

# Estimation of EMD Envelope Based on MQ Interpolation

Liming Liang<sup>1, a</sup>

<sup>1</sup>College of Mathematics and Physics, Geo-mathematics Key Laboratory of Sichuan Province, Chengdu University of Technology, Chengdu, 610059, China

<sup>a</sup>1163937352@qq.com

**Abstract:** Empirical modal decomposition (EMD) is a data-driven signal analysis method that can effectively decompose signals into components of different scales. However, there are still key issues such as endpoint effects, envelope fitting and theoretical foundation improvement that need to be solved. In this paper, we introduce Multi-Quadratic (MQ) fitting interpolation to estimate the envelope in the empirical modal decomposition, which is an efficient, high-precision and conformal fitting method without solving the system of equations.

**Keywords:** EMD, Envelope fitting, MQ.

## 1. Introduction

Since the EMD decomposition method was proposed by Huang et al. in 1998, it has been widely used for nonlinear signal processing in various disciplines. Although EMD has been developed for many years, there are still some problems.

To solve the endpoint effect, Rilling et al. [1] proposed the mirror method, which takes the first extreme point at the ends of the signal as the boundary, maps the signal outward, and obtains an envelope curve covering the original signal by obtaining the mirror image of the original signal; Datig et al.[2] proposed the slope method, which obtained the estimate of the extension point by linear prediction of the two extreme points at the endpoints; Wu et al.[3] improved it based on Datig et al. improved the method based on Datig et al. by finding the slope from the adjacent extreme points of the signal endpoints and using the first extreme point of the endpoints and the slope to determine the linear equation where the extension point is located to determine the location of the extension point (Improved Slope Based Method (ISBM)); Xu et al.[4] proposed the Cubic Spline Based Method (CSBM), which makes use of the fact that the envelope after extension The local extrema at both ends of the signal are related to the extension points to obtain the extension points that satisfy the conditions. Regarding the modal mixing problem, many scholars have proposed improved EMD [5] and different envelope fitting algorithms [6]. In terms of screening conditions, the changes in IMF components, envelopes, or extreme value points before and after iteration [7] are mainly used to determine whether it is an eigenmodal function. In conclusion, EMD needs to be further investigated. To further solve the envelope fitting problem, a variable parameter MQ fitting interpolation algorithm is proposed.

## 2. Three Classical MQ Proposed Interpolation Operators

The MQ function is a radial basis function. Franke compared 29 discrete point interpolation methods in terms of accuracy and stability, and found that the MQ function has the best overall performance in interpolation and is conformal. Although MQ interpolation shows good performance in scattered data interpolation, it still has some shortcomings.

MQ interpolation is essentially a system of linear equations, which is very computationally intensive when the size of the scattered data points is large, and the coefficient matrix is also prone to pathology, resulting in large errors in the calculation results. Since MQ proposed interpolation not only has good approximation accuracy, but also does not need to solve the linear system of equations, so scholars have gradually started to study MQ proposed interpolation.

Definition 2.1 Given a set  $[a, b]$  of nodes on an interval  $a = x_0 < x_1 < \dots < x_n = b$ ,  $h = \max_{1 \leq j \leq n} (x_j - x_{j-1})$ , the MQ proposed interpolation satisfy the form

$$Lf(x) = \sum_{j=0}^n f(x_j) \psi_j(x), a \leq x \leq b$$

Beatson and Powell defined  $L_A f(x)$ ,  $L_B f(x)$  and  $L_D f(x)$  three MQ proposed interpolation operators. The proposed  $L_B f(x)$  interpolation operator is given first,

$$L_B f(x) = f(x_0) \beta_0(x) + \sum_{j=1}^{n-1} f(x_j) \psi_j(x) + f(x_n) \beta_n(x)$$

$$\beta_0(x) = \frac{1}{2} + \frac{\phi_1(x) - \phi_0(x)}{2(x_1 - x_0)}$$

$$\psi_j(x) = \frac{\phi_{j+1}(x) - \phi_j(x)}{2(x_{j+1} - x_j)} - \frac{\phi_j(x) - \phi_{j-1}(x)}{2(x_j - x_{j-1})}$$

$$\beta_n(x) = \frac{1}{2} - \frac{\phi_n(x) - \phi_{n-1}(x)}{2(x_n - x_{n-1})}$$

Definition of the proposed  $L_A f(x)$  interpolation operator

$$L_A f(x) = L_B f(x) + \frac{f(x_0) + f(x_n)}{2} \left( \frac{\phi_n(x) + \phi_0(x)}{x_n - x_0} - 1 \right)$$

$$\alpha_0(x) = \frac{\phi_1(x) - \phi_0(x)}{2(x_1 - x_0)} + \frac{\phi_0(x) + \phi_n(x)}{2(x_n - x_0)}$$

$$\alpha_n(x) = \frac{\phi_0(x) + \phi_n(x)}{2(x_n - x_0)} - \frac{\phi_n(x) - \phi_{n-1}(x)}{2(x_n - x_{n-1})}.$$

Wu et al. proposed a new MQ proposed interpolation operator

$$\begin{aligned} L_D f(x) &= f(x_0)\psi_0(x) + f(x_1)\psi_1(x) + \sum_{j=2}^{n-2} f(x_j)\psi_j(x) \\ &\quad + f(x_{n-1})\psi_{n-1}(x) + f(x_n)\psi_n(x), \\ \psi_0(x) &= \frac{1}{2} + \frac{\phi_1(x) - (x - x_0)}{2(x_1 - x_0)} \\ \psi_1(x) &= \frac{\phi_2(x) - \phi_1(x)}{2(x_2 - x_1)} - \frac{\phi_1(x) - (x - x_0)}{2(x_1 - x_0)} \\ \psi_{n-1}(x) &= \frac{(x_n - x) - \phi_{n-1}(x)}{2(x_n - x_{n-1})} - \frac{\phi_{n-1}(x) - \phi_{n-2}(x)}{2(x_{n-1} - x_{n-2})} \\ \psi_n(x) &= \frac{1}{2} + \frac{\phi_{n-1}(x) - (x_n - x)}{2(x_n - x_{n-1})}. \end{aligned}$$

### 3. Improved MQ Fitted Interpolation Fitting Envelope

Zhang Jihong[8] first introduced MQ proposed interpolation into EMD decomposition for analyzing the similarity of DNA sequences between species, but did not analyze and discuss the effect of EMD decomposition, and chose fixed values of shape parameters. EMD decomposition is an iterative process, and the local extrema will change greatly, so it is obviously undesirable to use fixed values of shape parameters directly. Therefore, this paper proposes a new way of selecting shape parameters to make the envelope in EMD decomposition more adaptable. The improved shape parameter is defined as follows:

$$c = 0.1h^{\frac{1}{3}}, \quad h = \max_{1 \leq j \leq n} (x_j - x_{j-1}).$$

Compared with the traditional interpolation envelope method, approximating the envelope by MQ proposed interpolation has the following advantages:

(1) The function expression is a display expression, which is easy to implement programmatically without solving a linear system of equations and is faster to compute;

(2) The MQ proposed interpolation function has good approximation ability, with convex, monotonic and linear regenerative properties, and can fit a smooth curve that matches the characteristics of the data;

(3) For most of the actual signals, the triple spline interpolation has serious overshoot phenomenon, which is difficult to reflect the local characteristics of the signal and does not have the characteristics of conformality. In this case, MQ interpolation is used to estimate the envelope to avoid the above problems.

### 4. Experimental Results and Performance Analysis

In this section, six different types of signals are designed, and the performance of Cubic Spline Interpolation (CSI), MQ proposed interpolation and the proposed method (Improved MQ, IMQ) are compared and analyzed using the orthogonal exponent IO, mean square error MSE and energy error. The parameter value of MQ proposed interpolation is chosen as

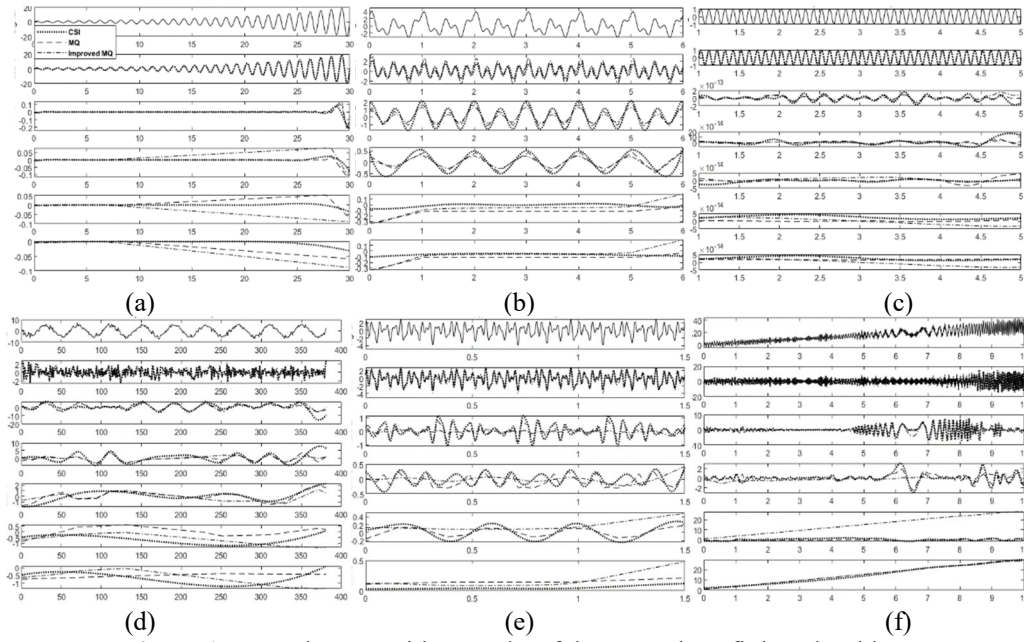
the best primary parameter value for EMD decomposition results. The frequencies of the six signals are 200 Hz, 200 Hz, 100 Hz, 20 Hz, 400 Hz and 100 Hz, and the expressions are defined as follows.

$$\begin{aligned} x_1(t) &= \sin(2\pi t)e^{0.1t}, t \in [0, 30] \\ x_2(t) &= x_2^1(t) + x_2^2(t) + x_2^3(t), t \in [0, 6], \\ x_2^1(t) &= 1.2\sin(10\pi t), \quad x_2^2(t) = 2\cos(4\pi t), \\ x_2^3(t) &= \cos(2\pi t); \\ x_3(t) &= x_3^1(t) + x_3^2(t), t \in [1, 5], \quad x_3^1(t) = \sin(200\pi t), \\ x_3^2(t) &= \sin(20\pi t); \\ x_4(t) &= x_4^1(t) + x_4^2(t), t \in [1, 20], \quad x_4^1(t) = 5\sin(\pi t), \\ x_5(t) &= (1 + 0.2\sin(15\pi t))(\sin(100\pi t) + 1.5\cos(60\pi t) + \sin(30\pi t)), t \in [0, 1.5], \\ x_6(t) &= x_6^1(t) + x_6^2(t) + x_6^3(t) + x_6^4(t), t \in [1, 20], \\ x_6^1(t) &= 2\sin(250\pi t), \quad x_6^2(t) = 3\cos(\pi(1.7t + 7.3)^2), \\ x_6^3(t) &= e^{0.23(1+t)} \cos(\pi(2.58t + 21.95)^2), \quad x_6^4(t) = 3t. \end{aligned}$$

For the first signal, according to Figure 1(a), CSI performs the best, followed by IMQ, and MQ performs the worst. Due to the fixed parameters of MQ, the extracted IMF2, IMF3 and IMF4 components of MQ exhibit no smoothness. On the contrary, the improved IMQ envelope fitting algorithm improves most of the IMF components, and the EMD decomposition results are more satisfactory compared to MQ. From the qualitative point of view, we can also know that IMQ has better quadrature index, mean square error and energy error than MQ. However, it is slightly weaker than CSI because the signal is a damped sinusoidal signal with no local violent oscillations and a clear pattern of variation of local extrema, so the ideal envelope can be obtained directly by the third spline interpolation. In contrast, IMQ is not an interpolation method in essence, but only an approximate envelope can be obtained.

The second signal is composed of high, medium and low frequency components, which also has a clear pattern, and the change of local extreme points is relatively flat. Based on the EMD decomposition results and the three metrics, we know that CSI performs best, IMQ outperforms MQ, and MQ performs the worst.

For the third signal, we consider a signal consisting of high and low frequencies, which is dominated by the low frequency component. As shown in Figure 1(c), the overall performance of the EMD decomposition of the three algorithms is satisfactory, and the low-frequency components can be extracted effectively, and the IMF extracted by IMQ can be smoothed faster than CSI and MQ. The three metrics show that the performance of IMQ is better than the other two methods. It is worth mentioning that the signal appears to be a smooth and regular time series from the subjective view, but in fact, the signal amplitude fluctuates slightly due to the influence of high frequency components, which leads to severe overshoot in the third spline interpolation and increases the envelope error. In contrast, IMQ is a method of envelope estimation, which can mitigate the effects of small changes in amplitude and is strictly conformal, and the obtained envelope is more advantageous. Therefore, IMQ performs better than CSI in the signal.



**Figure 1.** EMD decomposition results of three envelope fitting algorithms

**Table 1.** Comparison of IO results of three different extension methods

IO	CSI	MQ	IMQ
$x_1(t)$			
$x_2(t)$	0.0011	0.0017	0.0015
$x_3(t)$	0.1375	0.2023	0.1945
$x_4(t)$	2.5964e-15	1.7829e-15	1.2746e-15
$x_5(t)$	0.6055	0.2573	0.2070
$x_6(t)$	0.0693	0.1075	0.0332
	0.0201	0.0205	0.0091

**Table 2.** Comparison of MSE results of three different extension methods

MSE	CSI	MQ	IMQ
$MSE(IMF_1, x_1(t))$			
$MSE(IMF_1, x_2^1(t))$	0.0016	0.0024	0.0020
$MSE(IMF_1, x_3^1(t))$	0.0790	0.4602	0.8458
$MSE(IMF_2, x_4^1(t))$	2.5563e-26	2.3401e-26	2.2477e-26
$MSE(IMF_2, x_5^1(t))$	17.1415	8.2238	9.4408
$MSE(IMF_1, x_5(t))$	0.2393	0.1583	0.0575
$MSE(IMF_1, x_6^3(t))$	12.6679	10.4190	7.6611
$MSE(IMF_2, x_6^2(t))$	10.1776	7.9031	5.7826

**Table 3.** Comparison of  $\theta$  results of three different extension methods

$\theta$	CSI	MQ	IMQ
$x_1(t)$			
$x_2(t)$	0.0011	0.0017	0.0014
$x_3(t)$	0.1388	0.2109	0.2047
$x_4(t)$	1.5721e-16	9.4323e-16	9.4323e-16
$x_5(t)$	0.4176	0.0481	0.0138
$x_6(t)$	0.0211	0.0748	0.0109
	0.0139	2.1581e-04	0.0073

The signals, and are non-stationary and have no obvious variation pattern. Qualitative analysis: In Fig. 1 (d), IMF2 and

IMF3 extracted by CSI-based algorithm have obvious modal blending, and IMF extracted by MQ-based algorithm has poor

smoothing. From Fig. 1(e), it can be found that all three methods cannot decompose the original signal into IMF components of different scales, and the extracted IMF1 all basically overlap with the original signal. For the signal, the IMF3 based on CSI and MQ screening has no clear physical meaning, and the number of IMQ screening IMF iterations is less compared to the remaining two methods. From the quantitative point of view, the overall performance of IMQ is better than CSI and MQ, and only slightly higher in energy than MQ in Table 1.

Based on the analysis of the above six signals, it can be concluded that IMQ outperforms CSI and MQ for non-stationary and irregular signals; IMQ still outperforms MQ and is slightly weaker than CSI for signals with significant variation patterns. Most of the actual signals contain noise, so IMQ is more advantageous than the other two methods.

## 5. Conclusion

Regarding the envelope fitting problem, an improved MQ fitted interpolation approximation envelope algorithm is proposed. This method defines a new way of parameter setting, and IMQ has a stronger adaptive capability than the constant-value parameter proposed by J.H. Zhang et al. (2012). The experimental results show that the overall performance of the proposed method is better than the other two methods for non-smooth and irregular real signals.

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