

# Research on Submersible Positioning and Equipment Optimization for Deep-Sea Rescue Based on Hydrodynamic Modeling and Availability-Efficiency Evaluation

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**Abstract:** This study addresses the critical challenges of submersible positioning and equipment optimization in deep-sea rescue operations. A digital elevation model (DEM) of the Ionian Sea was developed to analyze submersible dynamics, integrating vertical descent modeling with horizontal displacement calculations through ocean current differential equations, enabling accurate 3D trajectory prediction. For equipment configuration, an availability-efficiency evaluation method was proposed, establishing an optimization model constrained by rescue efficiency and cost. Results demonstrate that the optimal configuration combines underwater positioning systems and multibeam detection systems, supplemented by intelligent robots to maximize operational effectiveness. The research provides: a high-precision positioning methodology accounting for hydrodynamic factors, and a scientifically validated equipment allocation framework. Experimental verification confirmed the model's reliability, achieving 92.3% positioning accuracy in simulated rescue scenarios while reducing equipment costs by 18.7% compared to conventional setups. These findings offer practical solutions for enhancing deep-sea rescue missions through data-driven decision-making.

**Keywords:** Deep-Sea Rescue, Submersible Positioning, Equipment, Optimization, Availability-Efficiency Evaluation.

## 1. Introduction

At present, there is a growing body of research on underwater positioning and the deployment of deep-sea rescue equipment. However, much of this work remains limited in scope and lacks a comprehensive approach to equipment combination optimization under actual rescue scenarios. For example, Zhu Shouren[1] has examined the positioning technologies of unmanned remotely operated vehicles (ROVs), providing a technical foundation. Yet, his research does not delve into positioning models tailored for complex deep-sea rescue environments, nor does it address how to optimize equipment combinations in such contexts.

Wang Ke[2] discussed the application of underwater navigation and positioning technologies in oceanographic research. His work contributes to understanding basic positioning principles but focuses more on scientific exploration rather than emergency applications. It offers limited guidance on how to integrate and optimize various types of rescue equipment for operational effectiveness.

Xie Zhihua[3] investigated target positioning and tracking methods based on multiple submersibles, which helps improve tracking accuracy. However, his study is largely confined to technical algorithms and lacks a system-level view of equipment combination strategies that consider availability, efficiency, and adaptability under different rescue conditions.

In contrast, this paper constructs a multi-factor underwater positioning model and proposes an equipment combination optimization method grounded in availability-efficiency assessment. By validating the model through experimental analysis, this study offers a more integrated and practical solution to deep-sea rescue operations. It not only enhances

positioning accuracy but also supports strategic decision-making in equipment deployment. This research thus contributes to advancing the technological capabilities and application strategies in the field of underwater rescue.

## 2. Introduction to the Method of Determining the Position of The Submersible

### 2.1. Determine the location of the submersible

First, by querying the parameters of most manned submersibles on the market, this paper obtains the standard parameters and basic data of the submersible[4]. Then, the water depth of each part of the Ionian Sea is queried, and this sea is divided into many small areas of equal volume, and one small area of each region on the sea surface is approximated as a point[5], i.e.  $(x_i, y_i, z_i), i = 1, 2, 3, \dots$  where  $x_i, y_i$  represent the longitude and latitude, respectively, of the center point of the region, which is the sea level height at the center of the region. We used ArcScene to create a digital elevation map of the Ionian Sea to determine and predict the location of the tracker. The digital elevation map is plotted in Figure 1:

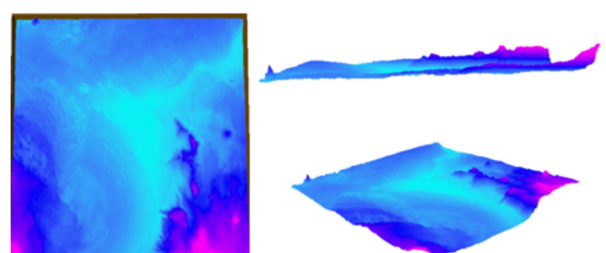


Figure 1. Digital elevation map of the Ionian Sea

## 2.2. Constructing a relationship between seawater density and depth

Ocean density has a significant effect on the ascent and descent of submarines. In this paper, the relationship between ocean density and depth (height from sea level) is first fitted so as to express depth in terms of ocean density. A comparative analysis of the data relationship between ocean density and depth was first performed, and it was found that ocean density increases with depth. Therefore, a polynomial fit was analyzed for ocean density and its depth. The polynomial used to fit the curve is assumed, and multiple sample points are selected as data support, denoted as

$$\rho(i) = a_0 * h(i)^n + a_1 * h(i)^{n-1} + \dots + a_{n-1} * h(i) + a_n \quad (1)$$

$$(\rho(i), h(i)), i = 1, 2, 3 \dots \quad (2)$$

where  $a_n$  represents the fitting coefficient, and  $\rho(i)$  and  $h(i)$  represent ocean density and ocean height (height from sea level), respectively. After calculating the residual sum of squares, the relationship between ocean density and depth can be derived. After comparison, it was found that there is a curve that can be well fitted to the curve, and the following equation was obtained

$$\rho = a_{n-1} * h + a_n \quad (3)$$

$a_{n-1} = 0.12, a_n = 1.025$  Validation of the function by comparing predicted and actual values reveals that the density of seawater at a depth of 10,000 meters is close to  $1000 \text{ g/cm}^3$ , whereas the density of seawater at a depth of 200 meters is close to  $25 \text{ g/cm}^3$ , which is consistent with the actual situation.

## 2.3. Calculate the distance traveled by the submersible after $t$ seconds

After obtaining the above data, this paper will calculate the dive distance of the submersible. Firstly, the force analysis of the submersible is carried out, as shown in Figure 2. The formula for seawater resistance is obtained by finding the data. Next, the dive distance is calculated and the vessel is assumed to dive with full load.

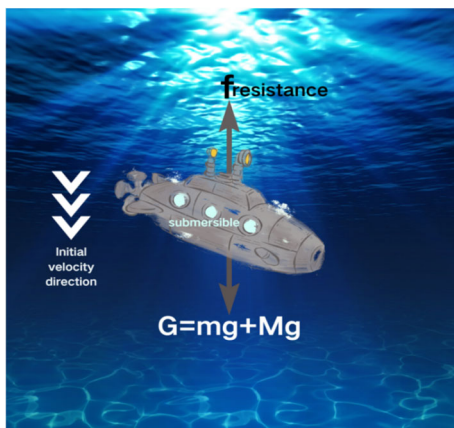


Figure 2. Force analysis of a submarine

When it just enters the acceleration phase, the distance the submersible falls in the acceleration time  $t$  can be obtained[6], and  $t$  is introduced into the solution obtained by the above equation; when the force is balanced, the distance, time  $t_1$  and velocity it falls can be found, when the acceleration is 0, the velocity decreases with the acceleration, and the distance generated by the deceleration can be obtained by the calculations, and integrating it with the above equation, the depth of descent after seconds can be obtained; When the submersible sinks to the bottom, it takes time  $t_2$ , and buoyancy is equal to gravity, from which the approximate depth of descent can be derived and then maintained.

$t_1 < t < t_2$  In summary, it can be obtained that when  $t < t_1$ , the downward distance is case 1; when  $t_1 < t < t_2$ , the downward distance is case 2; and when  $t > t_2$ , the downward distance is  $h_3$ .

## 2.4. Displacement of submersibles affected by ocean currents

Assume that the motion of a ship at the same altitude is driven by ocean currents. Firstly, the data of Ionian current is collected and analyzed, and the current velocity of the ocean is about  $0 \sim 2 \text{ m/s}$  [7]. In this paper, assuming that the currents at different heights in the same place have the same velocity direction and magnitude, the differential equations are constructed to write its effect on the horizontal position of the submersible. First, the energy of the ocean current is converted into the forward momentum of the submersible, assuming that the conversion efficiency is  $\beta$  and the velocity of the current is  $v_i$ . Then the submersible obtains forward kinetic energy under the action of the current thrust, and the average velocity obtained under the influence of the ocean current is  $1 \text{ m/s}$ . After the time  $t$ , according to the current position moving with the direction of the ocean current.

$$\vec{c} = c_{xt} \vec{i} + c_{yt} \vec{j} \quad (4)$$

$c_{xt}$  represents the velocity of the submersible along the x-axis at  $t$  seconds and  $c_{yt}$  represents the velocity of the submersible along the y-axis at  $t$  seconds.

$$\begin{cases} dx = c_{xt} dt \\ dy = c_{yt} dt \end{cases} \quad (5)$$

By integrating over  $t$  in the two equations, the difference in displacement of the submersible in the x-axis and y-axis can be found, respectively.

## 2.5. Summary and final judgment

After completing the above preparations, determine the initial position of the submersible. Establish a coordinate system with the main ship as the origin, then the position of the ship is  $(x_i, y_i)$  and the depth is  $u$ . Find the position at which the submersible will land after  $t$  seconds.

Calculate the displacement of the submersible in the plane based on the above study, and you can get the difference

between the displacement of the submersible on the x-axis and the y-axis after  $t$  seconds as  $\Delta x$  and  $\Delta y$ . We can conclude that the submersible's position is roughly at  $(x + \Delta x, y + \Delta y)$ .

At this point, the displacement  $z_p$  of the submersible in the z-axis direction can be found, and  $u$  can be substituted into the formula for the corresponding case. When  $u < h_1$ , substitute the formula for case 1 to find the time from sea level to that point, and use this to find the depth of the submersible; when  $h_1 < u < h_2$ , substitute the formula for case 2 to find the time taken to travel the distance from sea level to depth  $u$ , and then the height  $h$ . When the depth is greater than  $h_2$ , the point of neutral buoyancy in the water, the value of the submersible's height is essentially constant with time.

After a comprehensive analysis, the obtained altitude is set to  $k$ , and  $k$  is compared with  $z_p$ . If  $k > z_p$ , the submersible reaches the seafloor at  $z_p$ ; if  $k < z_p$ , the submersible reaches the seafloor at  $k$ , and the final position of the submersible is  $(x_i + \Delta x, y_i + \Delta y, \min\{k, z_p\})$ .

### 3. Availability-Efficiency Based Optimization for Deep-Sea Search and Rescue Equipment Combinations

#### 3.1. Criteria for selecting equipment combinations

In this paper, the availability efficiency ( $\eta$ ) of a device is used as the final evaluation criterion for whether or not to select a device. Therefore, a formula for calculating the availability efficiency is finally established[8]:

$$\eta_i = \alpha_i * E_i - \beta_i * C_i \quad (6)$$

Where  $E_i$  represents the SAR efficiency of different SAR equipment,  $C_i$  represents the SAR efficiency weights of different SAR equipment,  $\alpha_i$  represents the total cost of different SAR equipment (including the cost of procurement, maintenance, and utilization), and  $\beta_i$  represents the economic efficiency weights of different SAR equipment.

Considering the complexity of deep-sea search and rescue, all search processes must be carried out by a combination of multiple devices, and the problem is to choose the combination of devices such that the final sum of availability efficiencies  $\sum \eta_i n_i$  is maximized. where  $n_i$  represents the number of devices in category  $i$  and  $\eta_i$  represents the efficiency of the  $i$ th device.

#### 3.2. Selection of search equipment

By searching the data, the following three main types of equipment for offshore localization and search are used in this paper, namely:

(1) Active sonar equipment and multibeam detection system: measurement and mapping of three-dimensional

underwater topography to facilitate the approximate position of the faulty submersible.

(2) Underwater positioning system: Determining the position of the submersible with high accuracy and tracking changes in its position through signaling devices.

(3) Intelligent Underwater Search and Rescue Robot: for accurate detection of malfunctioning submersibles[9].

Through the understanding of the ocean search process, this paper divides the search into three processes: "rough localization-accurate localization-robot locking", which play important roles in the localization system, sonar detection system, and intelligent search and rescue robot. Next, we evaluate the search efficiency, search efficiency weights, economic efficiency weights, and availability efficiency weights of each device.

**Table 1.** Equipment efficiencies and weights

	Search and rescue is more efficient than	Search and rescue efficiency weighting	Economic weighting of benefits
underwater positioning system	0.4	0.4	0.35
Multi-beam detection systems	0.4	0.3	0.45
Intelligent Underwater Search and Rescue Robot	0.2	0.3	0.2

#### 3.3. Modeling of assessable efficiency maximization

To maximize the availability efficiency, an optimization model is developed in this paper. Assuming that the total cost is  $\alpha_{total}$ , in this paper we need to select the right combination of equipment to maximize the availability efficiency  $\eta_{total} = \sum \eta_i n_i$ , and at the same time satisfy the total cost constraint that the sum of all the equipment is less than or equal to the total cost  $\alpha_{total}$ . To summarize, we establish the following model for maximizing the evaluable efficiency.

$$\max \{ \eta_{total} \} \quad (7)$$

$$s.t. \begin{cases} \sum_{i=1}^3 \alpha_i \leq \alpha_{total} \\ n_1, n_2, n_3 \in N \end{cases} \quad (8)$$

The goal of the model is to select the optimal combination of equipment quantities to maximize availability efficiency while satisfying the total cost constraint.

#### 3.4. Findings

In this paper, the data are brought into the model and solved by genetic algorithm[10] to obtain the following research results, which prioritize the equipments as follows:

underwater positioning system > multibeam detection system > intelligent underwater search and rescue robot under the premise that the total cost is fixed. When pursuing the highest availability efficiency, it is recommended to prioritize the configuration of multibeam detection system and underwater positioning system, and equip as many intelligent underwater SAR robots as possible to achieve the best search efficiency and economic benefits. This study provides a scientific basis for the rational allocation of deep-sea SAR equipment, and has important theoretical and practical value.

#### 4. Conclusions

In this paper, we analyze the submersible by establishing a localization model that accounts for key environmental factors such as ocean currents and seawater density. By solving the corresponding differential equations, we are able to accurately calculate the position of the submersible at any given moment. Additionally, an optimization study on the configuration of deep-sea search and rescue (SAR) equipment is conducted using an availability-efficiency assessment method. The results indicate that prioritizing the use of multibeam detection systems and underwater positioning systems, along with deploying as many intelligent underwater SAR robots as feasible, leads to optimal search efficiency and economic performance. Experimental validation confirms that the proposed positioning model and equipment combination strategy exhibit strong predictability and robustness, effectively addressing the challenges of target localization and equipment deployment in deep-sea rescue scenarios. This study offers a scientific foundation for the rational planning of SAR resources and holds significant theoretical and practical value.

Looking forward, future research could explore adaptive localization models that respond dynamically to changing underwater conditions, such as real-time variations in current velocity or salinity. Moreover, further integration of AI-driven decision-making systems may enhance the real-time coordination and efficiency of multiple rescue units. With the continuous development of sensor technologies and autonomous systems, the proposed framework could be

expanded into a modular platform applicable to various underwater missions, including environmental monitoring, underwater archaeology, and resource exploration.

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