

Distortion Effect Analysis of Corrugated Steel Web Composite Box Girders under Eccentric Lane Loading

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Abstract: To accurately analyze the distortional effects of corrugated steel web composite box girders with mid-span diaphragms under eccentric lane loading, the general solution of the distortional governing differential equation is derived via the energy variational method and expressed using Krylov functions. The anti-distortional effect of the mid-span diaphragm is equivalently modeled as a concentrated distortional moment, yielding an analytical solution for the distortional response. Validation through numerical examples and ANSYS finite element simulations confirms the method's accuracy.

Keywords: Composite box girder, distortion effect, energy variation method, mid-span diaphragms, Krylov function.

1. Introduction

Prestressed concrete box girder bridges have been widely adopted in modern bridge construction due to their superior strength, structural integrity, and crack resistance. With the development of new technologies and the introduction of advanced materials, the wall thickness of box girders has gradually decreased, leading to increasingly pronounced distortion effects under eccentric loading [1-3]. In recent years, the analysis of distortion effects in box girders has become a focal point of research both domestically and internationally [4-7]. Xu Xun et al [8] based on the mixed variational principle, developed a torsion-distortion analysis theory for thin-walled beams with hybrid open-closed cross-sections. They demonstrated that the warping moment inertia ratio of cantilever flange plates to the entire section significantly affects the distortional behavior of box girders. Wang Chenguang et al [9] investigated the impact of constrained torsion on box girder distortion effects under concentrated load, further elucidating the underlying coupling mechanism between torsional and distortional behaviors. The installation of internal diaphragms has been widely favored by many researchers as an effective measure to address this issue. Zhang Yuanhai [10] extended the initial parameter method by replacing the mid-span diaphragm with a corresponding constrained distortion moment, thereby deriving an analytical solution for simply supported box girders with mid-span diaphragms under uniformly distributed load.

In summary, existing studies on distortional effects of box girders have predominantly focused on conventional concrete-web designs under single-load conditions. However, composite box girders in practice are subjected to more complex loading patterns. This paper develops an analytical method for corrugated steel web composite box girders with mid-span diaphragms, specifically investigating their distortional behavior under eccentric lane loading.

2. Equivalent Cross-section and Distortional Displacement

2.1. Equivalent cross-section

Owing to its material properties, the corrugated steel web must first be equivalently modeled as an orthotropic plate for

distortional analysis, followed by conversion to an equivalent concrete slab. The transformed box-girder cross-section is illustrated in Figure 1.

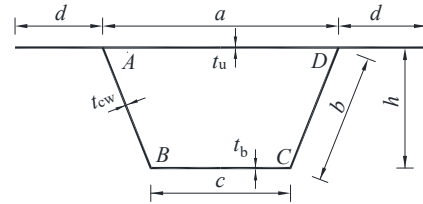


Figure 1. Cross Section of Trapezoidal Box Girder

The geometric configuration of the corrugated steel web is shown in Figure 2.

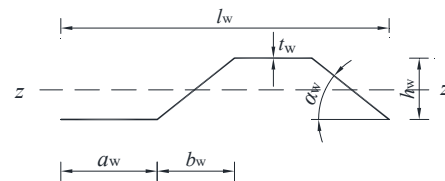


Figure 2. Geometry of Corrugated Steel Webs

The transverse apparent elastic modulus E_x of the corrugated steel web:

$$E_x = \frac{a_w + b_w \sec \alpha_w}{a_w + b_w} E_c \quad (1)$$

where: E_c is elastic modulus of steel.

Due to the accordion effect, the apparent axial elastic modulus of corrugated steel webs is significantly lower than that of flat steel plates, with a maximum value not exceeding 2% of the plate's modulus. The longitudinal stiffness can be entirely neglected when calculating axial forces in corrugated webs.

Based on the principles of material mechanics, the out-of-plane bending moment of inertia per unit length for

4. The Initial Parameter Solution for Composite Box Girders with Intermediate Diaphragms Under Lane Loading

The lane loading can be decomposed into a concentrated load M_d and a uniformly distributed load m_d over the full span. The anti-distortion effect of the mid-span diaphragm on the box girder is represented by a concentrated distortion moment L . Based on the initial parameter solution, the distortion displacement and internal forces at any section of the box girder can be expressed.

$$\begin{aligned} \gamma(z) = & \gamma_0 \Phi_1(\lambda z) + \frac{\gamma'_0}{\lambda} \Phi_2(\lambda z) - \frac{B_0}{\lambda^2 EI_\omega} \Phi_3(\lambda z) - \\ & \frac{M_0}{\lambda^3 EI_\omega} \Phi_4(\lambda z) + \frac{m_d}{EI_R} [1 - \Phi_1(\lambda z)] + \\ & \frac{L}{\lambda^3 EI_\omega} \Phi_4\left(\lambda z - \frac{\lambda l}{2}\right) + \frac{M_d}{\lambda^3 EI_\omega} \Phi_4[\lambda(z - z_i)] \end{aligned} \quad (11)$$

$$\begin{aligned} \gamma'(z) = & -4\lambda \gamma_0 \Phi_4(\lambda z) + \gamma'_0 \Phi_1(\lambda z) - \frac{B_0}{\lambda EI_\omega} \Phi_2(\lambda z) - \\ & \frac{M_0}{\lambda^2 EI_\omega} \Phi_3(\lambda z) + \frac{m_d}{\lambda^3 EI_\omega} \Phi_4(\lambda z) + \\ & \frac{L}{\lambda^2 EI_\omega} \Phi_3\left(\lambda z - \frac{\lambda l}{2}\right) + \frac{M_d}{\lambda^2 EI_\omega} \Phi_3[\lambda(z - z_i)] \end{aligned} \quad (12)$$

$$\begin{aligned} B_D(z) = & 4\lambda^2 EI_\omega \gamma_0 \Phi_3(\lambda z) + 4\lambda EI_\omega \gamma'_0 \Phi_4(\lambda z) + \\ & B_0 \Phi_1(\lambda z) + \frac{M_0}{\lambda} \Phi_2(\lambda z) - \frac{m_d}{\lambda^2} \Phi_3(\lambda z) - \end{aligned}$$

$$\frac{L}{\lambda} \Phi_2\left(\lambda z - \frac{\lambda l}{2}\right) + \frac{M_d}{\lambda} \Phi_2[\lambda(z - z_i)] \quad (13)$$

$$M_D(z) = 4\lambda^3 EI_\omega \gamma_0 \Phi_2(\lambda z) + 4\lambda^2 EI_\omega \gamma'_0 \Phi_3(\lambda z) -$$

$$4\lambda B_0 \Phi_4(\lambda z) + M_0 \Phi_1(\lambda z) - \frac{m_d}{\lambda} \Phi_2(\lambda z) -$$

$$L \Phi_1\left(\lambda z - \frac{\lambda l}{2}\right) + M_d \Phi_1[\lambda(z - z_i)] \quad (14)$$

where: l denotes the span length of the box girder, polynomial terms containing L are only considered when $z > l/2$; z_i is the application point of concentrated loads.

According to the deformation compatibility condition at the mid-span diaphragm section (where the distortional angle equals zero), the governing equation is derived as:

$$\gamma(l/2) = 0 \quad (15)$$

The four initial parameters are determined by the boundary conditions at both ends of the box girder:

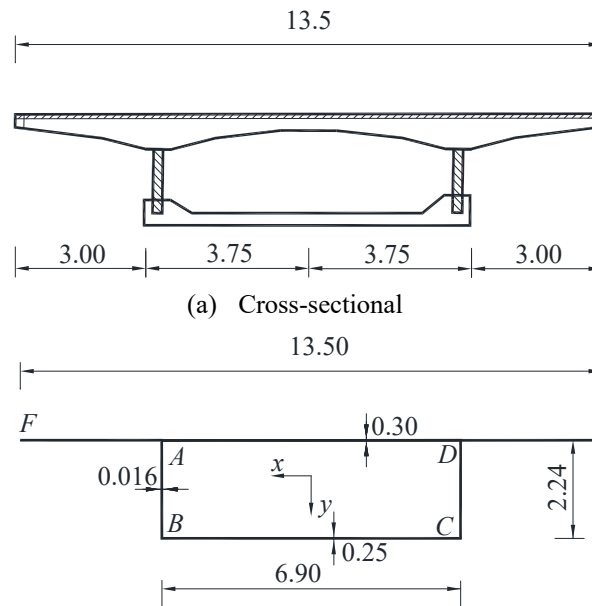
For fixed ends: $\gamma = \gamma' = 0$;

For simply-supported ends with diaphragms: $\gamma = \gamma'' = 0$;

For free ends with diaphragms: $\gamma = \gamma''' = 0$.

5. Numerical Case Analysis

The numerical example selects a constructed river-crossing corrugated steel web composite box girder bridge, adopting its side span length and mid-span cross-section to calculate distortional effects under lane loading. The actual bridge section and simplified section are shown in Figure 4.



(a) Cross-sectional
(b) Calculated Cross-section Diagram
Figure 4. Composite Box Girder

The simply-supported girder bridge has a span length of 50m, with diaphragms installed at both supports and the midspan. The corrugated steel webs utilize Grade 1600 steel with the following material properties: elastic modulus $E = 210$ GPa, shear modulus $G = 81$ GPa, and Poisson's ratio 0.3. The web geometry is defined by: $a_w = 215$ mm (corrugation amplitude), $b_w = 370$ mm (wavelength), $h_w = 220$ mm (projected height), and $t_w = 16$ mm (thickness). The concrete top and bottom flanges employ C50 concrete with $E = 35$ GPa, $G = 14.6$ GPa, and Poisson's ratio 0.2. The bridge is designed for Highway-Class I loading. To emphasize distortional effects, two traffic lanes are arranged transversely (see Fig. 5). The lane load consists of: concentrated load $P = 360$ KN positioned at $l/4$ (selected because the midspan diaphragm restricts distortional rotation), uniformly distributed load $q = 10.5$ KN/m applied along the entire span length.

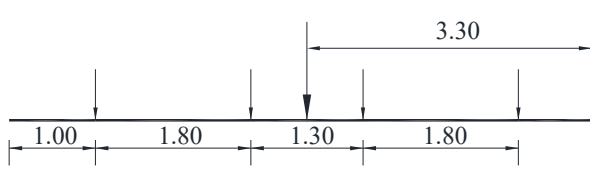


Figure 5. Load Arrangement Diagram for Lanes, Unit: (m)

As shown in Figure 5, the eccentricity under dual-lane loading configuration is $e = 3.3$ m. After comprehensive consideration of lane number and transverse reduction factors, the equivalent design eccentricity can be determined as: Concentrated Load:

$$P = 2p_k = 360000 \times 2 = 720000 \text{ N};$$

Uniformly Distributed Load:

$$q = 2q_k = 10500 \times 2 = 21000 \text{ N};$$

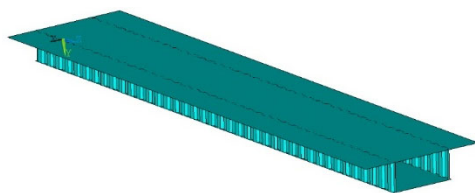
Concentrated Distortional Load:

$$M_d = \frac{Pe}{2} = \frac{720000 \times 3.3}{2} = 1188000 \text{ N} \cdot \text{m};$$

Distributed Distortional Load:

$$m_d = \frac{qe}{2} = \frac{21000 \times 3.3}{2} = 34650 \text{ N} \cdot \text{m}.$$

To verify the correctness of the formulas derived in this paper, a numerical simulation was conducted using the general finite element software ANSYS, with shell element Shell 181 used to create the model, as shown in Figure 6.



(a) Finite Element Model



(b) Mesh Configuration

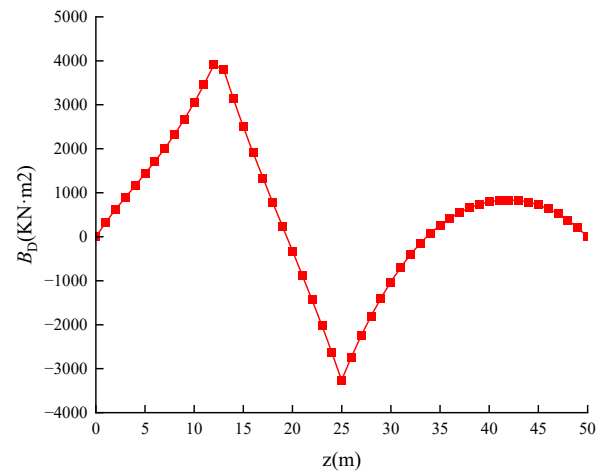
Figure 6. Finite Element Model and Mesh Configuration.

The cross-sections at $l/2$ and $l/4$ were selected as key sections, and the distortion warping normal stress at point A , located at the junction of the top plate and the web, was compared. As shown in Table 1, the analytical solutions are generally in good agreement with the ANSYS finite element numerical results, indicating that the derived expressions in this paper are correct and reasonable.

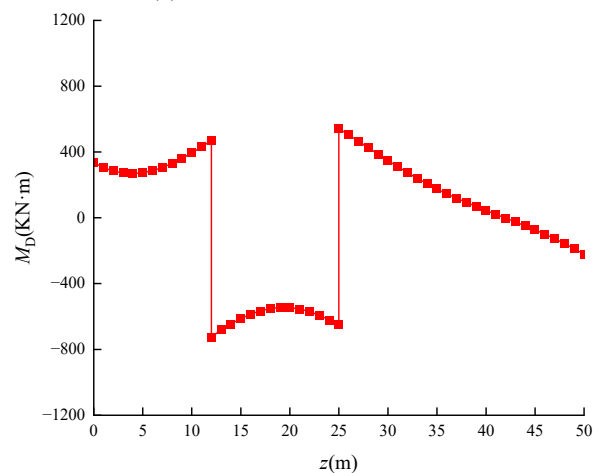
Table 1. Comparison of distortion stresses at corner A (kPa).

Section	Analytical	ANSYS	Error (%)
$l/2$	-145.97	-157.69	8.10
$l/4$	1370.44	1454.48	-6.13

Figure 7 illustrates the distortional bimoment and distortional moment distributions along the span of the composite box girder under lane loading.



(a) Distortional Bimoment



(b) Distortional Moment

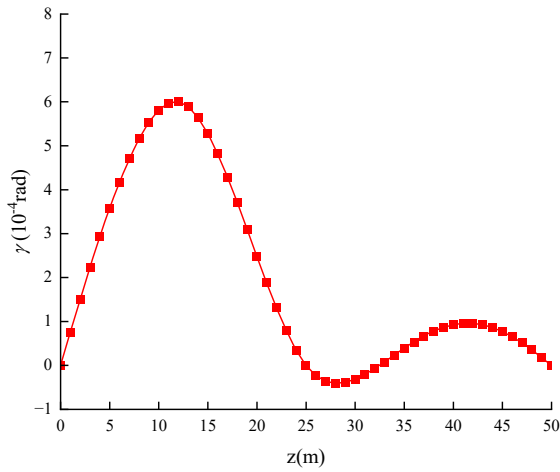
Figure 7. Distortional Bimoment and Distortional Moment

As observed in Fig.7(a), the distortional bimoment under lane loading exhibits inflection points at both the concentrated load location and midspan diaphragm, with distinct trend variations on either side. Across the entire span, the left half-span shows abrupt changes while the right half-span demonstrates relatively smoother variations, peaking at the quarter-span location.

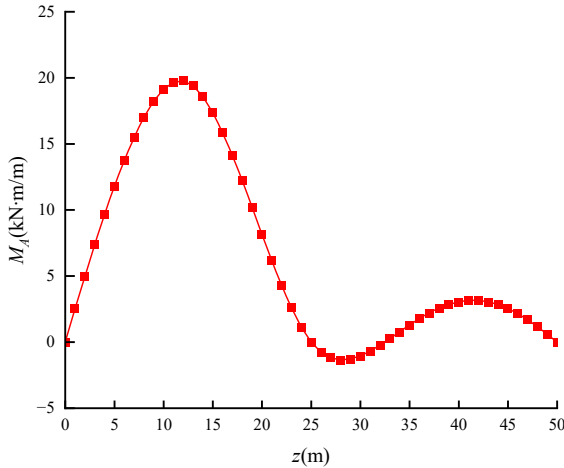
Fig.7(b) reveals that the distortional moment undergoes sudden changes at the concentrated load and midspan

diaphragm positions. The right half-span displays gentler variations due to the absence of concentrated loads and diaphragms.

Figure 8 presents the variations of distortional angle and transverse bending moment at Point *A* along the span of the composite box girder under lane loading.



(a) Distortional Angle



(b) Transverse Bending Moment at Point *A*

Figure 8. Distortional Angle and Transverse Bending Moment at Point *A*

As shown in Fig.8, under lane loading, both the distortional angle and transverse bending moment at Point *A* peak in the half-span containing the concentrated load, reaching their maximum values at the load application point with abrupt variations. In contrast, the opposite half-span exhibits significantly smaller peak magnitudes and gentler gradients.

6. Conclusion

(1) This study proposes an equivalent constrained distortional moment to replace the conventional anti-distortional effect of internal diaphragms. Through rigorous mathematical derivation, we obtain the analytical solution for distortional behavior of corrugated steel web composite box

girders with midspan diaphragms under eccentric lane loading. A corresponding ANSYS finite element model is established, and the computational results of our method show excellent agreement with numerical simulations, demonstrating the reliability of the derived formulations.

(2) Under lane loading, the distortional bimoment exhibits inflection points at both the concentrated load location and midspan diaphragm, showing opposite variation trends on either side. The left half-span displays abrupt changes while the right half-span varies more gradually, peaking at the quarter-span. Similarly, the distortional moment undergoes sudden transitions at these critical sections but remains smooth in the right half-span due to the absence of concentrated loads and diaphragms. Both the distortional angle and transverse bending moment at Point *A* demonstrate sharp peaks in the loaded half-span, reaching maximum values with rapid fluctuations, whereas the unloaded half-span exhibits significantly diminished and slowly varying responses.

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