Academic Journal of Science and Technology
ISSN: 2771-3032 | Vol. 14, No. 2, 2025

Decision-Making Research on Supply Chain Quality Inspection Based on Binomial Distribution and Monte Carlo Simulation

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Abstract: In modern industrial production, enterprises face the dual challenge of ensuring supply chain quality reliability while controlling costs. Traditional single-link quality detection methods are inadequate for multi-link collaborative decision-making in supply chains. This study proposes a quality inspection method combining binomial distribution and Monte Carlo simulation for enterprise production processes. Integrating hypothesis testing based on normal distribution determines whether spare part defective rates exceed supplier-declared standards, guiding batch acceptance decisions. The model introduces a random allowable error E to adjust significance levels dynamically and errors, enhancing performance analysis. The study also considers inspection, dismantling costs, and substandard product exchange losses. Using Monte Carlo simulation, a revenue model is constructed for supply chain decision-making, providing accurate success rate solutions for parts, semi-finished, and finished products, aiming to maximize enterprise profit. Results show that precise quality control and cost management significantly improve economic efficiency and market competitiveness. For instance, selling a single product yields maximum profits increased from 9.60 yuan to 19.36 yuan (102% increase), with the second-best option reaching 15.19 yuan, a gap of 4.17 yuan compared to the optimal solution. This study offers robust theoretical guidance for quality control in enterprise production processes.

Keywords: Monte Carlo Simulation, Z-test, Binomial Distribution, Hypothesis Testing, Profit Function.

1. Introduction

With the expanding scale of modern industrial production and the increasing complexity of the supply chain system, the quality control of enterprises in procuring spare is particularly important [1]. Effective quality inspection is not only related to the final quality of the product but also directly affects the operating costs, production efficiency, and market competitiveness [2]. Full inspection operation often brings excessive labor and material costs, while too simple sampling inspection may lead to unacceptable quality risks [3]. Therefore, how to design a scientific and reasonable sampling inspection program under the premise of ensuring quality reliability has become a key issue that needs to be solved by current industrial enterprises. Traditional quality inspection research mainly focuses on the design of a sampling scheme for a single link, with less consideration for the systematic and economic nature of the whole supply chain, and lacks a unified theoretical framework and quantitative analyzing methods. At the same time, various uncertainties in production practice, such as the fluctuation of the defective rate of spare parts, the inspection of changes in measurement costs, supplier reliability, and shifts in market demand all pose additional challenges to an organization's quality decisions. To meet these challenges, companies need to adopt a comprehensive quality control strategy that combines advanced statistical methods and simulation techniques to achieve accurate monitoring and optimal management of each link in the supply chain. In addition, the introduction of dynamic adjustment mechanisms to adapt to changing internal and external conditions is essential to maintain efficient and economical quality control. In this way, companies can ensure product quality while effectively controlling costs and enhancing their long-term competitiveness in the market.

Aiming at the multiple decision-making challenges faced by enterprises in the product production process, the study independently develops a Monte Carlo simulation-based quality inspection and revenue optimization strategy for spare parts under the consideration of inspection cost, dismantling cost, and market loss; at the same time, a hypothesis testing method based on the normal distribution is designed for deciding whether or not to accept spare parts provided by the supplier at different confidence levels, respectively, and the study is carried out through the introducing the volatility of sampling inspection, the Monte Carlo simulation method was used to model the success rate of each part, semi-finished product and finished product in the production process. Confidence intervals for the success rate of each step of the decision-making process are calculated by utilizing z-tests, and Monte Carlo simulation methods are used to generate multiple random success rate values within the confidence intervals, based on which the expected returns in simple and complex decision-making problems are calculated. For the success rate generated by each simulation, this paper evaluates the gains of different detection and disassembly decisions and derives the expected gain through multiple simulations.

2. Quality Sampling Inspection Methods for Parts

The data for this study were obtained from https://cumcm.cnki.1net/.In enterprise supply chain management, the quality inspection program of sampling and testing of spare parts provided by suppliers constitutes an indispensable quality control strategy in industrial production.

With the help of accurate statistical sampling techniques, enterprises can not only efficiently assess the overall quality level of batches to be accepted, but also quickly identify substandard parts, effectively prevent substandard parts from entering the production process, and significantly reduce manufacturing delays and additional cost burdens caused by substandard parts. Research has shown that companies want to decide whether to accept parts from suppliers through sampling, and the goal is to design a sampling program with as few tests as possible to ensure that the defective rate of parts exceeds a certain nominal value with a reasonable degree of confidence. The problem can be reduced to a hypothesis testing problem based on binomial distribution, which can be solved by hypothesis testing. In this paper, the defective rate provided by the supplier is regarded as a hypothesis and is tested by the results of sample testing to determine whether to accept or reject the spare parts [4].

To be closer to the real engineering situation, two scenarios are set up in this study, assuming that a supplier claims that the defective rate of a batch of spare parts will not be more than 10%. The company is going to use sampling and testing to decide whether to accept the spare parts from the supplier or not. The cost of testing the purchased spare parts will be borne by the enterprise. Case 1 is recognized at a 95% confidence level. If the defect rate of the spare part exceeds the nominal value, the spare part is rejected; in case 2, if the defect rate of the spare part is found not to exceed the nominal value at a 90% confidence level, the spare part is accepted.

2.1. Hypothesis testing method based on binomial distribution

The binomial distribution is a discrete probability distribution model in probability statistics, whose core parameters are the total number of independently repeated experiments n and the probability of success of an experiment p. An enterprise needs to detect whether the defective rate of a batch of spare parts is more than the nominal value, which can be reduced to a hypothesis test based on the binomial p_0 distribution to realize [5]. Based on the binomial distribution model, this study set the overall spare parts defective rate of p (unknown), the supplier to provide the defective rate of nominal value p_0 (known), the need to test the sample size of n, the number of unqualified spare parts in the sample for X, then X obey the binomial distribution. The relationship concerning X is expressed in Formula 1.

$$X \sim (n, p) \tag{1}$$

The Z value corresponding to the commonly used confidence level is shown in Table 1.

Table 1. Z-values for common confidence levels

Confidence Level	80%	90%	95%	97.5%	99%	99.5%
Value Z	0.842	1.282	1.645	1.96	2.326	2.576

Afterward, the overall part defect rate p is estimated by calculating the defect rate $\hat{p} = \frac{X}{n}$ of the sampled detected defects in the sample. The following hypotheses were constructed and tested. At this point, the original hypothesis of this study is that the rate of substandard products does not exceed the nominal value p_0 and the alternative hypothesis is that the rate of substandard products exceeds the nominal value p_0.

When the sample size n is large, the calculation can be simplified by approximating the binomial distribution with a normal distribution. According to the central limit theorem, the binomial distribution about X can be approximated by a normal distribution with mean np_0 and variance $np_0(1\text{-}p_0)$, which is expressed in Formula 2.

$$X \sim N(np_0, np_0(1 - p_0)) \tag{2}$$

Afterwards, a normal distribution approximation obeying $p \sim N(np_0, np_0(1 - p_0))$ is known. The critical values of corresponding confidence intervals are used to determine whether to accept or reject the original hypothesis. For the 95% confidence interval confidence level, the critical value is $Z_{\frac{0.95}{2}} \approx 1.96$; for the 90% confidence level, the critical value is $Z_{\frac{0.90}{2}} \approx 1.645$.

2.2. Solve for minimum sample size

According to the formula of confidence interval, the sample size calculation can be obtained to satisfy Formula 3.

$$n = (Z_{(\alpha/2)}/E)^2 p_0 (1 - p_0) \tag{3}$$

Where E is the allowable error, which is the difference between the desired defective rate and the actual defective rate; p_0 is the nominal value of 10%; $z_{(\alpha/2)}$ is the critical value of the normal distribution; n is the sample size for calculating the minimum sampling, which in this paper is introduced as an allowable random error E, which is also in line with the rationalization of the existence of random errors in the factory production process. Set the allowable error E = 0.05 and solve for n as the minimum sample size for sampling test results.

At a 95% confidence level, the minimum sample size was n=139; at a 90% confidence level, the minimum sample size was n=98. After finding the minimum sample size required at 95% and 90% confidence level, according to the results of solving the problem, this study continuously increase the number of substitutes in the sample and find out the maximum number of substitutes that can be accommodated at 95% and 90% confidence level, as shown in Figure.1 respectively.

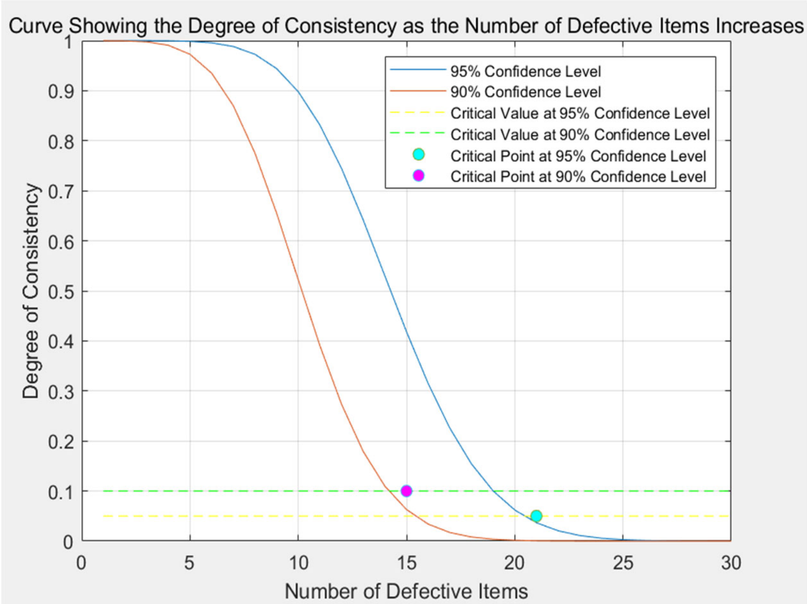

Figure 1. The change curve of consistency degree with the increase of defective quantity

At 95% confidence level, n=139, the maximum number of substitutions that can be accommodated is 20; at 90% confidence level, n=98, the maximum number of substitutions that can be accommodated is 14.

3. Production Decision Study Based on Monte Carlo Simulation

In the production process, enterprises must decide whether to inspect spare parts, semi-finished products, and finished products, as well as whether to disassemble defective semi-finished and finished products. Each of these decisions directly impacts the final product's profitability [6]. This study focuses on a scenario where two types of spare parts are assembled directly into a finished product. Given the known defect rates of the two spare parts and the finished product, the study provides decision-making guidelines for each stage of the production process. The defect rates of Spare Part 1, Spare Part 2, and the finished product, along with inspection costs and replacement loss expenses, vary. The research establishes a profit model based on Monte Carlo simulation, using this method to ensure the success rate of decisions. By comprehensively considering inspection and disassembly strategies, the model calculates the expected profits under various scenarios, ultimately determining the optimal solution [7].

During the procurement of spare parts, enterprises must decide whether to inspect the parts; untested spare parts proceed directly to the assembly stage, while any detected defective parts are discarded. After assembly, each finished product faces a decision point regarding inspection: uninspected products are released to the market, whereas inspected and qualified products are sold. Defective finished products that fail inspection are handled according to the company’s policies. For defective finished products, the enterprise must decide whether to disassemble them. Companies opting for disassembly will recover and re-inspect components to ensure they meet quality standards. Should customers receive defective products, the company commits to unconditional replacements and bears associated costs such as logistics and potential reputational damage. Returned defective products also undergo the disassembly and inspection process to assess further handling procedures. The corresponding production scenarios are shown in Table 2.

Table 2. Scenarios of Products Faced by the Company During Manufacturing

Cases	Part1			Part2			Finished Product				Unqualified Product	
	Defect rate	Unit cost	Inspect cost	Defect rate	Unit cost	Inspect cost	Defect rate	Assembly cost	Inspect cost	Market price	Exchange loss	Disassembly cost
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

3.1. Using z-tests to solve for confidence intervals for the success rate of decision-making

Since the success rate of each spare part, semi-finished product, and finished product in the production process of the enterprise is obtained through sampling, there is a certain error with the real success rate and these success rates are not fixed values, but their is a certain degree of volatility [8]. To more accurately reflect the volatility in the production reality, this paper calculates the confidence interval of each success rate by introducing the z-test, introduces Monte Carlo simulation within this interval, generates the success rate through 10000 times of random sampling, and finally takes the average value to get the final success rate, and then obtains the expected return. First, this paper uses the z-test to calculate the confidence intervals of the passing rates of spare parts, semi-finished products, and finished products, which are obtained through sampling and testing, and there is a certain degree of volatility. Specifically, the use of p~N($\hat{p}$, $\sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}}$) was calculated to obtain the confidence interval for the pass rate in Formula 4:

$$\left(\hat{p} - z_{(\alpha/2)}\sqrt{(p(1-p))/n}\,,\hat{p} + z_{(\alpha/2)}\sqrt{(p(1-p))/n}\right)(4)$$

where $\hat{p}$ is the pass rate obtained after n with sample size n=10000. The confidence interval obtained can be tabulated by assuming that the confidence level is 99% and the corresponding value = 2.576, which can be expressed in Formula 5:

$$\left(\hat{p} - 2.576\sqrt{\hat{p}\left(1-\hat{p}\right)/10000}\,,\hat{p} + 2.576\sqrt{\hat{p}\left(1-\hat{p}\right)/10000}\right)(5)$$

Based on the confidence interval, the study uses Monte Carlo simulations to generate multiple possible pass rate values [9]. In each simulation, the generated pass rates are used to decide whether or not Part 1, Part 2, and the finished product should be inspected and disassembled, and the corresponding expected returns are calculated. The total number of Monte Carlo simulations is denoted as N. In this way, it is ensured that the decision-making process fully takes into account the effects of fluctuations in the conformity rate, thus providing a scientific basis for the quality control of the enterprise.

3.2. Analysis of experimental results

There are three key decision-making components that companies need to consider in the quality control process [10]. First, the detection and success rate of parts 1 and 2. The study calculates a confidence interval for the success rate of the parts based on the z-test and generates multiple random success rate values within this interval through Monte Carlo simulations. In each simulation, if the inspection of the parts is selected, the associated inspection costs and substitution costs are factored into the total cost, and the success rate of the parts is updated. Second, the inspection of the finished product is just as important as the success rate. The success rate of the finished product is also generated by the Monte Carlo simulation, and the success rate of the finished product in each simulation determines whether or not a finished product inspection is required. Inspection of the finished product increases the cost of inspection but avoids the loss of swapping out failed finished products. Finally, for non-conforming finished products, the disassembly decision is critical. In each simulation, if the finished product inspection results in a failure, the recycled parts are recovered according to the value of the decision to disassemble or not. The disassembly decision relies on the recovery probability determined by the success rate generated by each simulation.

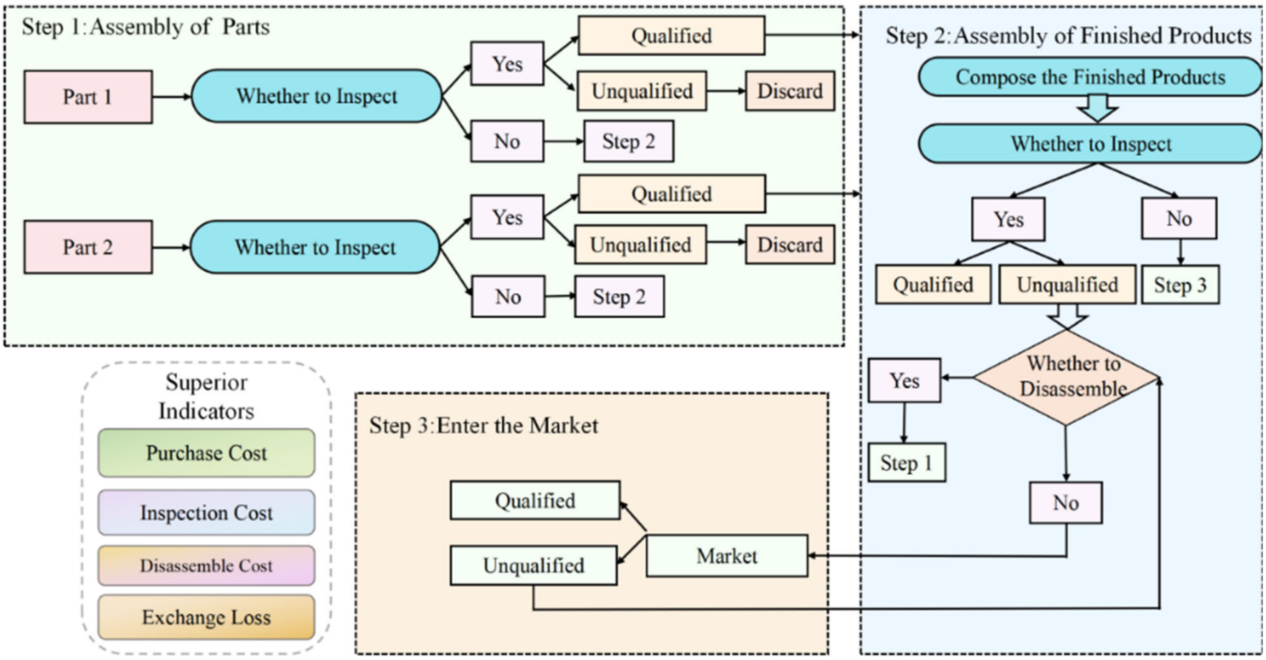

Figure 2. Industry Chain Production Decision-Making Flowchart

The total cost of the firm covers four main areas [11]. The first is the purchase cost of spare parts, the non-detectable zero accessories directly into the assembly process; second, testing costs, including the costs incurred in testing spare parts and finished products; third, dismantling costs, which are the additional expenditures incurred in dismantling unqualified finished products; and lastly, exchange losses, whereby unqualified finished products sold in the market will lead to exchange losses borne by the enterprise, such as logistic costs and reputational damages. By comprehensively considering these factors, enterprises can optimize the cost structure while ensuring product quality, thus enhancing economic efficiency and market competitiveness [12]. Figure 4 shows the flowchart of the above industrial chain production relationship.

3.3. Define the Profit Function

By randomly selecting the success rate of spare parts and finished products(within a given range) and calculating the profit under that strategy. The study stores the profit of each simulation in a corresponding list and finally calculates the average of the profits under each strategy and outputs the results. The profit for the i simulation is in Formula 6.

$$W^{(i)} = p^{(i)}c_f - \sum_{part} c_{part} - c_m - (1 - p^{(i)})c_d + c_{Disassemble} - c_{Inspect} \quad (6)$$

Expected profit is calculated by averaging all simulations in Formula 7.

$$E(W) = \frac{1}{N}\sum_{i=1}^{N} W^{(i)} \quad (7)$$

Where $p^{(i)}$ indicates the total qualified rate of finished products in the i simulation, determined by the success rate of each spare part and finished product; c_{part} indicates the purchase unit price of each spare part; c_m indicates the assembly cost of each finished product; c_d indicates the exchange loss of each unqualified finished product; $c_{Inspect}$ indicates the testing cost, including all semi-finished products and finished products testing costs; $c_{Disassemble}$ indicates the dismantling profit, when the finished product is unqualified and can be recycled after dismantling some spare parts, dismantling profit for the value of recycling minus the cost of dismantling; if the value of recycled spare parts is greater than the cost of dismantling, dismantling will be carried out; N is the total number of Monte Carlo simulations.

3.4. Analysis of results

In this paper, the solution steps are categorized into six corresponding scenarios based on six cases in Table 2. It can be seen that each scenario corresponds to 16 decision options, and a total of 6 scenarios corresponds to 96 decision options. The Monte Carlo simulation is performed to solve the 16 scenarios of scenario and the maximum profit of each scenario is obtained and visualized in a barchart, and the final decision-making scenario and optimal profit are as follows.

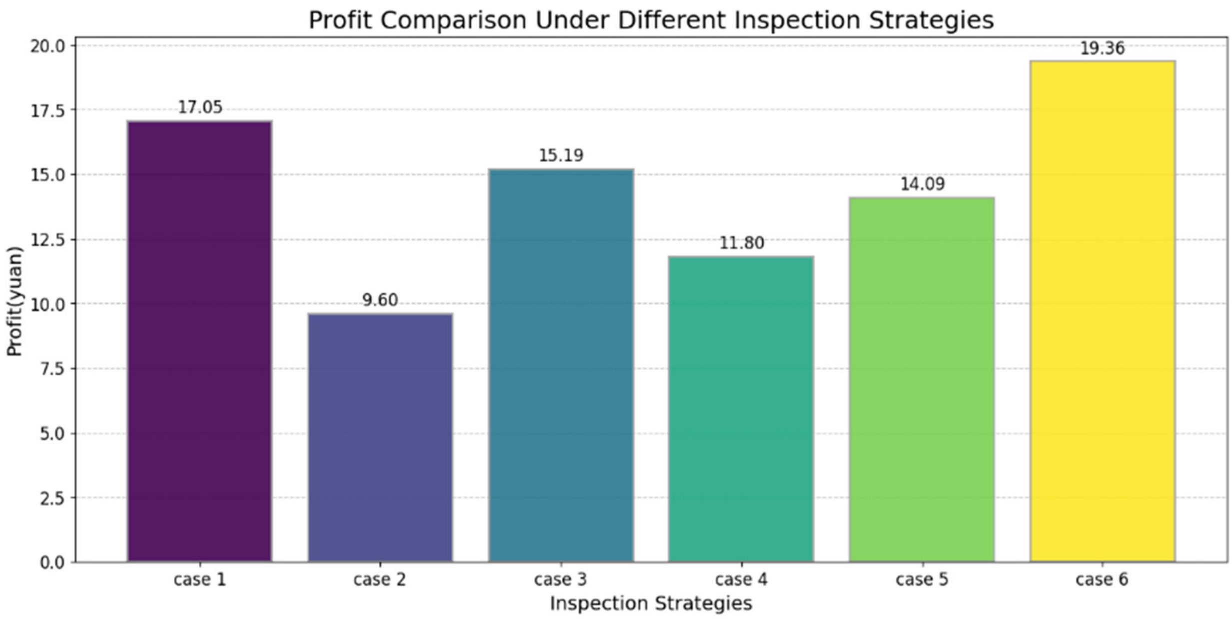

Figure 3. Profit Comparison Under Different Inspection Strategies

The study compares the profit impact of six different inspection and handling strategies and visualizes the optimal profit for different inspection decisions. Figure 5 shows that the optimal strategy is case 6, which is the inspection of Part 1 without disassembling the nonconforming finished product, and the average profit under this strategy reaches the highest value of \$19.36; the second best strategy is case 1, with an average profit of \$17.05 and a difference of 4.31 between the two cases. The worst strategy is case 2, whose average profit is only \$9.60, with a significant gap of \$9.76 with the optimal strategy, indicating that simplifying the testing process and avoiding unnecessary disassembling operations can effectively reduce production costs. The overall trend shows that the strategy of not disassembling unqualified finished products and appropriately selecting inspection spare parts can maximize profits. In addition, the data analysis shows that adding unnecessary inspection steps (e.g., inspecting Part 1 and Part 2 at the same time) and disassembling nonconforming finished products can lead to higher costs, which in turn can lower average final profits. Therefore, economic efficiency can be effectively improved by optimizing the inspection process and the handling of finished products [13]. The detailed decision-making is shown in Table 3.

Table 3. Decision-making strategy for maximum profit under Monte Carlo simulation

Number	Pass Rate of Monte Carlo	Optimal Profit (Yuan)	Inspect Part 1	Inspect Part 2	Inspect Finished Product	Disassemble Defective Product
Case 1	Part 1 Part 2 Product	17.05	yes	no	no	yes
Case 2	Part 1 Part 2 Product	9.60	yes	yes	no	yes
Case 3	Part 1 Part 2 Product	15.19	yes	no	yes	yes
Case 4	Part 1 Part 2 Product	11.80	yes	yes	yes	yes
Case 5	Part 1 Part 2 Product	14.09	no	yes	no	yes
Case 6	Part 1 Part 2 Product	19.36	yes	no	no	no

4. Conclusions

Focusing on the quality control challenges in modern industrial production, this study proposes a quality inspection method that combines binomial distribution hypothesis testing and Monte Carlo simulation, aiming to optimize the supply chain decision-making process. By introducing a random allowable error E and dynamically adjusting the significance level and random error, not only the model performance is improved, but also a revenue model based on supply chain decision-making is constructed, which realizes a more accurate solution for the success rate of each part, semi-finished product and finished product in the production process. The results of the study show that the maximum profit that can be obtained from selling a single product under the optimal solution increases from $9.60 to $19.36, an increase of 102%, which indicates that accurate quality control and cost management can significantly improve the economic efficiency and market competitiveness of enterprises.

This paper presents a methodological framework for quality management in complex supply chains, integrating binomial distribution testing with Monte Carlo simulation. The approach enhances model accuracy while optimizing quality control processes and reducing operational costs. This framework provides valuable guidance for enterprises across industries, helping them maintain product quality while maximizing profits and market competitiveness. The methodology demonstrates broad applicability and potential for industry-wide advancement. Future research could aim to extend the scope of application of the model, consider more realistic uncertainties, and empirically analyze it with real cases to further improve this theoretical framework.

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