

On the Solution of Five-position Synthesis for Stephenson II Six-bar Linkages Based on Solution Region Synthesis Method

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Abstract. In this paper, the five-position synthesis of Stephenson II six-bar linkages is studied based on the solution region synthesis methodology, in addition, a universal and effective defect determination method for complex linkages is presented. Firstly, five-position synthesis equations of Stephenson II six-bar linkages were established, then the equations are solved in Bertini. A numerical example of five-position synthesis for Stephenson II six-bar linkages was given to elaborate the method and process of establishing the solution region and the method for identifying defect linkages of complex six-bar linkages, and the correctness and effectiveness of the method for identifying defect linkages were proved. Finally, the feasible solution region of the five-position synthesis of six-bar linkages is presented.

Keywords: Planar linkages; circuits; branches; linkage synthesis.

1. Introduction

1.1. Application overview

The development of technology pushed the application for linkages in the fields of bionic robot, manipulator, medical rehabilitation machinery, aerospace and industrial production. Inspired by the jumping way of insects, Li et al. [1] designed a mini jumping robot, as shown in Fig. 1. Zhang et al. [2] designed a locust bionic robot based on a six-bar linkage with 1-DOF. The robot can keep its body posture almost unchanged during jumping, as shown in Fig. 2.

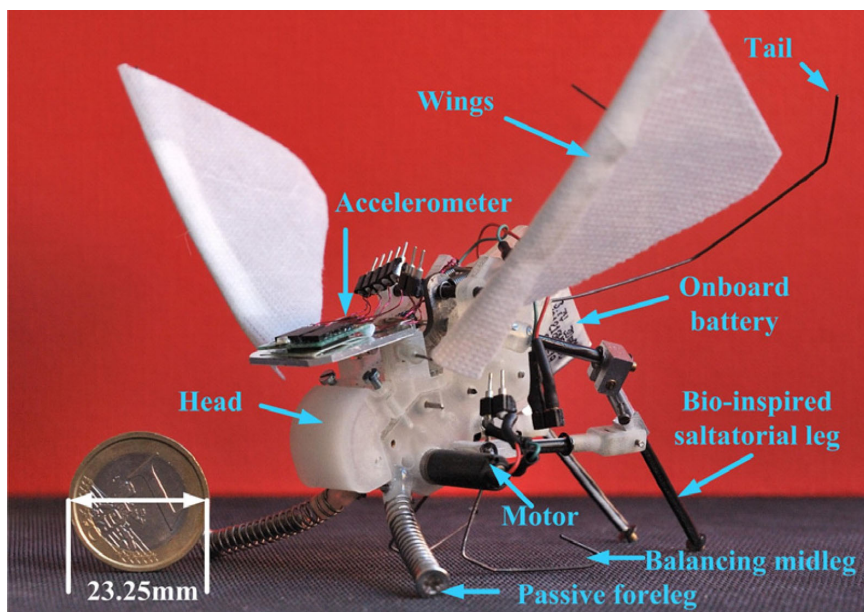


Figure 1. Bionic leafhopper

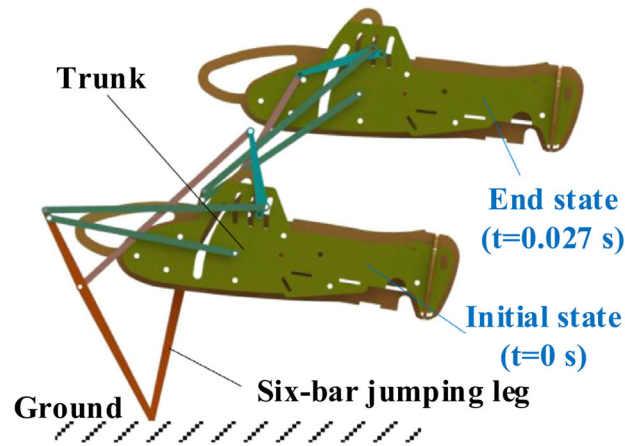


Figure 2. A type of Bionic locust

Gao and Zhang [3] designed a type of manipulator based on 1-DOF six-bar linkages according to the structure of human finger joints and the movements of fingers in the process of grasping objects. Rodríguez et al. [4-6] designed a three-finger manipulator based on 1-DOF six-bar linkages, as shown in Fig. 3. Wu et al. [7] designed an underactuated mechanical finger based on a six-bar linkage with 1-DOF, which can grasp objects of various sizes and shapes. However, the size of the bionic finger is quite than that of the real finger. In order to get the bionic finger of which size closes to the real finger, some scholars later designed and manufactured various mechanical fingers and rehabilitation machines using six-bar linkages with 1-DOF, which are not only small in size, but also fit well with real fingers [8-9], as shown in Fig. 4. Haldane and Plecnik et al. [10,11] presented 1-DOF eight-bar linkage leg capable of vertical jumping as shown in Fig. 5. Park and Jeon et al. [12-14] designed an industrial clamping device which can tighten steel plates of different thicknesses based on the six-bar linkage, as shown in Fig. 6. In the future, 1-DOF linkages can also be used for planetary exploration robots, because mechanical legs can more easily overcome obstacles and adapt to more terrain than wheeled or tracked exploration robots. With the development of modern machinery, linkage machineries will be more and more widely used.

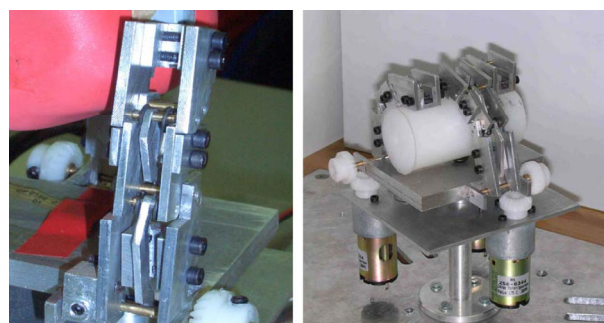


Figure 3. Six-bar manipulators

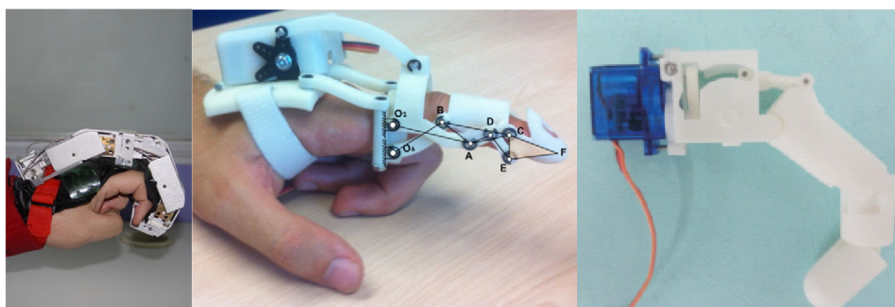


Figure 4. Exoskeleton rehabilitation fingers and robotic fingers

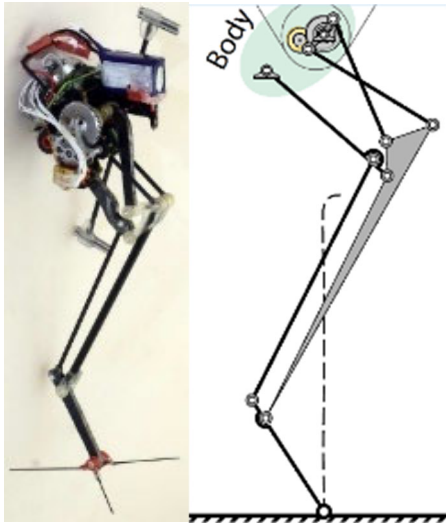


Figure 5. Vertical jump mechanical leg

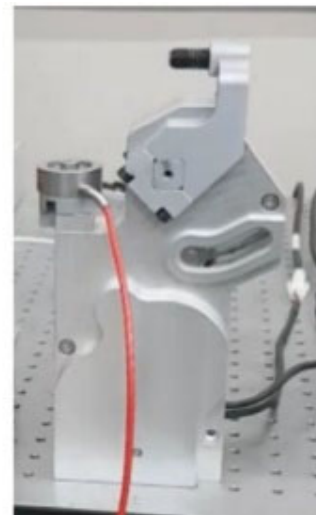
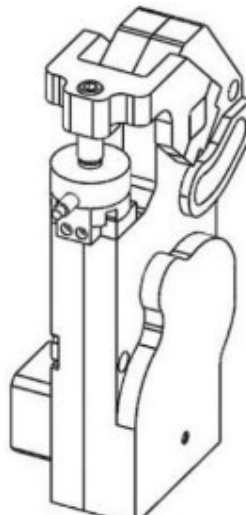


Figure 6. Large stroke industrial finger clamp device

1.2. Synthesis methods of linkages

In 1876, Burmester proposed the geometric method for linkage synthesis and proposed the famous Burmester Curve through the research of 3~5 finitely separated positions of four-bar linkages. References [15,16] presented the solution region synthesis method of four position synthesis of four-bar linkages based on the Burmester Curve. After that, the solution region synthesis method was widely used in the motion generation of planar four-bar linkages [17-20]. Later, Han and Yang [21,22] studied the four-position synthesis of planar six-bar linkages based on the solution region synthesis method, and used the Jacobian matrix determinant to determine whether six-bar linkages composed of the four-bar Assur Group have branch defects. However, complex linkages often have multiple branches in one circuit, and only the signs of Jacobian matrix determinants of two adjacent branches are different. Since, it does not guarantee that all of the linkages with branch defects are eliminated. Therefore, a universal and effective method to determine defect linkages for complex linkages is proposed in this paper. This method can not only solve the problems above, but also accurately determine whether the linkage has circuit defects or branch defects. Since branch defects are not fatal to the motion of the linkage, the linkage without circuit defects but with branch defects can be retained in the feasible solution region according to the specific situation.

At most five positions can be given for motion generation of four-bar linkages and just a finite number of linkages can be obtained. Owing to more design parameters, an infinite number of linkage solutions can be obtained theoretically for the five-position synthesis of six-bar linkages, which can provide a broader choice space for the design. Based on the solution region synthesis method, the five-position synthesis of Stephenson II six-bar linkages composed of four-bar Assur group II is studied in this paper.

2. Formula of Five Position Synthesis

Figure 7 shows the schematic of a Stephenson II six-bar linkage moving from its initial position to position j . The linkage is located on a complex plane, and the locations of joints are represented as complex points in the complex plane, where r and i represent real and imaginary units respectively. The dotted lines represent the initial configuration of the linkage. P_1 represents the first position of the rigid body and P_j represents the position j of the rigid body. β_j , θ_j , φ_j , μ_j and λ_j are the rotation angles of the linkage moving from initial position to position j .

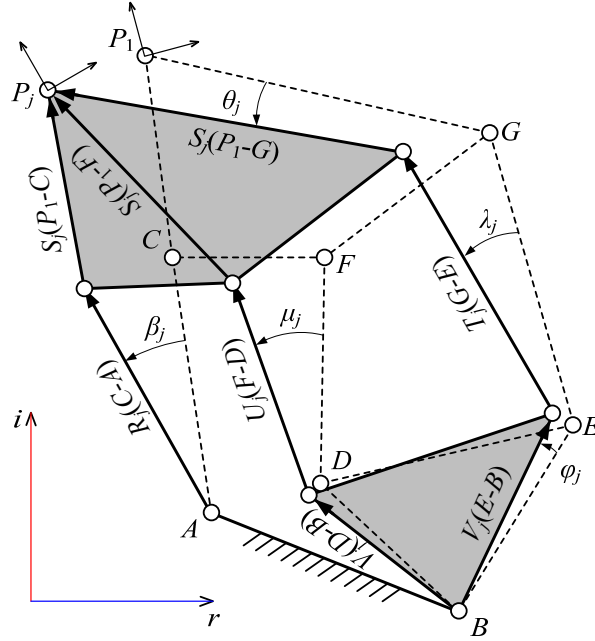


Figure 1. Schematic of the six-bar linkage moving from initial position to position j

When the six-bar linkage moves from initial position 1 to position j , three loop equations can be obtained:

$$A + R_j(C - A) + S_j(P_1 - C) - P_j = 0 \quad (1)$$

$$B + V_j(D - B) + U_j(F - D) + S_j(P_1 - F) - P_j = 0 \quad (2)$$

$$B + V_j(E - B) + T_j(G - E) + S_j(P_1 - G) - P_j = 0 \quad (3)$$

where $R_j = e^{i\beta_j}$, $S_j = e^{i\theta_j}$, $U_j = e^{i\mu_j}$, $T_j = e^{i\lambda_j}$, $V_j = e^{i\phi_j}$.

Conjugate equations of the loop equations should be introduced to synthesize linkages in the complex plane. Conjugate equations of the three equations above are as follows:

$$A^* + (C^* - A^*)/R_j + (P_1^* - C^*)/S_j - P_j^* = 0 \quad (4)$$

$$B^* + (D^* - B^*)/V_j + (F^* - D^*)/U_j + (P_1^* - F^*)/S_j - P_j^* = 0 \quad (5)$$

$$B^* + (E^* - B^*)/V_j + (G^* - E^*)/T_j + (P_1^* - G^*)/S_j - P_j^* = 0 \quad (6)$$

S_j is the known parameter in the position synthesis, and position variables $A, A^*, B, B^*, C, C^*, D, D^*, E, E^*, F, F^*, G, G^*$ and rotation angle variables R_j, T_j, U_j, V_j are the variables need to be solved. Position variables $A, A^*, B, B^*, C, C^*, D, D^*, E, E^*, F, F^*, G, G^*$ are considered as 14 independent variables in the isotropic coordinate system. Only when A and A^* , B and B^* , C and C^* , D and D^* , E and E^* , F and F^* , G and G^* all satisfy the conjugate relation at the same time, that is, the real parts are equal and the imaginary parts are opposite each other, the linkage solution is a solution with physical significance [37].

In order to reduce the number of variables and improve numerical solving speed, the equations above should be simplified. Coupling Eq. (1) and Eq. (4), the rotation angle variable R_j can be eliminated, and the following equation can be obtained:

$$S_j(C-A)(C^*-A^*) - ((P_j-A) - S_j(P_1-C))(S_j(P_j^*-A^*) - (P_1^*-C^*)) = 0 \quad (7)$$

Coupling Eq. (2) and Eq. (5), the rotation angle variable U_j can be eliminated, and the following equation can be obtained:

$$S_j V_j (F-D)(F^*-D^*) - ((P_j-B) - V_j(D-B) - S_j(P_1-F)) (S_j V_j (P_j^*-B^*) - S_j(D^*-B^*) - V_j(P_1^*-F^*)) = 0 \quad (8)$$

Coupling Eq. (3) and Eq. (6), the rotation angle variable T_j can be eliminated, and the following equation can be obtained:

$$S_j V_j (G-E)(G^*-E^*) - ((P_j-B) - V_j(E-B) - S_j(P_1-G)) (S_j V_j (P_j^*-B^*) - S_j(E^*-B^*) - V_j(P_1^*-G^*)) = 0 \quad (9)$$

where $j = 2, 3, 4, 5$.

Interestingly, looking at the Eq. (7), it can be found that it is actually the five-position synthesis equation of four-bar linkages. In other words, the formulas of position synthesis are common to the five-position synthesis of four-bar linkages and Stephenson II-B six-bar linkages. If four-bar linkages can satisfy design requirements, just solve Eq. (7). If linkages which meet design requirements cannot be found in four-bar linkages, six-bar linkages can be obtained by solving Eqs. (7) ~ (9) at the same time.

For five-position synthesis, the set of synthesis equations contains a total of 12 equations, which include 18 variables, namely $A, A^*, B, B^*, C, C^*, D, D^*, E, E^*, F, F^*, G, G^*, V_j (j=2, 3, 4, 5)$. Therefore, the values of six variables must be given to solve the 12 equations. In this paper, values of position variables B, B^* and rotation angle variables V_2, V_3, V_4 and V_5 are given. It is worth noting that although B and B^* are treated as two independent variables during numerical solving, the conjugate relationship of the two variables is preserved when the values of B and B^* are given in order to guarantee the physical meaning of the given variables.

Different values of B and B^* will result in different linkages. In theory, as long as enough number of values of B and B^* are given, an infinite number of linkages can be obtained which can pass through the given positions. Thus, it is possible to find more linkages without defects that can meet design requirements. Actually, an infinite number of linkages can be obtained by giving different values of V_2, V_3, V_4 and V_5 . In order to facilitate the study, the solution region for five-position synthesis of Stephenson II-B six-bar linkages is established through giving different locations of hinge point B .

3. Numerical example

A five-position synthesis of Stephenson II-B six-bar linkages example based on the solution region method is presented, and the given five positions are shown in Table 1.

Table 1. Five positions

No.	$r(\text{real})$	$i(\text{imaginary})$	Position angle (rad)
1	54.32	-122.9442	1.65994
2	70.22	-93.41596	1.84462
3	57.31	-28.08753	1.63798
4	39.43	23.51591	1.30471
5	10.95	42.70563	1.01899

For five-position synthesis, the set of synthesis equations contains a total of 12 equations, which include 18 variables. Because the number of equations is six lower than the number of variables, values of six variables need to be given to solve the equations. In this paper, values of position variables B, B^* and rotation angle variables V_2, V_3, V_4 and V_5 are given. If the step size of given values is infinitesimal or the value range is large enough, an infinite number of solutions can be obtained in theory. In order to facilitate the trials, eleven sets of values of B, B^* and a set of values of V_2, V_3, V_4, V_5 are given, as shown in Table 2. The conjugate relationship of values of B, B^* is preserved in order to guarantee the physical meaning. In Table 2, imaginary parts of variables B, B^* are zero. $V_j = e^{i\phi_j}$ ($j=2, 3, 4, 5$) are complex exponentials, and the values given in Table 2 are expressed by trigonometric functions.

Table 2. The given values of $(B, B^*), V_2, V_3, V_4, V_5$

variables	values
(B, B^*)	(300, 300) (320, 320) (340, 340) (360, 360) (380, 380) (400, 400) (420, 420) (440, 440) (460, 460) (480, 480) (500, 500)
V_2	0.990041-0.140779i
V_3	0.945867-0.324553i
V_4	0.914472-0.404649i
V_5	0.897412-0.441194i

After preliminary analysis, four defect-free six-bar linkages with cranks are obtained and they are obtained when the given value of (B, B^*) is (400, 400). This means that it may be easier to obtain defect-free linkages with cranks when the values of (B, B^*) near to (400, 400). Therefore, other 24 sets of values of (B, B^*) are given around (400, 400) in an attempt to find more linkage solutions without defects and with cranks. 24 sets of values of (B, B^*) are shown in Table 3.

Table 3 given values of (B, B^*)

values
(398, 398) (398+1i, 398-1i) (398+2i, 398-2i) (398-1i, 398+1i) (398-2i, 398+2i)
(399, 399) (399+1i, 399-1i) (399+2i, 399-2i) (399-1i, 399+1i) (399-2i, 399+2i)
(400+1i, 400-1i) (400+2i, 400-2i) (400-1i, 400+1i) (400-2i, 400+2i)
(401, 401) (401+1i, 401-1i) (401+2i, 401-2i) (401-1i, 401+1i) (401-2i, 401+2i)
(402, 402) (402+1i, 402-1i) (402+2i, 402-2i) (402-1i, 402+1i) (402-2i, 402+2i)

After eliminating linkages with circuit defects using the defect determinant method proposed, the solution region containing 107 linkages without circuit defects is obtained. Then the solution region containing 40 linkages without circuit defects and branch defects is obtained after eliminating linkages with branch defects, as shown in Fig. 8. Since branch defects are not fatal to the movement of linkages, linkages with branch defects but no circuit defects can also be used as alternative design. Among 40 defect-free linkages, there are 24 linkages with cranks as shown by red points in Fig. 8.

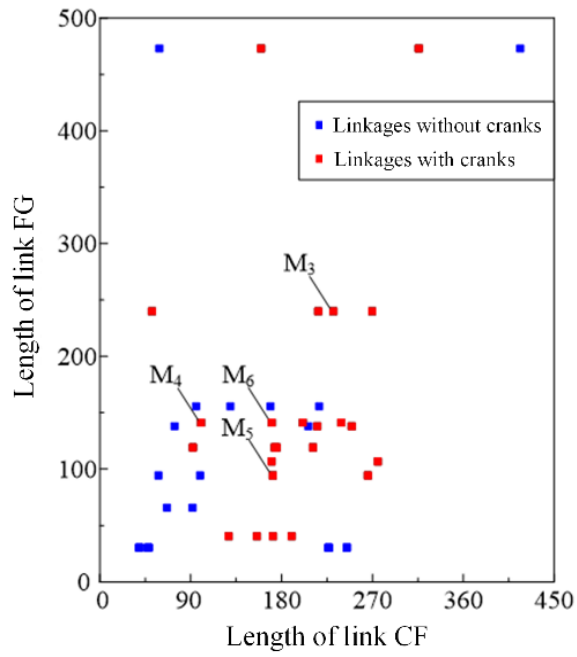


Figure 8. The solution region with linkages without defects

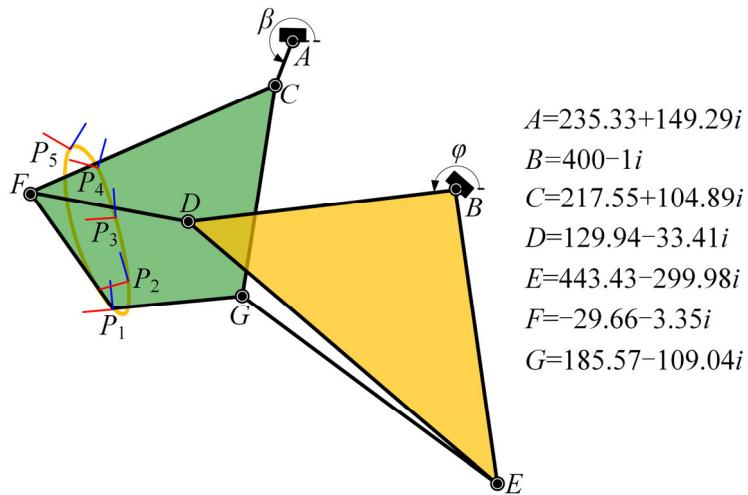


Figure 9.the linkage corresponding to point M3

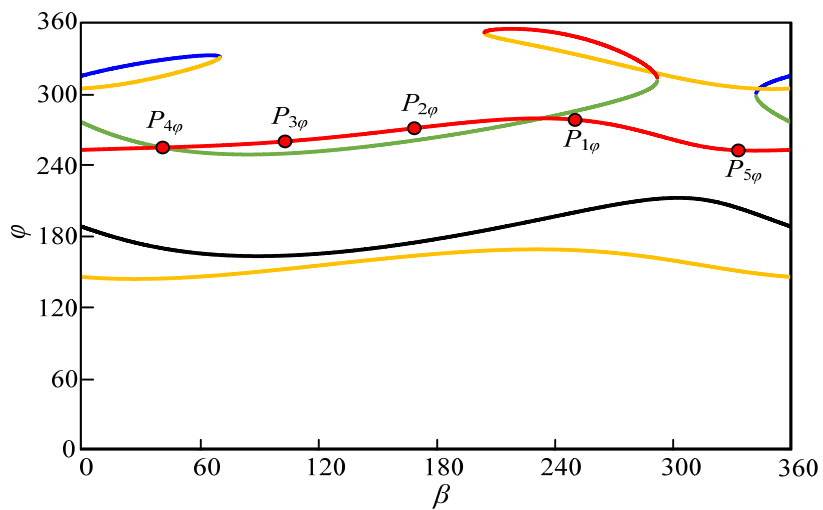


Figure 10. Angle displacement curves of point M3

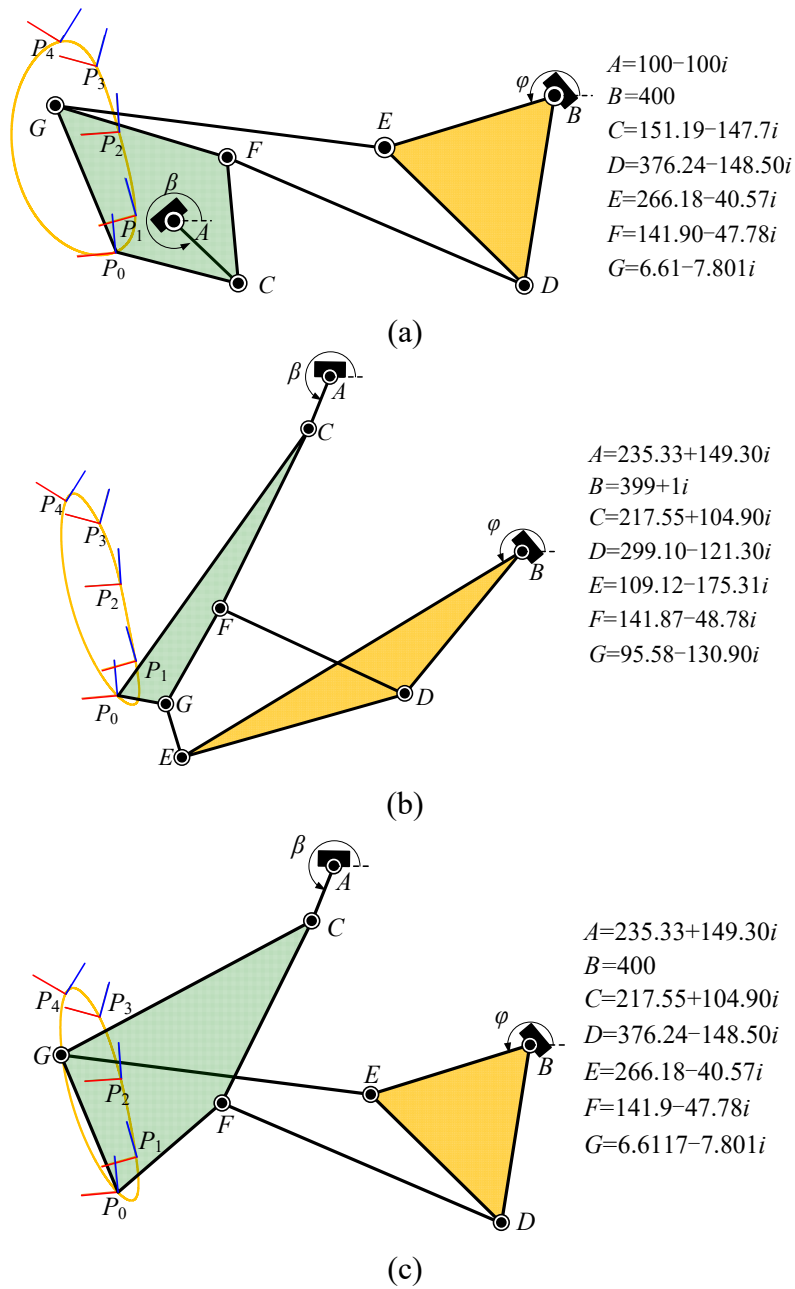


Figure 11. (a) The linkage corresponding to the point M_4 ; (b) The linkage corresponding to the point M_5 ; (c) The linkage corresponding to the point M_6 .

The linkage corresponding to point M_3 is shown in Fig. 9. Based on the continuity analysis method, the angle displacement curve of the position angle φ relative to the angle β of the driving link can be obtained, as shown in Fig. 10.

The six-bar linkage has four different circuits as shown in Fig. 10, and three of them where the linkage has a crank. Since $P_{1\varphi}, P_{2\varphi}, P_{3\varphi}, P_{4\varphi}, P_{5\varphi}$ are located in the same circuit and the linkage has a crank when the linkage moves in this circuit, the six-bar linkage is a defect-free linkage for the given five positions.

Other three defect-free linkages with cranks are displayed in Fig. 11. The results show that five-position synthesis of six-bar linkages can obtain a large number of linkages without defects and with cranks, which can provide a broader choice space for the design.

4. Conclusions

In this paper, the five-position synthesis equations of Stephenson II-B six-bar linkages are established, and the five-position of four-bar linkages can be solved using Eq. (7) alone. A universal and effective defect determinant method for position synthesis of six-bar linkages is proposed, and 24 defect-free linkages with cranks are obtained using this method. These 24 six-bar linkages are obtained under the given values of (B, B^*) and V_1, V_2, V_3, V_4 . However, these variables can be given infinite number of values. Therefore, an infinite number of linkages without motion defects can be obtained for the five-position synthesis of six-bar linkages in theory, which can provide a very broad choice space for the design of linkages. This is five-position synthesis of four-bar linkages cannot be achieved.

References

- [1] Li F, Liu W, Fu X, et al. Jumping like an insect: Design and dynamic optimization of a jumping mini robot based on bio-mimetic inspiration[J]. *Mechatronics*, 2012, 22(2): 167-176.
- [2] Zhang Z, Yang Q, Gui S, et al. Mechanism design for locust-inspired robot with one-DOF leg based on jumping stability[J]. *Mechanism and Machine Theory*, 2019, 133: 584-605.
- [3] Guo G, Zhang J, Gruver W A. Optimal Design of a Six-Bar Linkage with One Degree of Freedom for an Anthropomorphic Three-Jointed Finger Mechanism[J]. *Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine*, 1993, 207(3): 185-190.
- [4] Ceccarelli M, Rodríguez N, Carbone G, et al. An optimal design of driving mechanism in a 1 degree of freedom (dof) anthropomorphic finger[J]. *Applied Bionics and Biomechanics*, 2005, 2(2): 103-110.
- [5] Rodriguez N E N, Carbone G, Ceccarelli M. Optimal design of driving mechanism in a 1-DOF anthropomorphic finger[J]. *Mechanism and machine theory*, 2006, 41(8): 897-911.
- [6] Ceccarelli M, Tavolieri C, Lu Z. Design considerations for underactuated grasp with a one DOF anthropomorphic finger mechanism[C]. *2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2006: 1611-1616.
- [7] Wu L, Carbone G, Ceccarelli M. Designing an underactuated mechanism for a 1 active DOF finger operation[J]. *Mechanism and Machine Theory*, 2009, 44(2): 336-348.
- [8] Zhang F, Hua L, Fu Y, et al. Design and development of a hand exoskeleton for rehabilitation of hand injuries[J]. *Mechanism and Machine Theory*, 2014, 73: 103-116.
- [9] Bataller A, Cabrera J, Clavijo M, et al. Evolutionary synthesis of mechanisms applied to the design of an exoskeleton for finger rehabilitation[J]. *Mechanism and Machine Theory*, 2016, 105: 31-43.
- [10] Haldane D W, Plecnik M M, Yim J K, et al. Robotic vertical jumping agility via series-elastic power modulation[J]. *Science Robotics*, 2016, 1(1): eaag2048.
- [11] Plecnik M M, Haldane D W, Yim J K, et al. Design exploration and kinematic tuning of a power modulating jumping monopod[J]. *Journal of Mechanisms and Robotics*, 2017, 9(1): 011009.
- [12] Park S, Bae J, Jeon Y, et al. Optimal design of toggle-linkage mechanism for clamping applications[J]. *Mechanism and Machine Theory*, 2018, 120: 203-212.
- [13] Jeon Y, Chu K, Kim J, et al. Singularity-inducing compliant toggle linkage mechanism for large clamping range[J]. *Mechanism and Machine Theory*, 2019, 135: 40-53.
- [14] Jeon Y, Oh J, Kim J, et al. Finger clamping unit: A clamping device with a large clamping range[J]. *International Journal of Precision Engineering and Manufacturing*, 2021, 22(2): 313-327.
- [15] Han J, Qian W, Zhao H. Study on synthesis method of λ -formed 4-bar linkages approximating a straight line[J]. *Mechanism and machine theory*, 2009, 44(1): 57-65.
- [16] Han J, Qian W. On the solution of region-based planar four-bar motion generation[J]. *Mechanism and Machine Theory*, 2009, 44(2): 457-465.
- [17] Yang T, Han J, Yin L. A unified synthesis method based on solution regions for four finitely separated and mixed "Point-Order" positions[J]. *Mechanism and Machine Theory*, 2011, 46(11): 1719-1731.

- [18] Yin L, Han J, Mao C, et al. Synthesis method based on solution regions for planar four-bar straight-line linkages[J]. *Journal of mechanical science and technology*, 2012, 26(10): 3159-3167.
- [19] Yin L, Huang L, Huang J, et al. Solution-region-based synthesis approach for selecting optimal four-bar linkages with the Ball–Burmester point[J]. *Mechanical Sciences*, 2019, 10(1): 25-33.
- [20] Han J, Yang T. A novel synthesis method for three-position motion generation with planar four-bar mechanisms[C]. *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2012: 419-426.
- [21] Han J, Yang T. On the solution of region-based planar six-bar motion generation for four finitely separated position[C]. *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2012: 441-449.
- [22] Yang T, Han J. Solution region synthesis method of planar one-DoF six-bar linkages for specifying 4-positions of a 3R open chain[J]. *Journal of Mechanical Science and Technology*, 2016, 30(7): 3069-3077.