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Global Sensitivity Analysis of Random Buckling Characteristics Based on The Kriging Method

Zhen Cheng[1, *], **Qi Cheng**[1]

[1]Yangtze University, Jingzhou 434000, China
[*] Corresponding author

Abstract: As infrastructure construction gradually extends to regions such as plateaus and deserts, the complex and harsh natural environment will increase the risk of structural instability and collapse. The study of its buckling characteristics is of great significance for dynamic optimization design. Due to the influence of material production, construction processes and harsh outdoor environments, there is uncertainty in material properties (such as elastic modulus), which makes traditional deterministic analysis unable to meet actual needs. This paper takes an I-beam section column as an example, considers the randomness of the structural elastic modulus, and combines Latin hypercubic sampling and the Kriging method to establish a surrogate model between the first three order buckling loads and random parameters. Based on the surrogate model, the probability density distribution of the buckling load of the column structure was obtained, and the Sobol 'index was used to analyze the influence degree of the randomness of each parameter on the buckling characteristics of the support. The research results show that the first-order buckling load of this structure is easily affected by the bottom of the column. As the order increases, the parameters with greater influence gradually move towards the top, and the distribution range gradually expands. After considering the randomness of parameters, the critical buckling load of the structure will decrease, which indicates that the buckling load obtained based on deterministic analysis may lead to an underestimation of the actual failure risk of the structure. Therefore, stochastic analysis is very necessary. The method proposed in this paper has the characteristics of high stability and high efficiency in stochastic analysis.

Keywords: Buckling characteristics, Kriging method, Uncertainty, Global sensitivity.

1. Introduction

With the continuous development of the social economy, infrastructure construction has been constantly extending into complex and harsh outdoor environments, such as plateau permafrost areas, coastal highly corrosive zones, and desert areas ravaged by strong winds and sandstorms. A large number of key building structures such as Bridges, transmission towers, and offshore platforms have been established here. However, these outdoor environments are constantly subject to adverse conditions such as drastic temperature fluctuations, erosion by strong corrosive media, and frequent wind load impacts, which will continuously weaken the load-bearing capacity of the structure and easily lead to structural instability and collapse accidents. This not only causes huge economic losses but also may threaten the safety of people's lives. Therefore, conducting stability analysis on building structures in harsh outdoor environments, especially conducting precise research on the buckling characteristics directly related to structural safety, has become an important issue that urgently needs to be addressed in the field of civil engineering[1,2].

In traditional structural buckling analysis, it is usually assumed that parameters such as material properties, geometric dimensions, and load conditions are definite values [3,4]. The buckling loads obtained based on this often have significant deviations from the structural buckling characteristics in actual engineering. Upon delving into the reasons, on the one hand, during the material production process, subtle differences in raw material composition and fluctuations in processing techniques can lead to randomness in mechanical parameters such as the elastic modulus and Poisson's ratio of the material. On the other hand, construction uncertainties such as installation errors and component size deviations during the construction process will further alter the actual stress state of the structure. In addition, the dynamic changes in harsh outdoor environments, such as the degradation of material performance caused by corrosion and the random fluctuations of wind loads, will continuously increase the uncertainty of structural parameters. The coupling effect of these multi-source uncertain factors makes it difficult to accurately predict the actual buckling characteristics of the structure through deterministic analysis methods. Conducting research on the buckling load of the structure from the perspective of uncertainty has become a more practical analysis approach in engineering.

Under the framework of uncertainty analysis, clarifying the influence degree of each random parameter on the buckling load of the structure is a key prerequisite for optimizing the structural design and enhancing the anti-instability capacity of the structure. Global sensitivity analysis, as an effective tool capable of quantifying the impact of global parameter variations on output responses, has a more comprehensive analytical capability compared to local sensitivity analysis, which only reflects the influence of parameters near local points [5-7]. It can not only identify the key parameters that have a significant impact on buckling loads, but also reveal the influence laws of the interaction between parameters on buckling characteristics, providing a scientific basis for targeted control of key parameter fluctuations and reduction of the adverse effects of uncertain factors on structural safety in engineering practice. Among them, the variance-based global sensitivity index (i.e., the Sobol index) can describe the contribution of the variance of a random variable to the output variance. The index has a clear physical meaning and is simple to calculate, thus being widely applied.

Based on the above background, this paper takes an I-beam section column as an example, considers the randomness of the material's elastic modulus, and studies the first three order buckling loads of the column structure through Latin hypercubic sampling and the Kriging method. Finally, a global sensitivity analysis of the structural parameters was conducted in combination with Sobol 'to identify the main influencing parameters of each order of buckling characteristics, providing theoretical support for the stability design and safety assessment of building structures in harsh field environments.

2. Establishment of the Stochastic Buckling Characteristic Equation Based on The Kriging Method

Firstly, the deterministic buckling characteristic equation for column structures is as follows

$$\left(\mathbf{K}-\lambda\mathbf{K}_{\mathrm{G}}\right)\varPhi=0 \tag{1}$$

Here, $\mathbf{K}$ represents the overall stiffness matrix, $\mathbf{K}_{\mathrm{G}}$ represents the geometric stiffness matrix, λ represents the eigenvalue, that is, the buckling load factor, and $\varPhi$ represents the eigenvector, that is, the buckling mode.

When the uncertainty of the elastic modulus is taken into account, the input parameters become random variables or random fields, which can be expressed as follows

$$\left(\mathbf{K}(\delta)-\lambda(\delta)\mathbf{K}_{\mathrm{G}}(\delta)\right)\varPhi(\delta)=0 \tag{2}$$

Here, if δ is a random variable, then $\mathbf{K}(\delta)$ represents the random global stiffness matrix, $\mathbf{K}_{\mathrm{G}}(\delta)$ represents the random geometric stiffness matrix, $\lambda(\delta)$ represents the random buckling load factor, and $\varPhi(\delta)$ represents the random buckling mode.

(1) Introduce the Kriging surrogate model to construct the mapping relationship between random input and random output

$$\lambda(\delta)=f(\delta)^{\mathrm{T}}\beta+\mathbf{z}(\delta) \tag{3}$$

Here, $\lambda(\delta)$ represents the random buckling characteristic value, $f(\delta)^{\mathrm{T}}\beta$ is the deterministic buckling function, $f(\delta)$ represents the basis function vector, β represents the regression coefficient vector, and $\mathbf{z}(\delta)$ represents the stochastic process.

(2) Related functions

$$R\left(\delta^{(i)},\delta^{(j)};\theta\right)=\exp\left(-\sum_{k=1}^{d}\theta_k\left|\delta_k^{(i)}-\delta_k^{(j)}\right|^2\right) \tag{4}$$

Among them, θ is the relevant parameter vector and d is the input dimension.

(3) Optimal linear unbiased prediction

$$\hat{\lambda}(\delta)=f(\delta)^{\mathrm{T}}\hat{\beta}+\mathbf{r}^{\mathrm{T}}(\delta)\mathbf{R}^{-1}\left(\lambda-\mathbf{F}\hat{\beta}\right) \tag{5}$$

Here, $\mathbf{r}$ represents the correlation vector between the prediction points and the sample points, $\mathbf{R}$ is the correlation matrix of the sample points, and $\mathbf{F}$ is the design matrix.

(4) Estimation of regression coefficients

$$\hat{\beta}=\left(\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{F}\right)^{-1}\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\lambda \tag{6}$$

(5) Predictive variance

$$\sigma_{\hat{\lambda}}^2(\delta)=\hat{\sigma}^2\left[1-\mathbf{r}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{r}\right] \tag{7}$$

Here, $\hat{\sigma}^2$ is the estimated value of the process variance.

(6) Hyperparameter estimation

$$\varphi(\boldsymbol{\theta})=-\frac{1}{2}\left[n\ln\left(2\pi\hat{\sigma}^2+\ln|\mathbf{R}|\right)\right] \tag{8}$$

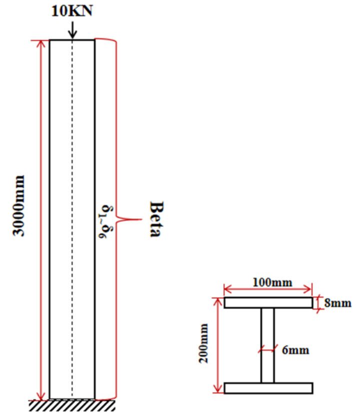
3. Global sensitivity expression of buckling characteristic values

The calculation method of the global sensitivity index S_i corresponding to the buckling characteristic value of the *i*-th random variable δ_i is as follows: First, generate two sets of sample matrices $A_{(G\times n)}$ and $B_{(G\times n)}$ with the number G for n random variables. Replace the *i*-th column of $B_{(G\times n)}$ with the *i*-th column of $A_{(G\times n)}$ to generate the sample matrix $C^i_{(G\times n)}$. Calculate the buckling eigenvalue samples $\lambda^{A}{}_{(1\times G)}$ and $\lambda^{C^i}{}_{(1\times G)}$ corresponding to the $A_{(G\times n)}$ and $C^i_{(G\times n)}$ sample matrices respectively by formula (5). Then, the calculation expression of S_i is as follows

$$S_i=\frac{\frac{1}{G}\sum_{k=1}^{G}\lambda^{A}(\delta)\lambda^{C^i}(\delta)-\left[\frac{1}{G}\sum_{k=1}^{G}\lambda^{A}(\delta)\right]^2}{\frac{1}{G}\sum_{k=1}^{G}\lambda^{A}(\delta)^2-\left[\frac{1}{G}\sum_{k=1}^{G}\lambda^{A}(\delta)\right]^2} \tag{9}$$

4. Numerical Example

This section takes an I-beam section column as an example, as shown in Figure 1. The column height is 3000mm, and there is a vertical downward axial load at the top, with a magnitude of 10kN. Suppose the elastic modulus of the column follows a Beta distribution with a mean of 0 and a mean square deviation of 0.1. The entire column is divided into 30 units, each with a length of 100mm. The randomness of the parameters within every 0.5m range of the column structure is represented by a random distribution, and the six parts are respectively represented by δ_1 to δ_6. If the elastic modulus of the column is expressed as E0(1+ δ), then the elastic modulus variability is 0.1.

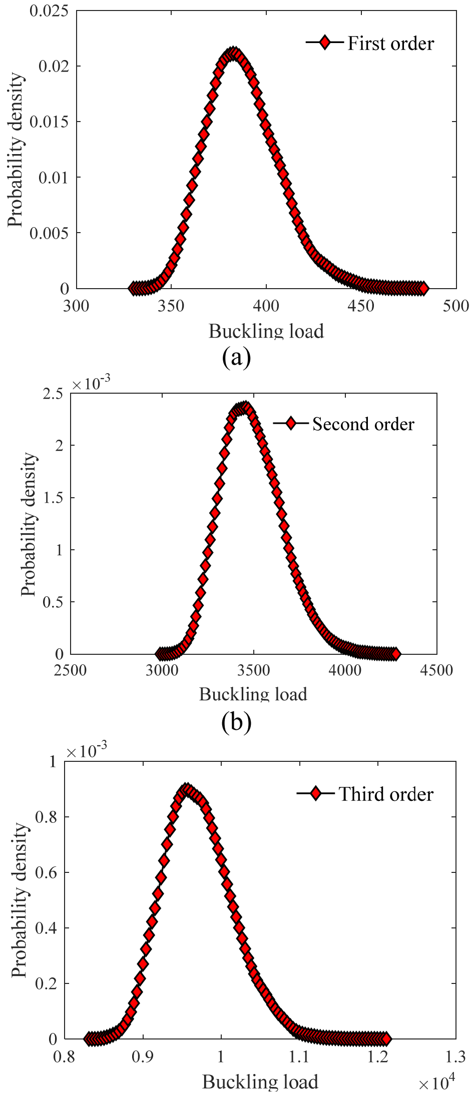


(a) (b)

Figure 1. Column cross-sectional dimension diagram and random variable distribution diagram

(1) Random buckling load analysis

The probability densities of the first three buckling loads of the column structure are shown in Figure 2, with their mean values being 387.7, 3492.1 and 9711.1 respectively. The design values of the first three buckling loads of the deterministic structure are 390.8, 3517.3 and 9770.4 respectively. It can be seen that after considering the randomness of parameters, the critical buckling load of the structure will decrease. This indicates that the buckling load obtained based on deterministic analysis does not take into account the variability of actual parameters. The corresponding safety factor may no longer meet the requirements under random conditions, and the actual failure risk of the structure is underestimated.



(a)

(b)

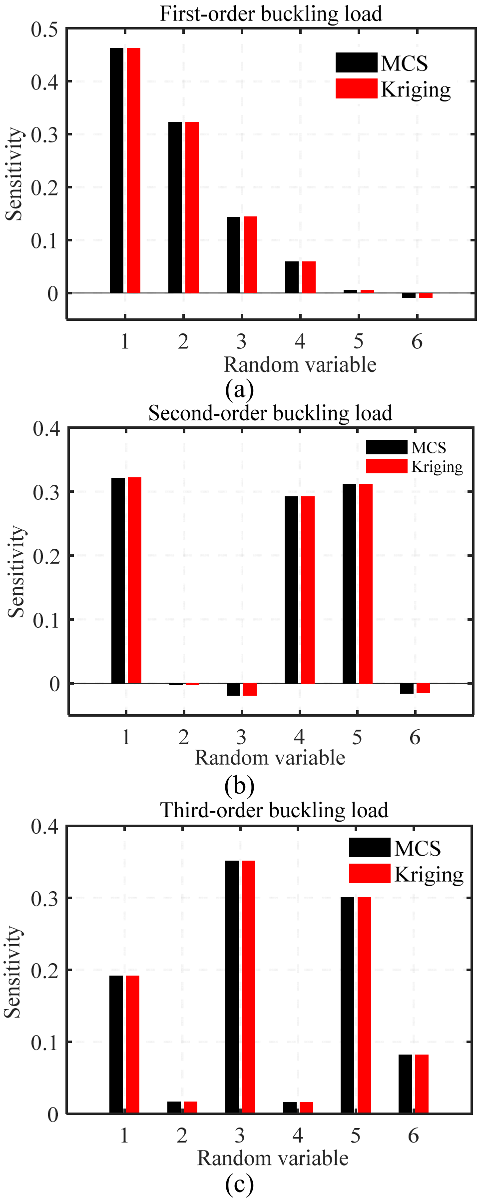
(c)

Figure 2. Probability density plots of the first three orders of buckling loads

It can also be seen from Figure 2 that the probability densities of each order of buckling load all show a unimodal distribution, and the samples at the right tail are more than those at the left tail, which is an asymmetric non-Gaussian distribution type.

(2) Global sensitivity analysis of buckling load

Figure 3 presents the global sensitivity results of the first three orders of buckling loads of the structure calculated by the Kriging method based on the MCS(Monte Carlo Simulation Method) calculation results. The sensitivities of the six random variables are represented as S1 to S6.



(a)

(b)

(c)

Figure 3. Sobol 'index of the first three orders of buckling load

It can be clearly seen that the sensitivity results of the method proposed in this paper are in good agreement with MCS. Meanwhile, the randomness of the first-order buckling load is most significantly affected by δ_1, δ_2 and δ_3. The sensitivity values of the second-order buckling load to δ_1, δ_4 and δ_5 are higher. The third-order buckling load is mainly affected by δ_1, δ_3 and δ_5. In summary, the first-order buckling load of this structure is easily affected by the bottom of the column. As the order increases, the parameters with greater influence gradually move towards the top, and their distribution range gradually expands.

Table 1 presents the time required to solve the global sensitivity of the column structure by two methods. Among them, Kriging established a surrogate model using 600 samples (100 times the variables). It can be seen that the calculation time of MCS is approximately ten times that of the method proposed in this paper. It is indicated that the method proposed in this paper has good computational efficiency while ensuring the calculation accuracy.

Table 1. Calculation schedule for the two methods

Methods	Kriging	MCS
Time (s)	20.8	216.5

5. Conclusions

(1) The sensitivity results show that the method proposed in this paper has good computational accuracy when conducting buckling analysis and global parameter sensitivity analysis on column structures, and has higher computational efficiency compared with MCS.

(2) It can be seen from the probability density plot that traditional deterministic analysis has limitations, while considering the uncertainty of parameters can better reflect the true situation of the structure. It provides theoretical references for subsequent scholars when conducting structural optimization design.

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