PID control used in this paper, i.e., $U(t) = -k_p w(L,t) - k_d \dot{w}(L,t)$, where the parameters k_p, k_d are $k_p = 9 \times 10^5$ and $k_d = 8 \times 10^5$,and the transverse vibration offsets of the oceanic flexible risers are shown in Fig. 4-2, and the control inputs of the PID control are shown in Fig. 4-3.

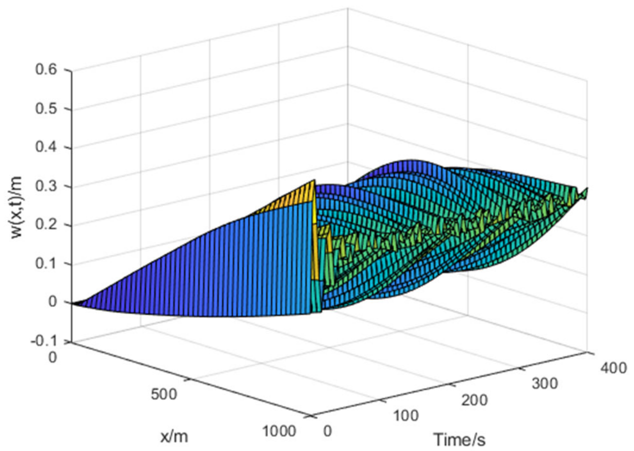

Figure 4-2. Lateral vibration excursion of the riser under PID control

3. With the robust adaptive boundary control

For the vibration problem of marine flexible riser with input saturation, Zhao(Zhao et al., 2017) et al. proposed an adaptive boundary control method.

$$\lambda_1 = -k_1 x_1 - \varsigma$$

Where $k_1 > 0$ and $\varsigma = Tz'(l,t) - EIz''(l,t)$.

$$\lambda_2 = \varsigma + d_a \dot{z}(l,t) + m\dot{\lambda}_1 - x_1 - k_2 x_2$$
$$= Tz'(l,t) - EIz''(l,t) + d_a \dot{z}(l,t) + m\dot{\lambda}_1 - x_1 - k_2 x_2$$

where $k_2 > 0$.

$$\dot{u}_i(t) = -ku_i(t) + \epsilon$$
$$\epsilon = N(\chi)\bar{\epsilon}$$
$$\bar{\epsilon} = \alpha k u_i + \frac{1}{m}\mu - \frac{1}{m}k_3 x_3 - \frac{1}{m}x_2 - \frac{1}{2m}\left(\frac{\partial \lambda_2}{\partial z_2}\right)^2 x_3$$
$$\alpha = \frac{\partial u_0(u_i(t))}{\partial u_i(t)} = \frac{4}{(e^{u_i(t)/u_m} + e^{-u_i(t)/u_m})^2} > 0$$

where $k_3 > 0$ and we define the Nussbaum function $N(\chi)$as

$$N(\chi) = \chi^2 \cos(\chi)$$
$$\dot{\chi} = \beta_\chi m x_3 \epsilon$$

with $\beta_\chi > 0$ and the Nussbaum function satisfying the following properties.

$$\lim_{k\to\pm\infty} \sup \frac{1}{k}\int_0^k N(s)\,ds = \infty$$
$$\lim_{k\to\pm\infty} \inf \frac{1}{k}\int_0^k N(s)ds = -\infty$$

The control parameters are selected as $k = 1500$, $k_1 = 10$, $k_2 = 40$,$k_3 = 40$ and $u_m = 3 \times 10^5$.

Using the robust adaptive boundary control, the riser transverse vibration offset is shown in Figure 4-4, and its control inputs are shown in Figure 4-5.

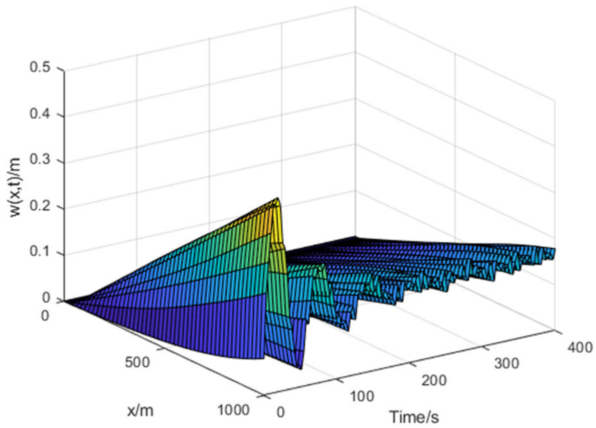

Figure 4-3. Lateral vibration excursion of the riser under boundary control

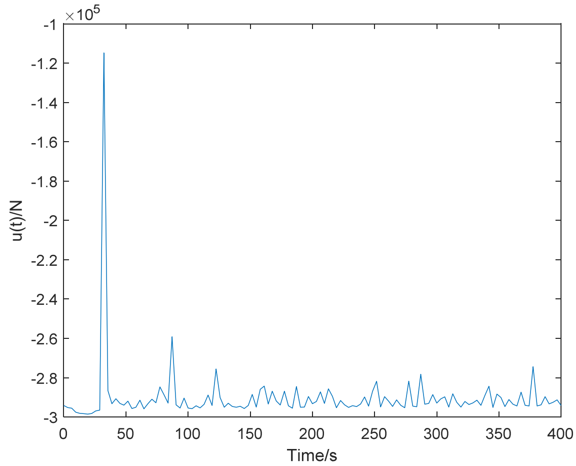

Figure 4-6. Proposed Control Inputs

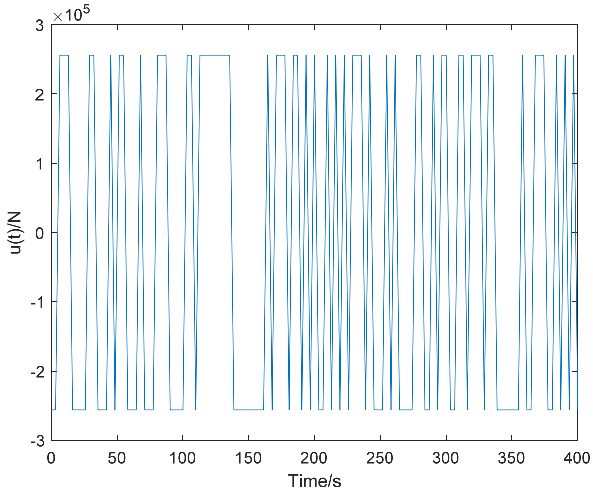

Figure 4-4. Boundary control inputs

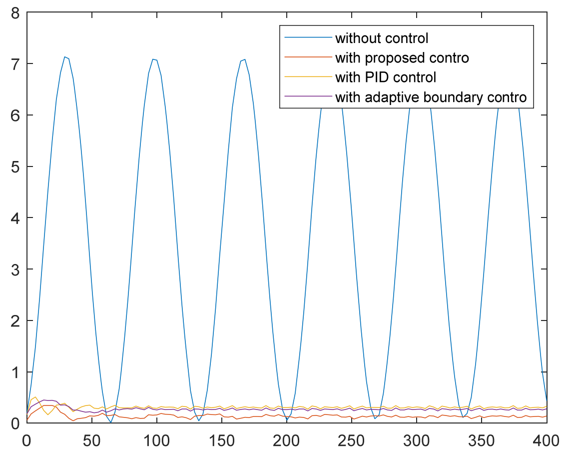

Figure 4-7. Comparison of riser end vibration offsets (vs. PID, input saturated boundary control and uncontrolled)

4. Controller proposed in this paper

The lateral vibration offset of the marine flexible riser under the action of the controller proposed in this paper is shown in Fig. 4-6, and its control inputs are shown in Fig. 4-7, with the control parameters selected as $k=2, k_1=1500$, $k_2=1000$ and $\ell=0.9$. The input saturation constraints are set to be $3.5*10^5$N.the initial value of $\hat{P}$ is set to be 0 and σ =0.1.

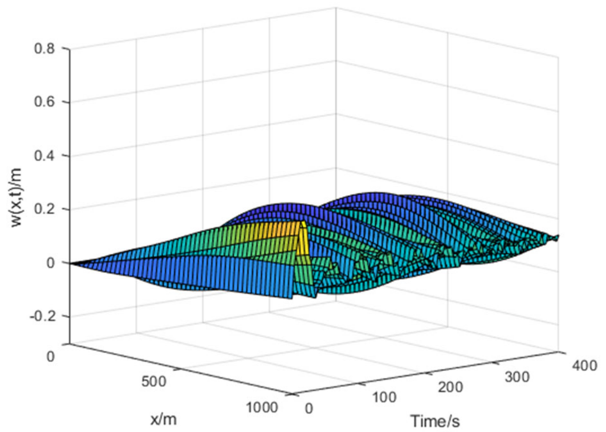

Figure 4-5. Lateral vibration deflections of risers under proposed control

As can be seen from Figures 4-2, 4-3, and 4-5, the PID control, adaptive boundary control, and the control method proposed in this paper can effectively control the riser end vibration offset and suppress the lateral vibration of the riser. The control method proposed in this paper can converge the riser end vibration in a limited time. The riser end vibration offset under the control of the three methods is superimposed on a graph in Figure 4-7, which can be more intuitively found that the riser end vibration offset under the control of the control scheme proposed in this paper is lower than that under the PID control and boundary control, and the convergence speed is faster than that of boundary control and PID control. Therefore, the control algorithm proposed in this paper has superiority.

5. Conclude

In this paper, a suitable control strategy is designed for the marine flexible riser tracking control problem affected by time-varying output constraints and input saturation. By introducing a finite-time stability criterion and a barrier Lyapunov function, the marine flexible riser vibration is limited to a specific range,and the convergence rate is improved. The input saturation nonlinearity is approximated using the hyperbolic tangent function and the approximation error is compensated by designing adaptive parameters. The stability of the closed-loop system is demonstrated by the Lyapunov direct method, and finally,the effectiveness of the proposed controller is experimentally verified by numerical simulation, and the comparison with other controllers reflects the superiority of the controller designed in this paper.

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