

Pure Azimuth Passive Positioning Model for UAV Formations

Mingqin Yao^{1, a}, Chonghan Wang², Yufei Zhang³, Congyin Hu⁴

¹College of Materials Science and Engineering, Shandong Jianzhu University, Jinan, 250101, China

²College of Mechanical Engineering and Automation, Northeastern University, Shenyang, 110819, China

³College of Information, Mechanical and Electrical Engineering, Shanghai Normal University, Shanghai, 201418, China

⁴College of Electronics and Information, Science and Technology College of Nanchang Hangkong, Jiujiang, 332020, China

^a625052555@qq.com

Abstract: In recent years, with the rapid development of the UAV industry, the traditional positioning equipment has been unable to carry out effective control processing. This paper mainly studies the pure azimuth passive positioning in the attempted formation flight of UAV. On the basis of in-depth analysis of the signals received by the passive receiving signal UAV, assuming that the deviation of the UAV position at the receiving point is under the controllable error range, analyzing the relationship between the known number and the size of the circular angle, obtaining the passive receiving point positioning model, and checking the results by the least squares method, the results show that the model sought in this paper. The results show that the model is of good accuracy. This paper is of great importance for the position positioning and adjustment of UAV cluster flight by pure azimuthal passive positioning method.

Keywords: UAV flight, Localization model, Least squares method.

1. Introduction

In recent years, with the rapid development of the UAV industry, the "low, slow and small" targets represented by them have become an increasing security threat in the air, and traditional positioning equipment has been unable to effectively control the processing. Nowadays, the passive positioning system has the advantages of good signal concealment, long action distance, and high survivability compared with traditional active positioning, and has wide applications in both civil and defense fields [1]. Therefore, it is of great research value to explore the position positioning and adjustment of UAV cluster flight using pure azimuth passive positioning.

This paper considers the geometric problem in the plane, the signal transmitting point of the drone one is located in the center of the circle, the other two are located in two of the nine equal points of the circumference of the circle, due to the fixed position of these three points, each of which two combined with their position and the angle of the line with the signal receiving point can determine the signal receiving point is located in the two points for the line of the chord of the circle, and then select the other two points the same way can determine another circle, the intersection of the two circles, the Thus, the location of the signal receiving point can be uniquely determined, and the specific calculation can be done with the help of the plane right angle coordinate system, and the specific geometric relationship can be drawn in detail with the help of the circle system square and the intersection of curves to find the intersection point.

2. Acquisition of Data and Assumptions

The data used in this paper are all obtained from question B of the National University Student Mathematical Modeling Competition of Gaoxia Cup 2022. In order to facilitate the

analysis of the problem, the following assumptions are made on the data used in this paper: (1) it is assumed that the impact of a slight deviation of the UAV position on the received signal is within 4°; (2) it is assumed that the impact of external conditions on the received signal is zero; (3) it is assumed that the relative position relationship between the passive receiving UAV and other UAVs in the formation does not change with time; (4) it is assumed that there is no delay in the passive positioning technology.

3. Positioning Model of Passive Receiving Signal UAV

3.1. Model analysis

In order to maintain electromagnetic silence as much as possible and reduce the signal emission to the outside world, the pure azimuth passive positioning method can be used to determine and adjust the position of the UAVs when the UAV cluster is attempting to fly in formation. It is known that each UAV in the formation has a fixed number, and passive positioning extracts directional information for positioning by transmitting signals from several UAVs and passively receiving signals from the remaining UAVs, and then adjusts the position of the UAVs [2].

Figure 1 below shows the known angles of α_1 , α_2 , and α_3 received by the UAV numbered FY04 when the signals are transmitted by the UAVs numbered FY01, FY02, and FY03.

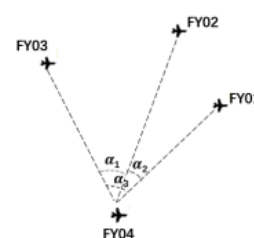


Figure 1. Schematic diagram of the directional information received by the drone

The positioning model of the passive receiving signal UAV is established, and the passive receiving signal UAV position relationship is established based on the signals emitted by three UAVs with known numbers and no deviation in position (one of which is located at the center of the circle). In the case of slight deviation in the position of the receiving UAV, it can be assumed that its acquisition angle error is within 4 degrees, and according to the received signal two two angle multiplier relationship to determine which is the UAV transmitting signal at the center of the circle. According to the condition that 10 UAVs form a circular formation, a geometric relationship of 40 degrees can be derived for the angle of the center of the circle of adjacent UAVs on the circle, and by analyzing the angle of the signal at the remaining launch points, a relationship of 20 degrees k ($k=1,2,3,4$) times its angle can be derived from the geometric law[3]. According to this relationship, the size of the circle angle of the two emission points on the circumference can be found. Then, by the number of the two transmitting points and the size of their circumcentric angles, the specific number of the UAV receiving the signal is derived by using the geometric law, so as to establish the positioning model.

3.2. Establishment of the model

According to the figure below, let the signals received by the UAV receiving signals are sign1, sign2, sign3, and the two angles α_1 , α_2 , α_3 can be obtained.

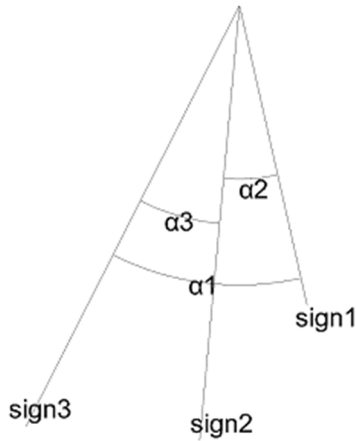


Figure 2. UAV received signal pinch angle diagram

Take the receiving point as the coordinate origin and establish a plane right angle coordinate system. Let $360^\circ > \text{sign1} > \text{sign2} > \text{sign3} \geq 0$, it is known that.

$$\begin{cases} \alpha_1 = \text{sign1} - \text{sign3} \\ \alpha_2 = \text{sign1} - \text{sign2} \\ \alpha_3 = \text{sign2} - \text{sign3} \end{cases} \quad (1)$$

$$\alpha_1 = \alpha_2 + \alpha_3 \quad (2)$$

First, assuming that all points from FY01 to FY09 are on the circle and uniformly distributed, randomly connect three points on the circle with the center of the circle, here FY01 and FY02 as the transmitting points, and FY03 as the passive receiving point as an example, we can obtain 2 and 3, as in Figure 3.

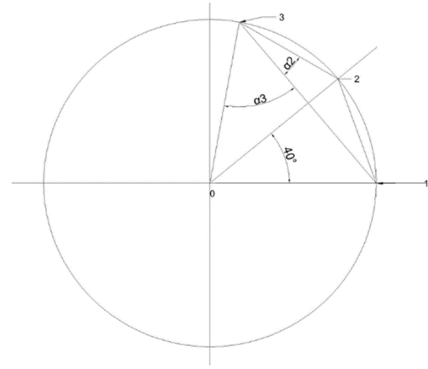


Figure 3. Angular relationship between FY01 and FY02 as transmitting points and α_3 as passive signal receiving points

Figure 3 takes FY01 and FY02 as the transmitting points and FY03 as the passive receiving points, and is solved according to the geometric relationship, get $\alpha_2 = 20^\circ$, $\alpha_3 = 50^\circ$.

Taking α_4 as the passive signal receiving point, FY01, FY02 and FY01, FY03 are considered as the transmitting points for calculation[4].

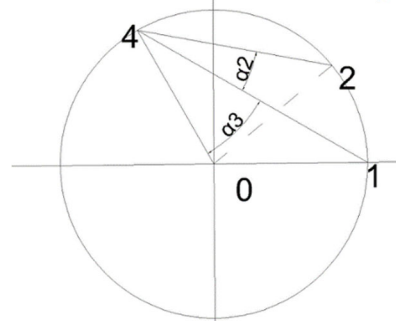
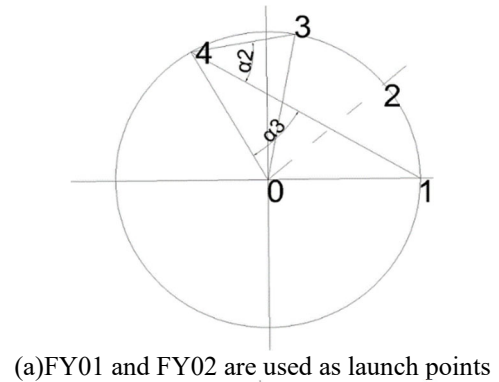


Figure 4. α_4 as the passive signal reception point angle relationship

According to the geometric relationship, solving for respectively $\alpha_2 = 20^\circ$, $\alpha_3 = 30^\circ$ of Figure 4(a) and $\alpha_2 = 40^\circ$, $\alpha_3 = 30^\circ$ of Figure 4(b). By analogy, the law can be obtained: if any of the signals composing the angle α comes from FY00, then we have

$$\alpha = \sum_{k=1}^{i=2} \cdot 10 \quad (k \leq 7) \quad (3)$$

If the signals that make up angle α come from two points on the circle, then we have

$$\alpha = \sum_{k=1}^{i=1} \cdot 20 \quad (k \leq 4) \quad (4)$$

If the signals of FY00 form two pinch angles α and β , then we have

$$\alpha = \sum_{k=1}^{i=2} \cdot 10 \quad (k \leq 7) \quad (5)$$

$$\beta = \sum_{k=1}^{i=2} \cdot 10 \quad (k \leq 7) \quad (6)$$

$$\alpha + \beta = \sum_{k=1}^{i=1} 20 \quad (k \leq 4) \quad (7)$$

The receiving point is on the symmetry axis when and only when $\alpha = \beta$. Furthermore, for a pair of transmitting point groups (a, b) ($a < b$) that possess the axis of symmetry, there are always multiple transmitting point groups ($a + i, b - i$) ($a + 2 \times i < b$) that possess the same diagonal if we set $a + b = c$ [5].

4. Model Solving and Results

4.1. Algorithm steps

Step1: Input the emission source number and the angle of the three signals obtained from the three passive receiving drones relative to the coordinate system with the receiving point drone as the origin.

Step2: Based on the input angles, calculate the angles α_1 , α_2 and α_3 , and determine whether the angles α_2 and α_3 are the same.

Step3: When the angles α_2 and α_3 are not equal, if one of them satisfies Equation (4), it can be determined which beam is the circular emission signal.

Step4: The angle satisfying the Equation (4) is divided by 20° to find the specific value of k, which is used to launch the specific location of the emission point on the circumference of the two circles[6].

Step5: According to the Equation (3), determine the specific number of the passive receiving UAV.

Step6: When α_2 and α_3 are approximately equal (the error of the two angles are within 4°), then the two launch points must be axisymmetric about the position of the passive receiving drone, after which the law of axisymmetry can be applied to find the number of the passive receiving drone.

4.2. Calculated results

By accurately calculating the signal received when the UAV's position is accurate, the application manually adds noise to the data source to simulate the angle information received by the UAV's sensors in real-world situations.

Table 1. Received signal drone number

| A transmitting source number | A transmitting source number | A passive received signal angle | A passive received signal angle | A passive received signal angle | The number of the UAV receiving the signal in the case of an angular error of 4° |
|------------------------------|------------------------------|---------------------------------|---------------------------------|---------------------------------|---|
| 1 | 4 | 302 | 248 | 182 | 3 |
| 1 | 4 | 142 | 221 | 169 | 2 |
| 1 | 2 | 70 | 91 | 142 | 9 |
| 1 | 3 | 71 | 48 | 22 | 8 |

5. Test of the Model

Figure 5 shows the angular relationship representation of the information of three UAVs with known numbers.

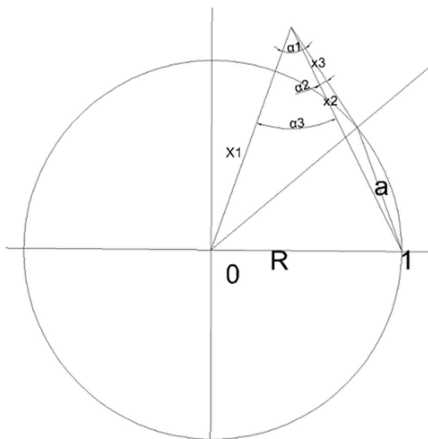


Figure 5. Known number of three UAV information angle relationship representation

For the three UAVs with known numbering information and exact positions, this paper assumes that the radius R of the circular formation and the two angles α_2 and α_3 set above, and establishes the corresponding equations for the edge-angle relationship.

$$a = \sqrt{2R^2 - 2R \cos(k \cdot \frac{40}{180} \Pi)} \quad (8)$$

$$\frac{x1^2 + \alpha x3^2 - R^2}{2 \cdot x1 \cdot x3} = \cos(\alpha_1) \quad (9)$$

$$\frac{x1^2 + x3^2 - a^2}{2 \cdot x1 \cdot x3} = \cos(\alpha_2) \quad (10)$$

$$\frac{x1^2 + x2^2 - R^2}{2 \cdot x1 \cdot x2} = \cos(\alpha_3) \quad (11)$$

When $\alpha_2=20^\circ$, $\alpha_3=30^\circ$ and $\alpha_1=50^\circ$, assuming that the transmitting signal drones are numbered 1 and 3, then according to the least squares method [7] using MATLAB the solution gives $x_1=0.653$, $x_2=1.879$, $x_3=1.286$ and $k=8$.

This value is consistent with the results obtained in this paper, but the model adopted for validation is not effective in determining the acceptance point for the solution obtained by the algorithm at a circular angle of 120° between the two emission points[8]. Therefore, after verification, the model in this paper is more accurate and reliable.

6. Conclusion

This paper focuses on pure azimuth passive positioning in attempted UAV formation flight. For a circular formation of 10 UAVs, the angular relationship between two different UAVs is established. Under the assumption that the deviation of the UAV position at the receiving point is within the controllable error, the angle multiplier relationship between the received signals can be analyzed to determine which is the signal emitted at the center of the circle[9]. And compare the relationship between the angle formed by the rest of the emitted signals and the multiplier of 20 can establish the size of the emitted point on the circumference of the circle, and finally analyze the relationship between the known number and the size of the aforementioned center of the circle to obtain a passive receiving point positioning model, and the results were tested by the least squares method, and the results showed that the model sought in this paper has good accuracy.

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