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Overview of Oil-water Two-phase Flow Pressure Drop Theory

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Abstract: This is a review of the existing pressure drop theories for dispersed and annular flow in oil and water phases in straight wells is presented. The flow classification criteria, pressure drop calculation methods are outlined. For the dispersed flow, the homogeneous flow model and the two-fluid model are introduced. The key to the pressure drop of the homogeneous flow is the calculation of the frictional resistance, and the two-fluid model focuses on the drag force between the two phases. And for the core-annular flow, the two-fluid model is used to focus on the interference between the two-phase interface.

Keywords: Oil-water two-phase flow, Pressure drop calculation, Homogeneous flow model, Two-fluid model.

1. Introduction

The research work of oil-water two-phase flow law originated from the study of drag reduction transport of thick oil in the petroleum industry in the early twentieth century. in 1904, Isaacs and speed proposed a patent for water ring transport of thick oil. in 1950, Clark and Shapiro patented the injection of water containing emulsion breakers into thick oil.Charles et al. and Russell et al. found that adding water to oil could reduce the pressure drop.

In 1961, G.W. GOVIER conducted oil-water two-phase experiments in a vertical pipe. He concluded that although the slip velocity equation proposed by Hlore and Wilde was written for a gas-liquid system, a similar equation was applicable to two systems of unmixed liquids, and he regarded liquids as high-density bodies in his experiments. Oil-water two-phase experiments were conducted using three different viscosities of oil to investigate the effect of oil phase viscosity on water holding rate, flow regime and pressure drop, and it was found that the viscosity of the mixture had little effect on the holding rate.

In 1966, Charles and Lilleleht applied the Locldlan-Maninelli formula for predicting the pressure drop in gas-liquid two-phase flow to the prediction of the pressure drop in oil-water two-phase flow, and found that the prediction was not satisfactory. In 1980, Theissing attributed this deviation to the relatively small density difference between oil and water, and he modified the Maninelli equation to derive an empirical equation suitable for the pressure drop of oil-water and gas-liquid two-phase flows. In 1994, Sapelberg and Mewes used the Locldlan-Maninelli method to predict the frictional pressure drop for oil-water two-phase flows with different pipe diameters. They found that the prediction results for small pipe diameters were similar to those of Charles 1966, while the method could not be used to predict the frictional pressure drop for various oil-water two-phase flow types for large pipe diameters.

In 1998, Neima Brauner [1] proposed to use the homogeneous flow model to calculate the pressure drop of oil-water mixtures, and also summarized the equations of Zuber-Findlay (1965) and Harmathy (1960), et al. to propose a complete method for calculating oil-water two-phase flow, which divides the flow regime according to the holding rate and calculates the pressure drop of stratified and dispersed flow, and the method was widely accepted.

In 2001 Antonio C. Bannwart [2] et al. proposed a phenomenological model to predict the pressure drop during oil-water annular flow through a vertical pipeline, which considered the effect of turbulence in the annular fluid and wavy interface, and also considered the effect of buoyancy on the vertical system proposed a method for calculating the pressure drop in oil-water two-phase core-annular flow.

In 2007 Arun K. Jana [3] et al. conducted oil-water two-phase experiments using kerosene and classified the flow patterns into four types: dispersed bubble flow, bubble flow, churning flow, and core-annular flow. And a suitable theoretical model was used to analyze the liquid holding rate and pressure drop of each flow pattern. The analysis shows that the homogeneous model is applicable to the dispersed bubbly flow, while the drift flux model can better predict the bubbly and churning-turbulent flow patterns. On the other hand, the separation flow model (N. Brauner, 1991) was used to accurately predict the pressure drop in the core circulation. However, this experiment only considered the effect of flow velocity and did not take into account the effect of antiphase and the effect of temperature on fluid viscosity.

In 2018 Mohammad J. Hamidia et al. chose water and kerosene with a viscosity of 1.49 mPa-s and a density of 780 kg/m^3 as unmixed-phase fluids and used high-speed photography to identify the flow pattern. The pressure drop and heat transfer coefficient (HTC) of oil-water two-phase flow in a horizontal tube were experimentally measured to establish a new correlation to predict the HTCs of STMI flow pattern, D w/o flow pattern and o/w flow pattern.

In 2021, Yang Hengqi et al. conducted a thick oil-water at high temperature and pressure experiment to derive and modify a two-phase pressure drop calculation model for thick oil-water in straight wells using Einstein's suspended viscosity equation.

From the research, it can be seen that the calculation factors affecting the pressure drop in the well are as follows:

1. the division of fluid flow patterns in the tubing;
2. calculation of mixture viscosity;
3. calculation of the fluid holding rate.

2. Oil-water Two-phase Veryical Flow Pattern Classification

The determination of the flow pattern is extremely important during the two-phase flow analysis. This is mainly because all design parameters (e.g., pressure drop, fluid holding rate, heat and mass transfer coefficients, retention time distribution, and chemical reaction rates) are strongly dependent on the flow pattern under consideration. For a given two-phase flow system, the flow pattern is mainly determined by a combination of wellbore characteristics (wall roughness, wellbore ID and wellbore inclination), media characteristics (liquid density, liquid viscosity and interfacial surface tension) and operating conditions (liquid flow rate, system pressure "p", system temperature T).

2.1. Dispersion Flow Flow Pattern Judgment

In 1997, Flores et al. conducted 121 sets of oil-water two-phase flow experiments in a vertical tube with an inner diameter of 50.8 mm using a conductivity probe, and defined two categories and six types of oil-water two-phase flow patterns, among which the flow patterns with water as the continuous phase include Dispersion o/w, Very Fine Dispersion o/w), transitional flow type of water-in-oil (o/w Churn); the flow type of oil in continuous phase includes transitional flow type of oil-in-water (w/o Churn), dispersion of water-in-oil (Dispersion w/o) and very fine dispersion of water-in-oil (Very Fine Dispersion w/o).

When the water is continuous phase, most of the discrete oil droplets are diffusely distributed in the water, which shows the dispersion of water-in-oil droplet flow; when the flow velocity of water phase is high (more than 0.9 m/s), the size and distribution state of oil droplets become very regular, and the slip phenomenon of two phases can be ignored, which is the fine dispersion of water-in-oil droplet flow; as the oil volume increases, the oil content increases, and the concentration of oil droplets increases to make them coalesce Merge phenomenon to form a large block of oil, at this time for the water-in-oil transition flow type. When the oil content increases again, the oil phase dominates, i.e. oil is the continuous phase and water is the dispersed phase, and the transitional flow pattern of water-in-oil transforms into the transitional flow pattern of oil-in-water, and gradually transitions into the dispersed flow of water-in-oil until the fine dispersed flow of water-in-oil with the further increase of the flow rate of oil phase. It is generally accepted by the researchers.

Harsan and Kabir et al. used the volume fraction of the dispersed phase $\epsilon_o \geq 0.25$ to determine the transition from small spherical bubbles to large oil bubbles and segment plugs earlier, and Lihua. Huang et al. used the ratio δ of the particle size of Sauter's average droplet to the pipe to classify the flow pattern:

Table 1. Reference range for determining the flow pattern for δ

Flow pattern	Dispersion Flow (DF)	Bubble flow (BF)	Stretch flow (SF)
Range of δ	>0.27	0.27~0.50	0.50~1.00

The maximum droplet size d_{max} is used to estimate the Sauter mean diameter of the droplet:

$$d_{32} = \sum n_i d_i^3 / \sum n_i d_i^2 \simeq d_{\max} / k_d \quad (1)$$

Azzopardi and Hewitt et al (1997). suggested a range of values for K_d from 1.5 to 3.

2.1.1. Calculation of the maximum droplet size

(1) Laminar flow

For the case where the viscosity of the viscous continuous phase is much larger than that of the dispersed phase, Taylor (1934) and Arivos (1978) proposed that elongated droplet rupture in axisymmetric strain motion can be used to estimate droplet size, and the maximum droplet size can be calculated using the following equation, which considers the maximum droplet size to be related to the surface tension of the continuous phase liquid:

$$\frac{d_{\max}}{D} = 0.296\frac{\sigma}{\mu_c \dot{\gamma} D}\left(\frac{\mu_c}{\mu_d}\right)^{1/6} = 0.074\frac{\sigma}{\mu_c U_m}\left(\frac{\mu_c}{\mu_d}\right)^{1/6}; \frac{\mu_d}{\mu_c} \ll 1 \quad (2)$$

$\dot{\gamma}$ is the average shear rate of the fluid $\dot{\gamma} = 4U_m / D$, μ is the viscosity of each phase of the fluid, and the subscripts c, d refer to the continuous and dispersed phases, respectively.

(2) Turbulence

In 1955 Kolmogorov and Hinze proposed that the size of the droplet depends on thecritical Weber number:

$$We_{\text{crit}} = \frac{\tau d_{\max}}{\sigma} = C[1 + F(On)] \quad (3)$$

For a dilute dispersion flow, taking into account the wall friction coefficient of the tube wall, Kubie and Gardner 1977 proposed that:

$$\left(d_{\max}\right)_o = \left(\frac{d_{\max}}{D}\right)_o = 0.55 We_c^{-0.6} f^{-0.4} \quad ; \quad \ell_k \ll d_{\max} < 0.1D \quad (4)$$

£k is the Kolmogorov microscale and 0.1D represents the inertial subscale (the length scale containing the energy of the vortex). N. Branner presented the limitations of the Kubie et al. model, supplemented the model for dense dispersions, and proposed the H-model for calculating the maximum droplet diameter as follows:

$$\left(\frac{d_{\max}}{D}\right)_o = 1.88 We_c^{-0.6} Re_c^{0.08} \quad (5)$$

$$\left(\tilde{d}_{\max}\right)_\epsilon = 7.61 \tilde{C}_H We_c^{-0.6} Re_c^{0.08} \left(\frac{\epsilon_d}{1-\epsilon_d}\right)^{0.6} \left[1 + \frac{\rho_d}{\rho_c}\frac{\epsilon_d}{1-\epsilon_d}\right]^{-0.4} \quad (6)$$

$$\tilde{d}_{\max} = Max\left\{\left(\tilde{d}_{\max}\right)_o \left(\tilde{d}_{\max}\right)_\epsilon\right\} \quad (7)$$

$\tilde{C}_H$ is an adjustable constant, $\tilde{C}_H = O(1)$, the model is applicable to the case of $1.82 Re_c^{-0.7} < \tilde{d}_{\max} < 0.1$ or Re_c>2100, For $\tilde{d}_{\max} > 0.1$, N. Branner proposed the K model:

$$\left(\tilde{d}_{\max}\right)_0 = 30We_c^{-1}Re_c^{0.2};\tilde{d}_{\max} > 0.1 \quad (8)$$

$$\left(\tilde{d}_{\max}\right)_\epsilon = 174C_K We_c^{-1}Re_c^{0.2}\left(\frac{\epsilon_d}{1-\epsilon_d}\right) \quad (9)$$

$$\tilde{d}_{\max} = Max\left\{\left(\tilde{d}_{\max}\right)_o\left(\tilde{d}_{\max}\right)_\epsilon\right\} \quad (10)$$

According to Hinze (1955), if the dispersed phase viscosity is large, then the effect of the dispersed phase viscosity μ_d on the maximum particle size d_{max} must be considered, and the effect of the dispersed phase viscosity is expressed by the Ohnesorge number, and the On number is increased $[1+F(On)]^{0.6}$ at the right end of both the k model and the H model when the On number is not negligible

If the viscosity of the dispersed phase is much greater than that of the continuous phase, consider the viscosity of the dispersed phase (o/w) Paul and Sleicher suggest using:

$$\frac{\rho_c U_c^2 d_{\max}}{\sigma}\left(\frac{\mu_c U_c}{\sigma}\right)^{0.5} = C\left[1+0.7\left(\frac{\mu_d U_c}{\sigma}\right)^{0.7}\right] \quad (11)$$

Where C=38~43

2.1.2. Core-annular flow

In 1996 Joseph et al. proposed the use of the effective viscosity method to determine the presence of a central annular flow:

$$\mu_{1,\text{ eff}} > \mu_{2,\text{ eff}} \qquad \varepsilon > 0.5 \quad (12)$$

$\mu_{ef}$is the effective viscosity of the fluid, ε is the volume number of the core, and the subscripts "1" and "2" are the core and annular phases

If the case of perfect core-annular flow is considered (two-phase fluid is assumed to be free of any interference) the following equation can be used:

$$\mu_1 > \mu_2 + 0.0005\rho_2 J_2 D \quad \text{for} \quad \frac{\rho_2 J_2 D}{\mu_2} > 2000 \quad (13)$$

ρ_2 is the density of the annular fluid and J_2 is the apparent flow velocity of the fluid.

In 2001 Bannwart applied the Eötvos number to supplement the criterion for the appearance of annular flow based on Branner's idea:

$$\frac{\pi\Delta\rho g D^2\varepsilon}{4\sigma} < 8$$

3. Pressure drop theory

3.1. Homogeneous flow model

Homogeneous flow models usually treat the two-phase mixture as a homogeneous single-phase fluid, and the frictional pressure drop is a key element in the accuracy of the total pressure drop calculation.

For a fully expanded dispersive flow in a vertical pipe and neglecting acceleration gradients, the total pressure gradient including gravitational and frictional pressure gradients can be calculated as (Brauner, 1998):

$$\left(\frac{dp}{dz}\right)_{tp} = \left(\frac{dp}{dz}\right)_g \pm \left(\frac{dp}{dz}\right)_f \quad (14)$$

where the "±" symbols correspond to vertical upward or downward flow, respectively.

The gravitational pressure gradient is:

$$\left(\frac{dp}{dz}\right)_g = \rho_M g \quad (15)$$

Frictional pressure gradients are:

$$\left(\frac{dp}{dz}\right)_f = 2f_{tp}\rho_M\frac{U_M^2}{D} \quad (16)$$

where f_{tp} is the friction coefficient, ρ_M is the density of the mixture, U_M is the flow rate of the mixture, D is the diameter of the pipe

The density of the mixture is determined by the weighted average of the volume fractions:

$$\rho_M = \varepsilon_o\rho_o + \varepsilon_w\rho_w \quad (17)$$

ε is the volume fraction, and the subscripts o and w represent the oil and water phases, respectively

The friction coefficient f_{tp} is calculated according to the Blasius equation:

$$f_{tp} = C\cdot\text{Re}_m^{-n} \quad (18)$$

For turbulent flow C=0.079, n=0.25 and laminar flow C=16, n=1.

If the effective viscosity of the mixture is known, the Reynolds number of the mixture Re_m is:

$$\text{Re}_m = \frac{\rho_M D U_M}{\mu_e} \quad (19)$$

In 1998 Flores et al. proposed a method to calculate the two-phase friction coefficient by considering the difference between the water-dominated and oil-dominated flows. The constants C and n in the above equation will be determined from experimental data by redefining Re_m:

$$\mathrm{Re}_m = \frac{\rho_M U_M D}{\mu_c} \tag{20}$$

where the subscripts "M" and "c" refer to the mixture and the continuous phase.

3.1.1. Effective viscosity model

Most current calculations of oil-water flow use the Dukler volume factor weighting method to calculate viscosity:

$$\mu_M = \mu_o\beta_o + \mu_w\beta_w$$

Oil-water two-phase flow differs from gas-liquid two-phase flow in that the viscosity of the dispersed phase in oil-water two-phase flow plays a large role in transmitting shear, and the deformation of the droplet near the wall (by shear) thus affects the flow of the droplet in the center of the pipe. Some models have been proposed such as Brinkman (1952) and Roscoe (1952) model:

$$\frac{\mu_e}{\mu_c} = \left(1-\varepsilon_d\right)^{-2.5} \tag{21}$$

Pal model (2001):

$$\left(\frac{\mu_e}{\mu_c}\right)\left[\frac{2\left(\frac{\mu_e}{\mu_c}\right)+5\left(\frac{\mu_d}{\mu_c}\right)}{2+5\left(\frac{\mu_d}{\mu_c}\right)}\right]^{1.5} = \frac{9}{8}\left[\frac{\left(K\cdot\varepsilon_d\right)^{\frac{1}{3}}}{1-\left(K\cdot\varepsilon_d\right)^{\frac{1}{3}}}\right] \tag{22}$$

3.1.2. Calculation of the holding rate

Due to the different densities of the two-phase fluids, the accuracy of the calculation of the holding rate directly affects the calculation of the pressure drop. The two mainstream methods for calculating the holding rate are the homogeneous model and the drift flow model.

1. Homogeneous model (ignoring the slip between the two liquid phases):

The in-situ holding rate is determined by the input volume flow rate of the two liquids

$$\epsilon_d = \frac{U_{ds}}{U_m} \quad ; \quad \epsilon_c = 1-\epsilon_d = \frac{U_{cs}}{U_m}; U_m = U_{ds} + U_{cs} \tag{23}$$

where U is the input volume flow rate and the subscripts d, c, ds, cs, and m refer to the volume fraction of the dispersed phase, the volume fraction of the continuous phase, the volume flow rate of the dispersed phase, the volume flow rate of the continuous phase, and the total flow rate of the mixture, respectively.

2. Drift flow model Zuber-Findlay (1965):

$$\frac{U_{\mathrm{sw}}}{Y_{\mathrm{w}}} = C_{\mathrm{w}}U_{\mathrm{m}} + U_{\mathrm{e}} \tag{24}$$

$$U_{\mathrm{e}} = U_\infty\left(1-Y_{\mathrm{w}}\right)^N \tag{25}$$

The drift flow model mainly considers the slip phenomenon due to the density difference between the two phases of the liquid.

Where: U_{sw} is the apparent velocity of water phase; C_w and N are the phase distribution parameters and particle size index, respectively; U_∞ is the limiting rise velocity of water droplets in oil continuum medium. C_o is a distribution parameter which explains the droplet velocity and concentration distribution. For a uniform droplet concentration, $C_o = 1$, $C_o > 1$ when the droplet tends to flow in the center, and $C_o < 1$ when the droplet concentration is higher near the wall. the value of N depends mainly on the droplet size. hassan and Kabir (1990) and and Flores et al. (1997) suggest that for large droplets (of the order of tube diameter), $N \approx 0$, while for liquid-liquid dispersions, N = 1.5 ~ 2.5.

Harmathy (1960) proposed a method to calculate the limiting rise velocity of droplets:

$$u_\infty = 1.53\left[\frac{g\sigma\,|\Delta\rho|}{\rho_c^2}\right]^{1/4} \tag{26}$$

where σ is the surface tension of the fluid. $\Delta\rho$ is the density difference between the two phases, and ρ_c is the density of the continuous phase. g is the acceleration of gravity.

Drift flow models applicable to specific flow types have been proposed as follows:

1.Nicolas model (suitable for dispersion flow):

$$\frac{U_{\mathrm{so}}}{1-Y_{\mathrm{w}}} = U_{\mathrm{m}} + U_\infty Y_{\mathrm{w}}^{N+1}, \quad N = 0.5 \tag{27}$$

2.Flores model (suitable for disturbance flow):

$$\begin{cases} \frac{U_{\mathrm{s0}}}{1-Y_{\mathrm{w}}} = 1.038U_{\mathrm{m}} + 0.142Y_{\mathrm{w}}^{2.5}\left(\mathrm{D\ o/w}\right) \\ \frac{U_{\mathrm{so}}}{1-Y_{\mathrm{w}}} = 1.1045U_{\mathrm{m}} + 0.142Y_{\mathrm{w}}^{1.5}\left(\mathrm{CF\ o/w}\right) \end{cases} \tag{28}$$

3.1.3. Two-fluid model

The two-fluid model was originally proposed by Taitel and Dukler for the stratified flow pattern, based on the assumption of separate flow of the two phases and the presence of shear stress at the phase interface; the two-fluid model for o/w dispersed flow was proposed by Poesio et al. based on the two-fluid model for stratified flow:

$$-A_o\left(\frac{dp}{dz}\right) - AF_{D,o/w} - \rho_o A_0 g\sin\theta = 0 \tag{29}$$

$$-A_W\left(\frac{dp}{dz}\right) - \tau_W S_W + AF_{D,o/w} - \rho_W A_W g\sin\theta = 0 \tag{30}$$

Where A and S_W denote the cross-sectional area of the pipe and the water wall wetting perimeter, respectively, τw is the wall shear stress of water, and θ is the inclination angle of the pipe.

The above two equations are combined to eliminate the pressure drop, so that the o/w dispersion flow model can be introduced:

$$\alpha\left(\rho_w-\rho_0\right)A_w g\sin\theta+\tau_W S_w\alpha-F_{D,o/w}A=0 \quad (31)$$

Similarly a two-fluid model for w/o dispersive flow can be introduced:

$$-A_W\left(\frac{dp}{dz}\right)-AF_{D,w/o}-\rho_W A_W g\sin\theta=0 \quad (32)$$

$$-A_O\left(\frac{dp}{dz}\right)-\tau_O S_O+AF_{D,w/o}-\rho_O A_O g\sin\theta=0 \quad (33)$$

Eliminate the pressure drop to obtain the w/o dispersion flow model:

$$(1-\alpha)\left(\rho_0-\rho_W\right)A_0 g\sin\theta+\tau_0 S_0(1-\alpha)-F_{D,w/0}A=0 \quad (34)$$

Assuming that the dispersed phase is not in contact with the tube wall:

$$S_W=S_0=\pi D \quad (35)$$

Assuming that F_D is the drag force of the phase of the continuous relatively spherical dispersed phase:

$$F_D=\frac{3}{4}\alpha\rho_c C_D\frac{\left(U_d-U_c\right)\left|U_d-U_c\right|}{d_d} \quad (36)$$

where C_D is the drag coefficient, U_d is the velocity of the dispersed phase, U_c is the velocity of the continuous phase, and dd is the effective diameter of the dispersed phase droplet. The drag coefficients are:

$$C_D=\begin{cases}\frac{24}{Re}\left(1+0.15Re^{0.687}\right) & Re\leqslant 800\\ 0.44 & \mathrm{Re}>800\end{cases} \quad (37)$$

where the droplet Reynolds number:

$$Re=\frac{\rho_c\left|U_d-U_c\right|d_d}{\mu_m} \quad (38)$$

Mixture viscosity using Einstein's suspension viscosity formula:

$$\mu_m=\mu_{\mathrm{c}}\left(1+2.5H_{\mathrm{d}}\right) \quad (39)$$

3.2. Core-annular air flow pressure drop calculation method

At present, for the calculation of annular air pressure drop, the most used is the two-fluid model. This model takes into account the interface interference between the phase-phase and phase-tube wall, and the error of pressure drop pre is relatively small.

3.2.1. Perfect Core-annual Flow

Taitel and Dukler first proposed a perfect core-annular flow two-fluid model for interfacial smoothing, and the pressure drop calculation model was obtained after eliminating the two-phase pressure drop phase:

$$\Gamma_{\mathrm{f}}=\frac{128\mu_2 Q}{\pi D^4\left[1-\varepsilon^2(1-m)\right]}-\frac{\left(\rho_2-\rho_1\right)g\varepsilon(1-\varepsilon)[1-\varepsilon(1-m)]}{\left[1-\varepsilon^2(1-m)\right]}\cong\frac{128\mu_2 Q}{\pi D^4\left(1-\varepsilon^2\right)}-\frac{\left(\rho_2-\rho_1\right)g\varepsilon(1-\varepsilon)}{(1+\varepsilon)} \quad (40)$$

where μ, ρ, Q,g are viscosity, density, flow rate, and gravitational acceleration, respectively. ε is the volume number of the core, and the subscripts "1" and "2" are the core and annular phases. $m=\mu_2/\mu_1$

ε is calculated by the following equation:

$$J_1(1-\varepsilon)-s_{\mathrm{i},0}J_2\varepsilon+V_{\mathrm{ref}}\varepsilon^{\frac{9-5n_i}{4-2n_{\mathrm{i}}}}(1-\varepsilon)^{\frac{n_{\mathrm{i}}}{2-n_{\mathrm{i}}}}\left(\frac{3-\varepsilon}{2}+\frac{\ln\varepsilon}{1-\varepsilon}\right)^{\frac{n_i}{2-n_i}}=0 \quad (41)$$

其中

$$V_{\mathrm{ref}}=\frac{g_{\mathrm{z}}\left(\rho_1-\rho_2\right)}{\left|g_{\mathrm{z}}\left(\rho_1-\rho_2\right)\right|}a_{\mathrm{i}}^{\frac{1}{\mathrm{i}^{-}-2}}\sqrt{\left|g_{\mathrm{z}}\right|D}\times\left(\frac{|\Delta\rho|}{\rho_2}\right)^{\frac{1}{2-n_{\mathrm{i}}}}\left(\frac{\rho_2\sqrt{\left|g_{\mathrm{z}}\right|D}D}{\mu_2}\right)^{\frac{n_{\mathrm{i}}}{2-n_{\mathrm{i}}}} \quad (42)$$

g_{z} is the axial component of gravitational acceleration, ai and ni take the values of:

$$\begin{cases}\left(\mathrm{Re}_k<2000\right): & a_k=16 \quad n_k=1\\ \left(Re_k\geq 2000\right): & a_k=0.079 \quad n_k=0.25\end{cases} \quad (43)$$

3.2.2. Considering wave characteristics and circulating air turbulence effects

2001 Bannwart predicts the pressure drop during oil-water annular flow through a vertical pipe by means of a phenomenological model that takes into account the effect of turbulence in the annular fluid and wavy interface, and also the effect of buoyancy on the vertical system, where the pressure drop gradient can be expressed as:

$$\Gamma_{\mathrm{f}}=b\left(\frac{\rho_{\mathrm{m}}JD}{\mu_{\mathrm{m}}}\right)^{-n}\frac{\rho_{\mathrm{m}}J^2}{2D}-C\left(\rho_2-\rho_1\right)g\varepsilon(1-\varepsilon) \quad (44)$$

The core volume fraction ε is calculated by the following equation:

$$J_1(1-\varepsilon)-s_{\mathrm{i},0}J_2\varepsilon-cV_{\mathrm{ref}}\varepsilon^2(1-\varepsilon)^m=0 \quad (45)$$

where c, m are adjustable parameters that can be adjusted experimentally to suit the wave velocity data, and Bai (1995) determined that the optimal data:

$$s_{i,o} = 1.5 \quad c = 0.02 \quad m = 2 \tag{46}$$

The constants b, n, c are the parameters to be adjusted in the experiment, n=0.25 for turbulent flow, n=16 for laminar flow (turbulent and laminar flow in smooth pipe walls), Bannwart recommends taking n=0.25, b=0.257, C=0.159

Core-annulal flow mixture density:

$$\rho_m = \varepsilon\rho_1 + (1-\varepsilon)\rho_2 \tag{47}$$

Core-annular flow mixture viscosity:

$$\frac{1}{\mu_m} = \frac{\varepsilon}{\mu_1} + \frac{1-\delta}{\mu_2} \cong \frac{1-\delta}{\mu_2} \tag{48}$$

The two-fluid model and the uniform flow model are based on different assumptions, from the assumptions, the uniform flow model is suitable for oil-water two-phase flow where one phase is continuous, which is applicable to the dispersed flow type, while the two-fluid model is suitable for the case where there is a clear interface between the two phases and both are in a continuous state. They compared the pressure drop data predicted by the uniform flow model and the two-fluid model in an inclined pipe and found that the uniform flow model could predict the inverse phase point while the two-fluid model could not, but the two-fluid model was more accurate in terms of the accuracy of the pressure drop prediction.

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